International Journal of Engineering Studies.
ISSN 0975-6469 Volume 8, Number 2 (2016), pp. 147-158

http://www.ripublication.com

The Numerical Solution of Chua-Nonlinear Tumor Model by Three Different Transform Methods

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Abstract

In this paper consider the Chua's nonlinear system of Ordinary differential Equation model as representation of tumor mathematical model and the model solution obtained by Differential Transform Method, Elzaki Transform method and Fractional order Multistage Homotophy perturbation method.The numerical results of the nonlineartumormodel are presentedand it is analyzed by graphically with the numerical solutions.The graphical analysis of the results is perfectly in agreement with the established basic principles of Tumor microenvironment.

1. Introduction:

Tumor growth is basically a hybrid dynamical system [7]. Even same types of tumors vary from patient to patient not only in size but also in temporal behavior. The complexity in the growth of a tumor is further aggravated by its multi-directional proliferation. The dynamical nature of a tumor growth can be mathematically modeled and represented in the form of system of Ordinary Differential Equation or Partial Differential Equation or as an exponential function.

The prime objective of this paper is to study typical Non-linear chaos theory Ordinary Differential Equation model namely Chua [5]. We represent these nonlinear Ordinary Differential Equation model as Tumor Mathematical model and analyzed the solutions of these models to obtain concrete inferences. The above system of non-linear Ordinary Differential Equation model are solved by employing by Differential Transform Method,Elzaki Transform method and Fractional order Multistage Homotophy perturbation method.

The system variables, *x, y, z* of the above said non-linear Ordinary Differential Equation model represents Host Cell, Effector Immune Cell and Tumor Cell respectively [10].We have analyzed these systems and graphically compared the numerical solutions.

Basically there are two stages of tumor growth namely Benin and Malignant. The first one corresponds to the initial stage of tumor and the later corresponds to spiteful stage of the tumor. In Benin stage the rate of growth of the cells in the tumor is slow and in the case of Malignant stage the growth is rapid and unpredictable. In this paper we study the nonlinear ordinary differential equation tumor model ascorresponds to Benin stage.

1.1. Introduction to Chua's System

We consider the following tumor growth Mathematical Model obtained from chaotic attractor as System of Nonlinear Ordinary Differential Equations. Chua's is a Mathematical term that describes complex behavior in deterministic dynamical systems, which has short-term predictability but is nevertheless unstable and unpredictable in the long term. Extensive studies on chaos in the 1980s clarified that chaos is ubiquitous not only in Mathematical modelsbut also in real-world systems. Any dynamical systems can be modeled by any of the following systems described [5].

A linear systemis a mathematical model of a system based on the use of linear operator. A non-linear system is a system which is not linear, that is a system which does not satisfy the superposition principle or whose output is not directly proportional to its input. The third effective system is a Hybrid system, which is a dynamic system that exhibits both continuous and discrete dynamic behaviorfor a system that can be described by a differential equation:

$$\frac{dx}{dt} = \alpha\big(y - x - f(x)\big) \qquad (1)$$

$$\frac{dy}{dt} = x - y + z \qquad (2)$$

$$\frac{dz}{dt} = -\beta y \qquad (3)$$

Where*x*, *y* and *z* are continuous state variables, 'α' and 'β' are positive parameters and f(*x*) is a piecewise smooth function of *x*.

Consider$\alpha = 10, \beta = 16.82. h(x) = -0.55x(|x+1| - |x-1|)$.

1.2. Introduction to DTM (Differential Transform method)

The Differential Transform Method is one of the approximate methods which can be easily applied to many linear and nonlinear problems and is capable of reducing the size of computational work. Exact solutions can also be achieved by the known forms of the series solutions. The concept of the differential transformation method has been introduced to solve linear and nonlinear initial value problems in electric circuit analysis The differential transformation method is a semi numerical analytic technique that formalizes the Taylor series in a totally different manner.

With this method, the given differential equation and related initial conditions are transformed into a recurrence equation that finally leads to the solution of a system of algebraic equations as coefficients of a power series solution. This method is useful for obtaining exact and approximate solutions of linear and nonlinear differential equations. There is no need for linearization or perturbations, large computational work and round-off errors are avoided [11, 14].

