

## **Analysis of High Current Sheet Beam Transportation**

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### **Abstract**

Sheet or planar beam that carry high current densities find tremendous applications in the field of high power microwave devices. In the present work, an analysis has been carried out for the stable transportation of sheet beam as a function of perveance and beam geometry. Exact value of axial space charge length is calculated as a function of input current and it is shown that diocotron instability can be controlled by normalizing the beam geometry parameters.

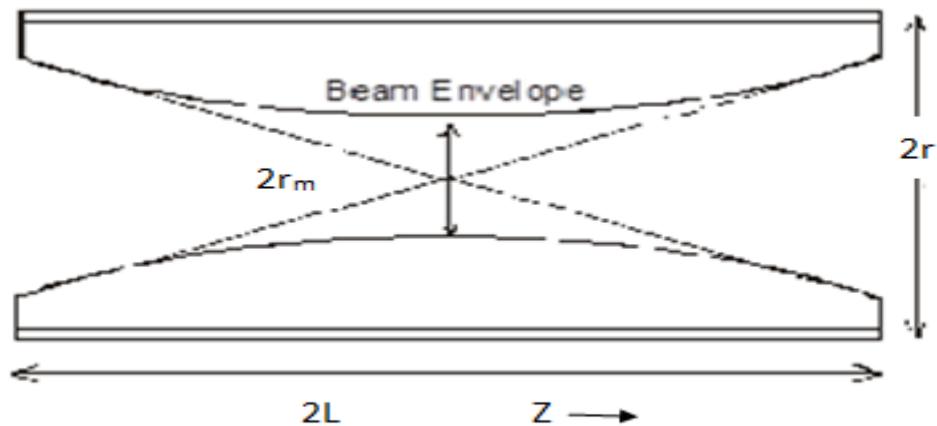
**Keywords:** Sheet beam, Beam control, Space-charge effects, Diocotron instability, Perveance, High power microwaves

### **1.INTRODUCTION**

Sheet beam is known to carry high current density charged particles (Booske et al., 1988). These are used in high energy sections of accelerators and other devices like BWO (Tusharika et al., 2014; & Zhaliang et al., 2013; ), TWT (Mark et al., 2010; Young et al., 2010) and FELs (Shiqiu C. et al., 1996; Marshall et al., 1985) where production of high power microwaves is done (Jinshu et al., 2007; Booske et al., 2011). There are several methods for stabilization of sheet beam that have been

theoretically predicted and experimentally demonstrated viz use of uniform magnetic field(Purna et al.,2013; Zhaliang et al., 2013), non uniform or cusped magnetic field(Booske et al., 1993 and 1994 & Basten et al.,1994), tailoring of beam edges (Jayshree P. et al.,1999; Arti et al.,2002 ) etc and concluded that with non uniform magnetic field ,the sheet beam transportation is an efficient option(Bruce et al.,2005&Arti et al., 2007.This has made the use of planar sheet beams important. Since, last eight decades space charge effect has caused serious limitations to beam transport (Haeff et al., 1939 ;Michael et al.,2006 &Wallace et al., 1973 ) and researchers have tried to find solutions to transport high current density with the use of sheet beams.Thus, analysis on stable transportation of sheet beam is often needed (Wallace et al., 1973 and Purna C et al., 2011) .The limitation to carry high current density is due to increase of space charge effect which results in an unstable beam. The general instability observed in high current sheet beam is diocotron instability which are “ac space charge” or kink-type modes (Pierce et al., 1956;Kyhl R.L. et al., 1956) or velocity shear effects(Antonsen et al., 1975) and stabilization of diocotron instability is important(Han S Uhm et al., 1994).

Most of the textbooks(Sisodia et al., 2009 ) explain cylindrical or annular beams which are the most common, well-defined beams.



**Fig. 1:** Conventional cylindrical beam with converging beam radius in focused and unfocused state.

*Ref:* M. L. Sisodia, 'Vacuum and solid state devices,' New Age International Publishers, New Delhi, Section 2

In the present paper the beam dynamics of sheet beam based on single electron model is being discussed for the first time.

