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Influence of Permeability lying along the wall on the Peristaltic Motion of Casson Fluid

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Abstract

In this article, we have presented effect of permeable lying along the wall on the peristaltic motion of Casson fluid. The governing equations are constructed under long wave length and low Reynolds number approximations. Governing equations are solved by analytic method and expressions for the stream function, velocity are obtained. The pertinent features of various physical parameters have been discussed graphically.

Keywords: Casson Fluid, Peristaltic Motion, Permeable.

I. INTRODUCTION

Peristalsis is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular this mechanism is involved in urine transport from kidney to bladder, movement of ovum in the fallopian tubes, in movement of chime in the gastro intestinal tract, in the transport of spermatozoa in the ductus efferent of the male reproductive tracts and in the cervical canal, in movement of ovum in the female fallopian tubes, in transport of lymph in the lymphatic vessels and in the vasomotion of small blood vessels. Engineers have developed pumps having industrial and physiological applications adopting the principle of peristalsis. In particular finger and roller pumps are frequently used for pumping corrosive or very pure material so as to prevent direct contact of the fluid with the pump's internal surfaces.

A number of studies on peristaltic flows of different fluids [1-5] have been reported. N.L.Bhikshu et. al. (6) investigated peristaltic flow of a conducting Williamson fluid in a vertical asymmetric channel with heat transfer through porous medium. S. Sreenadh et. al. (7) investigated the effect of slip on peristaltic pumping of a hyperbolic tangent fluid in an inclined asymmetric channel. K. Chakradhar et. al. (8) studied peristaltic transport of Bingham fluid in a channel with permeable walls. S. Sreenadh et. al. (9) studied the effects of slip and heat transfer on the peristaltic pumping of a Williamson fluid in an inclined channel. Das K. (10) studied heat transfer on peristaltic pumping of a Johnson-Segalman fluid in an inclined asymmetric porous channel. K. Vajravelu et. al. (11) studied second order slip flow and heat transfer over a stretching sheet with non-linear Navier boundary condition. S. Sreenadh et. al.(12) studied peristaltic transport of a power-law fluid in an asymmetric channel bounded by permeable walls. Govardhan P et. al.(13) Effects of Slip and Heat Transfer on the Peristaltic Pumping of a Williamson Fluid in an Inclined Channel. N.S. Akbar et. al. (14) investigated the peristaltic flow of a nano fluid with slip effects. A. Ebaid et. al. (15) studied the exact analytical solution of the peristaltic nano fluids flow in an asymmetric channel with flexible walls and slip condition. N.L. Bhikshu et. al. (16) studied the effects of MHD on the peristaltic flow of fourth grade fluid in an inclined channel with permeable walls. V.P. Rathod et. al. (17) studied the slip effects of peristaltic transport of a porous medium in an asymmetric vertical channel by Adomian decomposition method. M. Turkyilmazoglu, (18) investigated heat and mass transfer of MHD second order slip flow. K. Chakradhar et. al. (19) studied peristaltic transport of a power law fluid in a channel with permeable walls. A.V. Rosca et. al. (20) studied the flow and heat transfer over a vertical permeable stretching/shrinking sheet with a second order slip. Srinivas A.N.S (21) investigated peristaltic transport of a Casson fluid in a channel with permeable walls. Chakradhar K. et. al. (22)discussed peristaltic pumping of a Micro polar fluid in a tube with permeable wall S.V.H.N. Krishna Kumari, et. al. (23) studied unsteady peristaltic pumping in a finite length tube with permeable walls. Vajravelu K. et. al. (24) discussed peristaltic transport of a Micro polar fluid in channel with permeable walls.

The aim of the present paper motivated from the above analysis is to discuss the influence of permeability lying along the wall on the peristaltic motion of Casson fluid. Exact solutions are computed and expressions for velocity, stream function and volume flux are obtained. The importance of pertinent parameters entering into the flow modeling is discussed in detail through graphs.

II. MATHEMATICAL FORMULATION

Consider the peristaltic motion of a non-Newtonain fluid in a non- uniform channel whose walls are lined with the porous material.Consider a train of progressive infinite sinusoidal wave travelling in a channel with two dimensions having width 2d and velocity ‘c’ .Consider the coordinateshaving x axis along the propagation wave through central line and y axis perpendicular to the x-axis as shown in fig 1.

The region between y = 0 and y = y_0 is called plug flow region. In the plug flow region

$|\tau - xy| \leq \tau_0$ in the region y = y_0 to y = h, $|\tau - xy| > \tau_0$.