1.3. Introduction to ETM (Elzaki Transform Method)

The Elzaki transform method is an efficient method for solving the linear and non-linear system of ordinary differential equations. Elzaki transform is a useful technique for solving linear differential equations, however, Elzaki transform is totally incapable of handling nonlinear equations because of the difficulties that are caused by nonlinear terms. In this paper we use differential transform method to decompose the nonlinear term, so that the solution can be obtained by iteration procedure [13].

1.4. Introduction to FMPHM(Fractional Order Multistage Homotophy Perturbation method)

Perturbation theory comprises mathematical methods for finding an approximate solution to a problem, by starting from the exact solution of a related problem. A critical feature of the technique is a middle step that breaks the problem into "solvable" and "perturbation" parts. Perturbation theory is applicable if the problem at hand cannot be solved exactly, but can be formulated by adding a "small" term to the mathematical description of the exactly solvable problem.

The Homotophy Perturbation Method (HPM) is a universal one which can be applied to various kinds of linear and nonlinear equations. It usually needs only a few iterations to lead to the active approximate analytical solutions for a given system. The approximate solutions generally, as shall be shown in the numerical experiments, not valid for large t. The HPM treated as an algorithm in a sequence of intervals for finding accurate approximate solution to the system. The modified HPM, i.e. the Multi Stage Homotophy Perturbation Method, can give the valid solutions for a long time [2].

2.1. Solution of Tumor Model by Differential Transform Method

Apply the Differential transform method in the equations (1), (2) and (3), we get the following

$$\boldsymbol{X(K+1)=\frac{1}{k+1}\sigma[Y(K)-0.02X(K)]} \tag{4}$$

$$\boldsymbol{Y(K+1)=\frac{1}{k+1}X(K)-Y(K)+Z(K)} \tag{5}$$

$$\boldsymbol{Z(K+1) = -\frac{1}{k+1}\beta Y(K)} \qquad (\boldsymbol{6})$$

The solution of differential transform method described as power series is given bellow:

$$x(t) = \sum_{k=0}^{n} X(k)t^k \, ; y(t) = \sum_{k=0}^{n} Y(k)t^k ; z(t) = \sum_{k=0}^{n} Z(k)t^k .$$

For the initial values $X(0) = 0.01, Y(0) = 0.01$ & $Z(0) = 0.01$, in equations (4), (5) & (6) by using differential transform method, the values obtained for x, y and z are given in the following table

Table 1: Iteration values of x, y, z by DTM

Iteration	x	y	z
1	0.01	0.01	0.01
2	0.049	0.005	-0.0841
3	0.0201	-0.0201	-0.0421
4	-0.1025	-0.0010	0.1690
5	0.0053	0.0338	0.0084

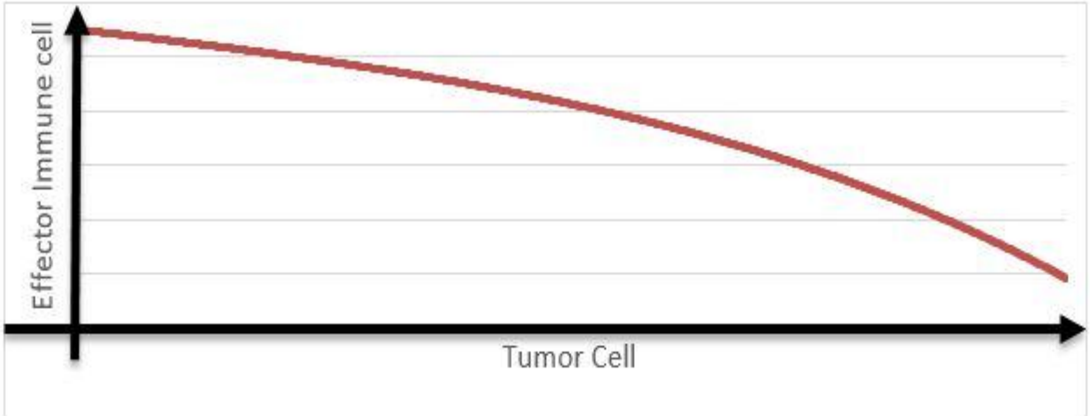
The system variables, *x, y, z* in the tumor model asnonlinear ordinary differential equation system represents host cell, effector immune cell and tumor cell respectively. For the initial values $X(0) = 0.01, Y(0) = 0.01$ & $Z(0) = 0.01$, we obtain the solution of the Chua's model by using Differential Transform Method as a function of t in the form of power series and for each value of 't' in a period of time interval (0.0001, 0.5) the results of *x(t), y(t), z(t).*By taking values for t∈(0.0001,0.5) and using equations(4),(5) and (6), the sample values obtained out of nearly 5000 values are tabulated for reference.