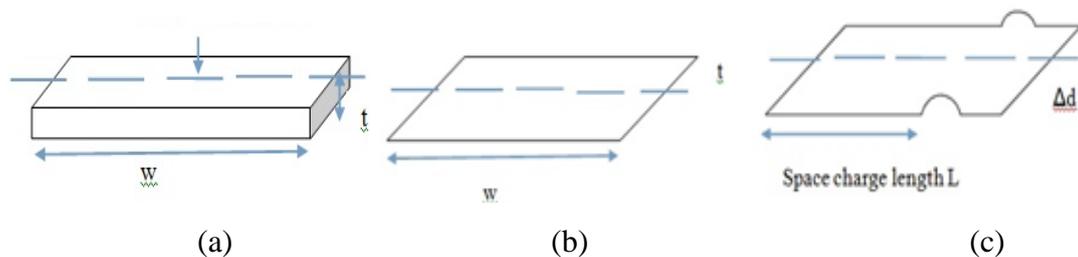
Fig.1 shows the conventional cylindrical beams with converging beam radius  $r$  and minimum beam radius  $r_m$ . The operation of sheet beam during transportation of high current density, in terms of input, i.e. perveance and beam parameters need to be understood, so that by controlling the various beam parameters, input can be maintained and instability can be overcome to a great extent. The need of clear understanding of sheet beam operation as a function of perveance has thus urged. Perveance is a notion used in description of charged particle beams and is a fixed value for a particular power output that is defined over a frequency regime.

The present work emphasizes to give the clear position of instability and beam dimensions. In Sec.2 of the paper, the beam dynamics using single electron model is explained. The field free region and field region has been shown with clear geometry in terms of space-charge length  $L$  and sheet beam thickness  $t$ . In Sec.3, the various important aspects of space-charge, diocotron instability and beam geometry of sheet beam has been analyzed and discussed in detail as a function of perveance. The space charge length  $L$ , implications to increase or decrease sheet beam dimensions  $w \times t$ , linear displacement length  $\Delta d$  that lead to diocotron instability have been discussed.

Later in Sec.4, the condition to maintain maximum value of  $L$ , keep  $\Delta d$  as low as possible and the implication to reduce and increase beam dimensions is shown. Conclusions are presented in Sec.5.

## 2. BEAM DYNAMICS OF SHEET BEAM

In this section, beam dimensions are modeled at various region that starts from emission point, moves to the dispersed state and come back to focused state. The focused or original state is achieved by the application of field to the sheet beam in the unstable or dispersed state. The assumption of beam motion is Brillouin. The electron gun is shielded from magnetic field. This Brillouin flow requires minimum magnetic field for maintaining the diameter of electron beam in interaction region and it is difficult to achieve in actual practice. Let the first consideration to beam motion be due to space charge force, that is, we consider the coulombs repulsion between the charged particle. Fig.2 shows the typical 2D planar sheet beam in 3D, 2D and rectilinear shift from the path.



**Fig. 2:** Typical 2D sheet beam of width  $w$ , thickness  $t$  where  $w \times t$  beam propagates along length  $L$  (a) 3D view (b) 2D view (c) Rectilinear shift from propagation path.

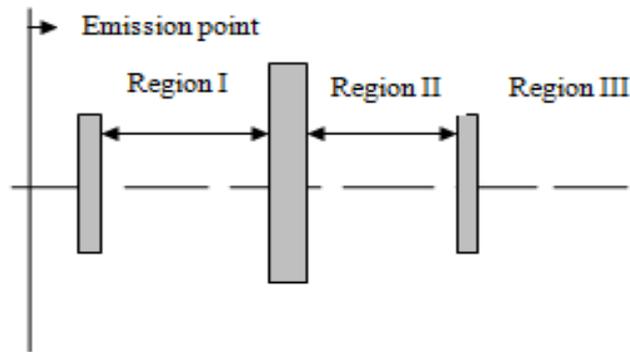
We define three regions corresponding to propagation axis in Fig.3. Region I is defined as  $Z \leq 0$ ; region II as  $0 < Z < L$  and region III as  $Z \geq L$  where  $Z$  is the propagation axis and  $L$  is the space charge length. The emission point of cathode is as shown in Fig.3.

In case of planar sheet beam, the converging beam radius  $r$  is the expanded thickness of the beam. It is denoted by  $t'$ . Consequently, the minimum radius  $r_m$ , is the original thickness  $t$  of sheet beam. So, the following can be written

$$2r = t' \quad (1)$$

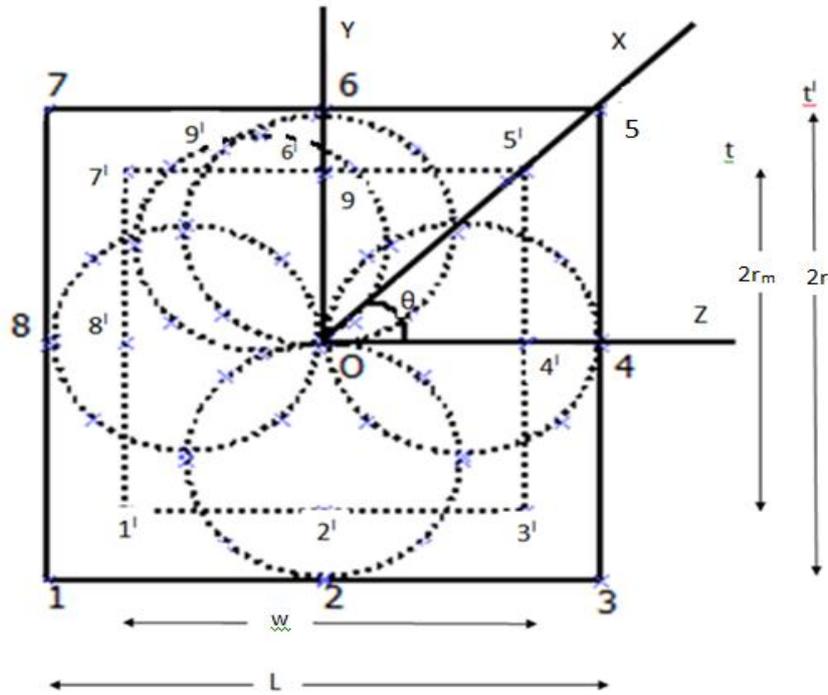
$$2r_m = t \quad (2)$$

The trajectory of charged particle will be straight line in region I and III as these are field free regions. The detailed analysis of charged particles is done for region II as it travels from unfocused state to focused state as shown in figure 3.



**Fig. 3:** Different regions of sheet beam during focused and unfocused state

In Fig. 4, the periphery of the cross sectional beam is shown by solid line as it enters the field, that is, region II. Eight particles are located at 1,2,3,4,5,6,7 and 8. The beam radius at this instant is  $r$ . After the beam travels a certain distance through field, these particles move, from location 1,2,3,4,5,6,7 and 8 to 1', 2',3',4',5',6',7' and 8'.  $r_m$  is the radial coordinate of 6' after the beam particle travels certain distance in field. Particle located at 9 of individual electron trajectory is initially inside the periphery of beam which corresponds to the instable or dispersed state.  $L$  is the distance at which field lines are supplied.



**Fig. 4:** Pictorial explanation of focusing of charged particle in 2D sheet beam. The solid boundary shows the electron beam as it enters the field. The dashed boundary shows the periphery of electron beam after it travels some distance in field. The small circles are the trajectories of individual electrons.