Table 2. Sample solutions of *x(t),y(t)* and *z(t) by DTM*

S. No	*x(t)*	*y(t)*	z(t)
1	0.01	0.01	0.00999159
2	0.0105	0.01005	0.00914647
3	0.01099	0.01009	0.008293954
4	0.01149	0.01013	0.007435056
5	0.01199	0.01017	0.006570783
6	0.01249	0.0102	0.005702135
7	0.013	0.01023	0.004830102
8	0.0135	0.01025	0.003955658
9	0.014	0.01027	0.00307976
10	0.0145	0.01029	0.002203345

2.2. Graphical representation of Solution Tumor Model by DTM

The results of *x(t)*, *y(t)*, *z(t)* are plotted pairwise. The basic tumor microenvironment principles and graphical representation are as follows: **“If the Host cell is growing and the Effector immune cell is declining then the Tumor cell is growing”.**



Firure 1: Comaprision of Tumor Cell and Effector Immune Cell

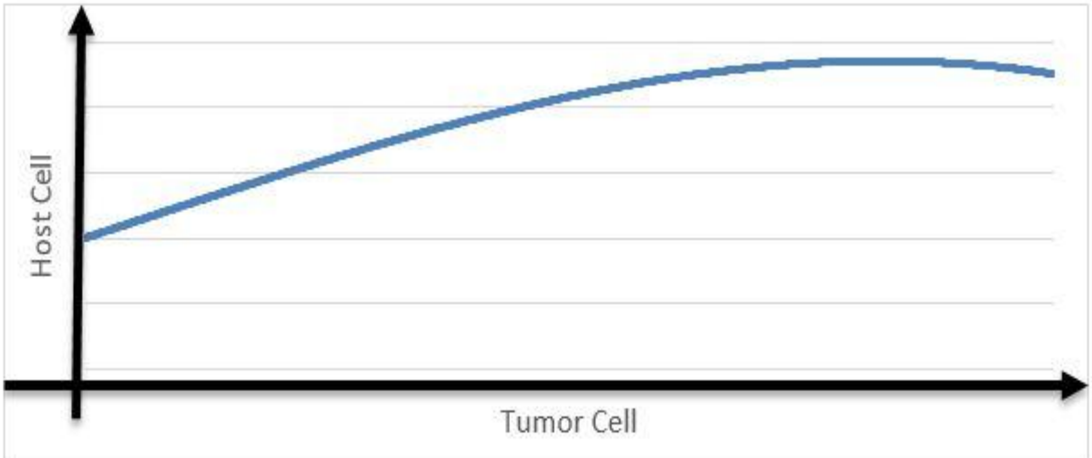


Figure 2: Comparison of Tumor Cell and Host Cell

3.1.Solution of Tumor Model by Elzaki Transform Method

Solve the nonlinear tumor model by Elzaki Transform Method, we get the recursive relations are

$$\boldsymbol{X(n+1) = \alpha E^{-1}[uE(y(n) - x(n)) - f(x)]} \quad \textbf{(7)}$$

$$\boldsymbol{Y(n+1) = E^{-1}[uE(x(n) - y(n) + z(n))]} \quad \textbf{(8)}$$

$$\boldsymbol{Z(n+1) = -\beta E^{-1}[uE(y(n))]} \quad \textbf{(9)}$$

The power series obtained is given bellow

$$x(t) = \sum_{k=o}^{n} X(k)t^k\, ; y(t) = \sum_{k=o}^{n} Y(k)t^k ; z(t) = \sum_{k=o}^{n} Z(k)t^k$$