### 3. BEAM ANALYSIS ON PERVEANCE

Perveance is a notion used in description of charged particle beams. The value of perveance indicates how significant the space charge effect is on the motion of beam. The term is used with electron beams where the motion is mostly dominated by space charge. We see space charge is discussed in detail in the following section.

#### 3.1 Space charge

The detailed analysis on thickness of the beam has been done next, when the coulomb force between electrons that eventually lead to significant space-charge have been taken into consideration.

##### 3.1.1 Analysis of expanded thickness t

Let us consider the right angled triangle 045 (from Fig.4) and by Pythagoras theorem

the value of changed sheet beam thickness  $t$  is given by

$$\frac{t'}{2} = \frac{L}{2} \tan \theta \quad (3)$$

where  $L$  is the propagation length of expanded sheet beam thickness  $t'$ . It is thus, the space-charge length where the magnetic field lines must be applied.

Thus, the value of following length  $L$  is given by

$$L = t' \cot \theta \quad (4)$$

where  $\theta$  is the angle made by electron beam with the axis of propagation. The maximum and minimum value of space charge length, for given set of operation, that use sheet beam can be found as per the above developed relation. It can be concluded that with electrons of the beam being perfectly aligned with propagation axis, maximum value of  $L$  is possible. It is interesting to note from above relation that maximum value of  $L$  will be equal to changed thickness  $t'$ .  $L$  is the propagation length for expanded sheet beam thickness which is the changed sheet beam thickness  $t'$ .

$$L_{\max} = t' \quad (5)$$

The minimum value of  $L$  will be zero. This signifies the immediate positioning of magnets ( $L=0$ ). At perpendicular position of charged particles, this situation will be true. The non-parallel position of charged particles caused due to beam defocusing, leads to the requirement of  $L$ .

During the sheet beam generation, the choice of space charge length  $L$ , which is, the distance at which field lines must be applied is important. With the developed relation in (4), the value of maximum focusing length can be found. However, there is going to be limitation on taking the maximum value of  $L$ . This is so because, the emission of electron beams will not be perfectly aligned with propagation axis in real situations. The value of space charge length can be calculated with (4) based on the angle at which beam exists after being dispersed due to space charge. The angle of emission electrons is defined by angle  $\theta$  and is given by

$$\theta = \theta_0 + L \frac{\omega_L}{v_z} \quad (6)$$

where  $\theta_0$  is initial value and is zero.  $\omega_L$  is Larmor frequency given by  $\omega_L = eB_0/2\gamma m$  where  $e$  is the magnitude of electronic charge,  $B_0$  is the magnitude of magnetic field,  $m$  is electrons rest mass,

$$\gamma = \sqrt{1 - \frac{v_z^2}{c^2}}$$

$\gamma$  is the usual Lorentz factor;  $v_z$  is the electron beam velocity in  $z$ -direction and  $c$  is

constant velocity of propagation of light in free space.

In order to maintain  $\theta$  as  $\theta_0$  the ratio of product of  $L$  and  $\omega_L$  to  $v_z$ , must be zero. This implies to choose the value of  $v_z$  large as compared to product of  $L$  and  $\omega_L$ .

It must be observed that the maximum value of  $L$  is equal to expanded sheet beam thickness  $t'$ . With the above relation, direct relation between current, operating voltage, and perveance  $K$  can be given. From Mathieu, 's equation

$$\frac{L}{r} \sqrt{\frac{3 \times 10^4 \times I_0}{V_0^{3/2}}} = 1.082 \quad (7)$$

Let us consider the maximum value of space charge length which is the changed thickness of sheet beam. ( $L_{\max} = t'$ )

Let us next from (1) substitute  $2r = t'$ . Consequently,

$$K = 9.756 \times 10^{-6} \quad (8)$$

The above relation (8) can be written in terms of current and voltage ratio in case of sheet beam as follows

$$\frac{I_0}{V_0^{3/2}} = 9.756 \times 10^{-6} \quad (9)$$

Above equations clearly states the exact value of perveance  $K$  for given current at a particular operating voltage in case of sheet beam. Consequently, with Richardson-Dushman equation, the calculation of exact value of current density is possible. Moreover, analysis of  $K$  by either varying current at particular operating voltage or vice-versa is possible.

### 3.1.2 Analysis of normalized beam radius $R$

The normalized beam radius  $R$  is defined as

$$R = r / r_m \quad (10)$$

From equation (1) and (2)

$$R = t' / t \quad (11)$$

and from geometry (fig. 4), it is given by

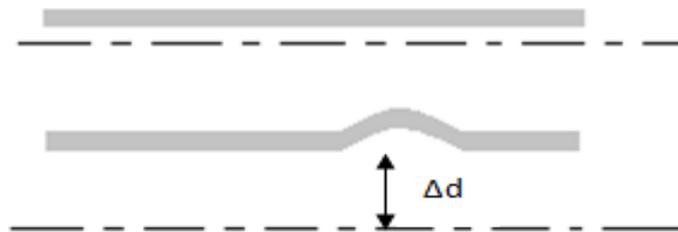
$$R = \sin \theta \quad (12)$$

At perfect alignment of electrons with propagation axis, normalized beam radius will not exist. It can be seen from (11) that with the value of R as unity, no instability of the beam can be implied. Practically, there will be some value of R that will exist.

However, the value of R should be as low as possible. This has two main reasons. First, it indicates lower application of magnetic field lines. Secondly, it must be noted that large value of minimum converging beam radius is desired, the need for larger sheet beam area can be seen. This means there will be decrease in current density of beam. We next move to analyze the most fundamental instability that takes place in sheet beam. This is diocotron instability.

### 3.2 Diocotron Instability

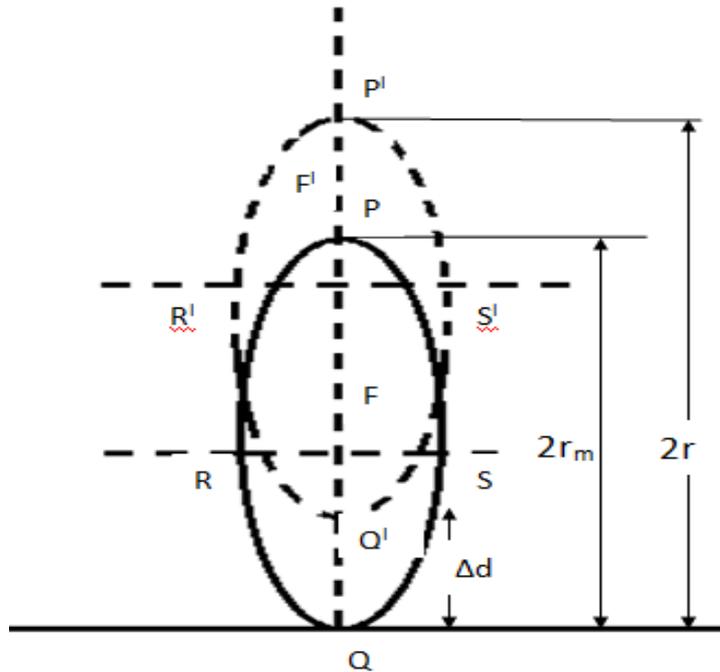
Sheet beam is large current density. If a close look on various kinds of instabilities are taken into consideration, apart from the instability caused due to space charge, the most dominant instability that will occur in sheet-beam is diocotron instability. This instability is created by two sheets of charge that slip past each other. It is shown in Fig.5. In simple words it is the rectilinear shift from the path of propagation. We next analyze the diocotron instability



**Fig. 5 :** The rectilinear shift of sheet beam along its axis of propagation due to linear displacement  $d$ .

#### 3.2.1 Analysis of linear shift $\Delta d$ (based on single particle linear theory)

The cross section, for sheet beam is taken elliptical since these elliptical cross-sections are easy to both focus and generate (Basten et al., 2007 & Ningfeng et al., 2015).