For the initial values $X(0) = 0.01, Y(0) = 0.01$ & $Z(0) = 0.01$, in nonlinear tumor model system by using Elzaki transform method, the values obtained for x , y and z are given in the following table

Table 3: Iteration values of x, y, z

s.no	X	Y	Z
1	0.01	0.01	0.01
2	0.098	0.01	-0.1682
3	0.0402	-0.0401	-0.0841
4	-0.13635	-0.00013	0.22483
5	0.00047	0.02215	0.00055

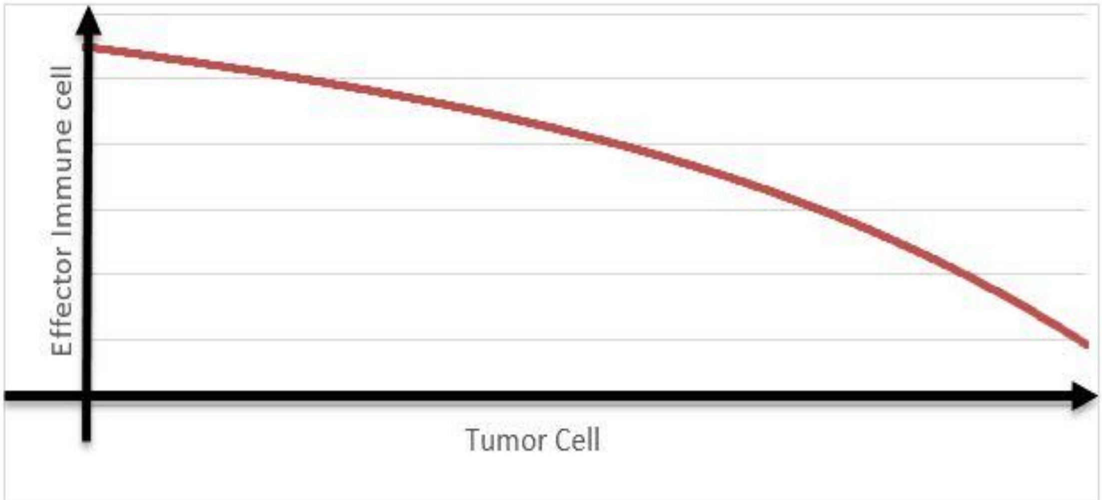
The system variables *x, y, z* in the tumor model as nonlinear ordinary differential equation system represents host cell, effector immune cell and tumor cell respectively. For the initial values $X(0) = 0.01, Y(0) = 0.01$ & $Z(0) = 0.01$, we obtain the solution of the system by using Elzaki Transform Method as a function of t in the form of power series and for each value of t in a period of time interval (0.0001, 0.5) the sample results of $x(t), y(t), z(t)$.By taking values for t∈(0.0001,0.5) and using equations(7),(8) and (9), the sample values obtained out of nearly 5000 values are tabulated for reference.

Table 4. Sample solutions of *x(t),y(t)* and *z(t) by ETM*

S.no	*x(t)*	*y(t)*	*z(t)*
1	0.01009804	0.01000996	0.00983172
2	0.01098406	0.01009599	0.00830981
3	0.01197571	0.01018395	0.00660416
4	0.01297412	0.01026389	0.00488438
5	0.01397848	0.01033581	0.00315182
6	0.01498797	0.01039972	0.00140783
7	0.01600179	0.01045564	-0.00034625
8	0.01701912	0.01050359	-0.00210909
9	0.01803915	0.01054359	-0.00387936
10	0.01906109	0.01057567	-0.00565573

3.2. Graphical Representation of solution for Tumor Model by Elzaki

we obtain the solution of the nonlinear tumor model by using Elzaki Transform Method as a function of t in the form of power series and for each value of 't' in a period of time interval (0.0001, 0.5) the results of $x(t), y(t), z(t)$ are plotted pairwise. The basic tumor microenvironment principles and graphical representation are as follows:**"If the Host cell is growing and the Effector immune cell is declining then the Tumor cell is growing".**