**Fig. 6 :** 2D sheet beam CS .Solid boundary is the periphery of beam CS with PQ as a major axis and RS a minor axis. Dotted boundary is the shifted periphery of beam.

Elliptical cross-sections are made by tailoring of the beams around the edges . It is one of the ways to control the sheet beam instability.

In the above figure, PQ is the CS of sheet beam and P'Q' is the shifted CS of sheet beam. The shift in focus F to F' is seen. There is a linear shift of the elliptical CS beam, which is vertical to the propagation axis. It must be observed that the original thickness PQ is same as the shifted diameter P'Q'. The original sheet beam thickness t or minimum converging beam radius  $r_m$  is to be achieved after focusing. Also, the expanded beam with thickness t' or converging beam diameter given by 2r is P'Q. It can be said that the linear displacement  $\Delta d$  from its original path will be given by

$$\Delta d = 2r - 2r_m \tag{13}$$

From fig.6,  $2r = P'Q$  and  $2r_m = PQ$

From equation (1) and (2)

$$\Delta d = t' - t \tag{14}$$

It is evident from above relation that at higher values of input current,  $t'$  increases and so does the diocotron instability. The value of thickness has to be kept high to match the value of  $t'$  as per above relation. Especially in cases where there is more instability or higher values of  $t'$  are observed, thicker beams that match  $t'$  should be taken. However, the  $w \times t$  ratio has to be maintained. This limits the choice to take any large value of  $t$ . As usual the current density is compromised too. Exact magnitude of diocotron instability can be calculated from (14).

From (5), the above relation can be written as

$$\Delta d = L_{\max} - t \quad (15)$$

The diocotron instability can completely be stabilized by increasing the beam thickness to the maximum value of space charge length.

#### 4. RESULTS AND DISCUSSIONS

The focusing of charged particles with the help of permanent magnets in wiggler arrangements is the most common and useful method as far as efficiency is concerned. The concept of electron beam focusing with alternating magnets involves the single magnet to be replaced by  $N$  small magnets of same length. This is to reduce the bulkiness. The crisscross or wiggler arrangements further reduces the bulkiness and helps to increase compactness. While designing experiments (Zhaliang et al., 2013; Ningfeng et al., 2015; Arti et al., 2011 and 2007) for a particular frequency band and power that use sheet beam, the exact value of focusing length  $L$  can be found as per (4). In practical situations, the value of  $\theta$  which is the emission angle of electron beams with respect to propagation axis cannot be always maintained at a situation when electrons will be aligned with the propagation axis. The choice of electron beam velocity over a particular frequency band has to be given consideration, though not much can be done. For given perveance, calculation of exact value of  $v_z$ , which will be required is however possible as shown in (6). The magnitude of saturation current that can flow for given geometry can be known as direct relation of input voltage and current exists given by (9). (11) can be used to discuss the figure of merit for beam propagation which has been shown as function of sheet beam thickness. To overcome the fundamental instability which is diocotron instability in case of sheet beam, thickness needs to be increased significantly as (14) points, which means lowering of current density. However,  $w \times t$  ratio needs to be maintained under any circumstances for given perveance. (15) points that diocotron instability can completely be stabilized by increasing the beam thickness to the maximum value of space charge length.

## 5. CONCLUSION

Sheet or planar beam that carry high current densities find tremendous applications in the field of high power microwave devices. Input voltage and input current have been related in case of sheet beam. Also, the normalized beam radius value which has been related to sheet beam thickness for desired value of current, can directly be used to comment on the stability of sheet beam propagation. In the present work, an analysis has been carried out for the stable transportation of sheet beam as a function of perveance and beam geometry. Exact value of axial space charge length is calculated as a function of input current in the first part. This exact value of  $L$  will be useful to decide the position of magnets required for focusing of the beam in non-uniform field. Consequently, it is found that diocotron instability can be controlled by normalizing the beam geometry parameters.

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