Firure 3: Comaprision of Tumor Cell and Effector Immune Cell

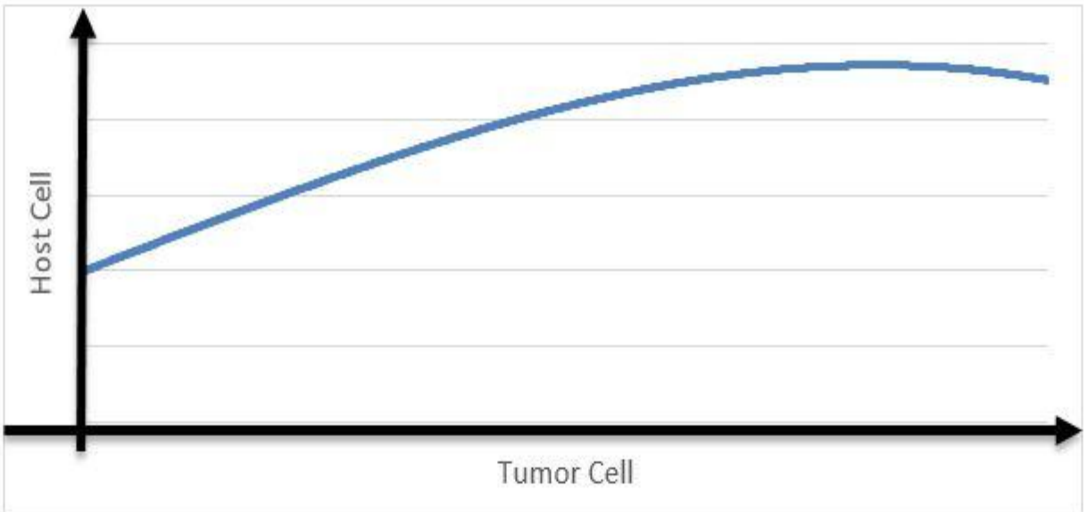


Figure 4: Comparison of Tumor Cell andHost Cell

4.1. Solution of Tumor Model by Multistage Fractional Order Homotophy perturbation method

The fractional order nonlinear tumor model described as

$$ {}_0\boldsymbol{D}_t^{\alpha_1}\boldsymbol{x}(\boldsymbol{t}) = \boldsymbol{\alpha}(\boldsymbol{y}-\boldsymbol{x}-\boldsymbol{a}\boldsymbol{x}) \quad \textbf{(10)} $$

$$ {}_0\boldsymbol{D}_t^{\alpha_1}\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{x}-\boldsymbol{y}+\boldsymbol{z} \quad \textbf{(11)} $$

$$ {}_0\boldsymbol{D}_t^{\alpha_1}\boldsymbol{z}(\boldsymbol{t}) == -\boldsymbol{\beta}\boldsymbol{y} \quad \textbf{(12)} $$

Subject to the initial conditions $x(0) = c_1, y(0) = c_2, z(0) = c_3$
Where $0 <\propto_i< 1, i = 1,2,3, x, y, z$ are state positive parameters.

$$ {}_0D_t^{\alpha_1}x(t) = P\big(\alpha(y-x-ax)\big) $$
$$ {}_0D_t^{\alpha_1}y(t) = P(x-y+z) $$
$$ {}_0D_t^{\alpha_1}z(t) == P(-\beta y) $$

Where $P \in [0,1]$ is an embedding parameter.

$$ P^0:\ {}_0D_t^{\alpha_1}x_0 = 0 $$
$$ P^1:\ {}_0D_t^{\alpha_1}x_1 = \propto y_0 - \propto x_0 - \propto ax_0 $$
$$ P^2:\ {}_0D_t^{\alpha_1}x_2 = \propto y_1 - \propto x_1 - \propto ax_1 $$
$$ P^3:\ {}_0D_t^{\alpha_1}x_3 = \propto y_2 - \propto x_2 - \propto ax_2 $$

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$$ P^0:\ {}_0D_t^{\alpha_2}y_0 = 0 $$
$$ P^1:\ {}_0D_t^{\alpha_2}y_1 = x_0 - y_0 + z_0 $$
$$ P^2:\ {}_0D_t^{\alpha_2}y_2 = x_1 - y_1 + z_1 $$
$$ P^3:\ {}_0D_t^{\alpha_2}y_3 = x_2 - y_2 + z_2 $$

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$$ P^0:\ {}_0D_t^{\alpha_3}z_0 = 0 $$
$$ P^1:\ {}_0D_t^{\alpha_3}z_1 = -\beta y_0 $$
$$ P^2:\ {}_0D_t^{\alpha_3}z_2 = -\beta y_1 $$
$$ P^3:\ {}_0D_t^{\alpha_3}z_3 = -\beta y_2 $$

The initial Condition $x_0 = x_0(0) = c_1 = 0.01$

$$ y_0 = y_0(0) = c_2 = 0.01 $$
$$ z_0 = z_0(0) = c_3 = 0.01, \propto = 10, \beta = 16.82 $$

By using the initial condition and the solution of nonlinear tumor model by Fractional order multistage Homotophy perturbationmethod, the approximate series solutions of system are expressed given bellow

$x(t) = 0.01 + -0.001008\, t^{0.98} + 0.06229\, t^{1.96} - 0.049974\, t^{2.94} - 0.068258687 t^{3.92} - 0.00585625 t^{4.90} + \cdots$

$y(t) = 0.01 + 0.010083 t^{0.98} - 0.092956\, t^{1.96} - 0.07850\, t^{2.94} - 0.052480503\, t^{3.92} + 0.008277324583\, t^{4.90} + \cdots$

$z(t) = 0.01 - 0.169626\, t^{0.98} - 0.08725\, t^{1.96} - 1.14588\, t^{2.94} + 0.8540245\, t^{3.92} + 0.00871754\, t^{4.90} + \cdots$

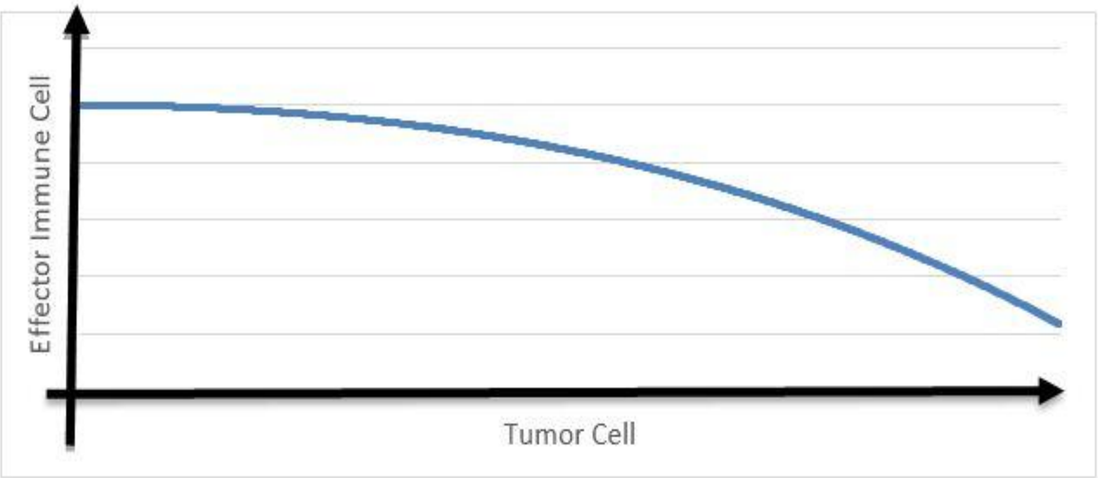
The system variables *x*, *y*, *z* in the tumor model as nonlinear ordinary differential equation system represents host cell, effector immune cell and tumor cell respectively. For the initial values $x(0) = 0.01, y(0) = 0.01$ & $z(0) = 0.01$, we obtain the solution of the system by using Fractional order Multistage Homotophy Perturbation Method as a function of t in the form of power series and for each value of t in a period of time interval (0.0001, 0.5) the sample results of $x(t), y(t), z(t)$.By taking values for t∈(0.0001,0.5) and using equations(10),(11) and (12), the sample values obtained out of nearly 5000 values are tabulated for reference.

Table 5. Sample solutions of *x(t)*,*y(t)* and *z(t) by FMHPM*

S. No	*x(t)*	*y(t)*	*z(t)*
1	0.00999988	0.009999877	0.009979605
2	0.009997511	0.009996075	0.009483662
3	0.009995939	0.009983266	0.00846086
4	0.010000018	0.009958026	0.007186047
5	0.010003229	0.00994587	0.006705236
6	0.010004098	0.009942859	0.006594165
7	0.010005014	0.00993977	0.006483042
8	0.010006142	0.009936069	0.006353329
9	0.010007687	0.009931155	0.006186434
10	0.010008961	0.009927213	0.006056527

4.2.Graphical Representation of Tumor Model by MHPM

The system variables *x*, *y*, *z* in the tumor model as nonlinear ordinary differential equation system represents host cell, effector immune cell and tumor cell respectively. For the initial values $x(0) = 0.01, y(0) = 0.01$ & $z(0) = 0.01$, we obtain the solution of the Chua's model by using Multistage Homotophy Perturbation Method as a function of t in the form of power series and for each value of 't' in a period of time interval (0.0001, 0.5) the results of $x(t), y(t), z(t)$ are plotted pairwise. The basic tumor microenvironment principles and graphical representation are as follows: **"If the Host cell is growing and the Effector immune cell is declining then the Tumor cell is growing".**



Firure 5: Comaprision of Tumor Cell and Effector Immune Cell

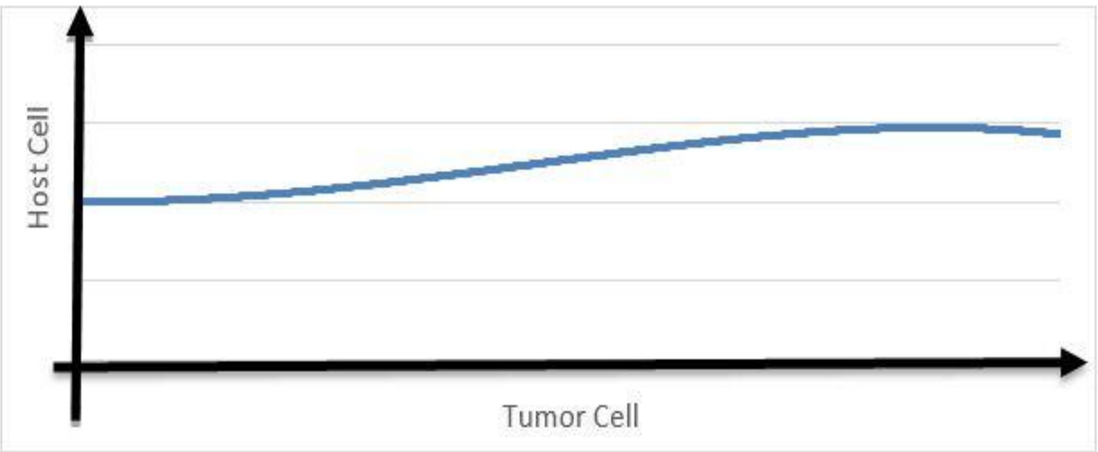


Figure 6: Comparison of Tumor Cell with Host Cell

Conclusion

In this nonlinear tumor model we apply three different methods such as Differential Transform Method, Elzaki Transform Method and Fractional order Multistage Homotophy Perturbation method we get the similar results. , it is observed that there is monotonic decline in the population of effector immune cells corresponding to the increase in the population of tumor cells whereas the population of host cells increases corresponding to the increase in the population of tumor cells.This graphical analysis of the results is perfectly in agreement with the established basic principles of Tumor microenvironment.

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