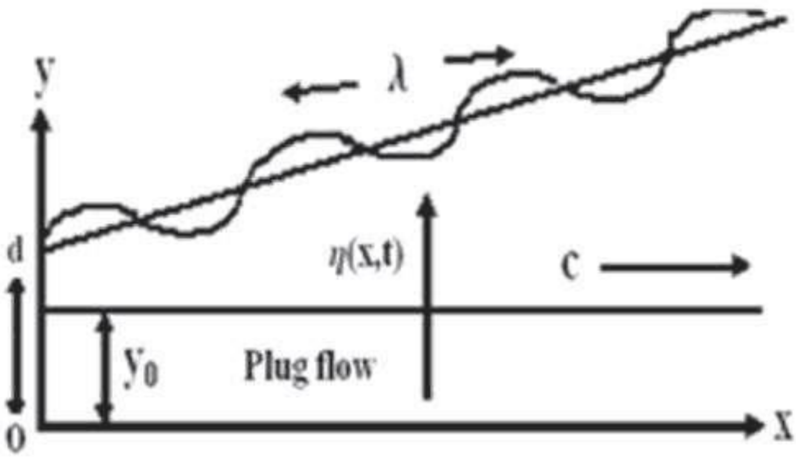

Fig. 1

The channel wall are governed by equation

$$Y=H(X,t)=d+a\ \mathrm{Sin}\frac{2\pi}{\lambda}(X-ct) \quad (1)$$

Where d=d’+ bx and a being considered as wave amplitude, λ is wave length and d half the width of a channel from inlet at any distance and d, being half width at the inlet . ‘b’ is the dimensional non uniformity of the channel. The flow becomes steady in the wave frame (x, y) moving with velocityc away from the fixed (laboratory) frame(X, Y). The transformation betweenthese two frames is given by

$$x = X - ct;\ y = Y;\ u(x, y) = U(X - ct, Y) - c;\ v(x, y) = V(X - ct, Y) \quad (2)$$

Using the non-dimensional quantities

$$\bar{u} - \frac{u}{c}; \bar{x} = \frac{x}{\lambda}; \bar{y} = \frac{y}{a}; \bar{h} - \frac{h}{a}; \bar{t} = \frac{ct}{\lambda};$$

$$\bar{\psi} = \frac{\psi}{ac}; \bar{p} = \frac{pa^2}{\lambda \mu c}; \bar{q} = \frac{q}{ac}; \phi = \frac{d}{a}; Da = \frac{k}{a^2}; \beta = \frac{\sqrt{Da}}{\alpha} \tag{3}$$

The governing equations of motion after dropping primes are as follows

$$\frac{\partial}{\partial y}(\tau_{yx}) = -\frac{\partial p}{\partial x} \qquad 0 \le y \le h \tag{4}$$

Where $\sqrt{\tau_{yx}} = \sqrt{-\frac{\partial u}{\partial y}} + \sqrt{\tau_0}$

And dimensionless conditions at the boundary are

$$\frac{\partial u}{\partial y} = 0 \qquad \text{at y=0} \tag{5}$$

$$u = -1 - \beta \frac{\partial u}{\partial y} \qquad \text{at } y=h-\varepsilon \tag{6}$$

III. SOLUTION OF THE PROBLEM

Solving equation (4) by using the boundary conditions (5) and (6) we get the velocity as

$$u = -1 - \frac{P}{2}[y^2 - (h-\varepsilon)^2 - 2(h-\varepsilon)\beta] - 2\tau_0[y - (h-\varepsilon) - \beta] + \frac{4\sqrt{\tau_0}}{3P}[(Py+\tau_0)^{\frac{3}{2}} - (P(h-\varepsilon)+\tau_0)]^{\frac{3}{2}} - 2\sqrt{\tau_0}\beta(P(h-\varepsilon)+\tau_0)^{\frac{1}{2}} \tag{7}$$

$$u_p = -1 - \frac{P}{2}[{y_0}^2 - (h-\varepsilon)^2 - 2(h-\varepsilon)\beta] - 2\tau_0[y_0 - (h-\varepsilon) - \beta] +$$
$$\frac{4\sqrt{\tau_0}}{3P}[(Py_0 + \tau_0)^{\frac{3}{2}} - (P(h-\varepsilon) + \tau_0)]^{\frac{3}{2}} - 2\sqrt{\tau_0}\beta(P(h-\varepsilon) + \tau_0)^{\frac{1}{2}} \tag{8}$$
$$0 \le y \le y_0$$

Integrating the equation (7) and (8), we obtain the stream function as

$$\psi = -y - \frac{P}{2}[\frac{y^3}{3} - (h-\varepsilon)^2 y - 2(h-\varepsilon)\beta y + \frac{2y_0^3}{3}] - 2\tau_0[\frac{y^2}{2} - (h-\varepsilon)y - \beta y + \frac{y_0^2}{2}]$$
$$+\frac{4\sqrt{\tau_0}}{3P}[\frac{2(Py+\tau_0)}{5P} - y(P(h-\varepsilon)+\tau_0)^{\frac{3}{2}} - \frac{2(Py_0+\tau_0)^{\frac{5}{2}}}{5P} + y_0(Py_0+\tau_0)^{\frac{3}{2}}$$
$$-2y\sqrt{\tau_0}\beta(P(h-\varepsilon)+\tau_0)^{\frac{1}{2}} \tag{9}$$

and

$$\psi_p = -y_0 - \frac{P}{2}[y_0^3 - (h-\varepsilon)^2 y_0 - 2(h-\varepsilon)\beta y_0] - 2\tau_0[y_0^2 - (h-\varepsilon)y_0 - \beta y_0]$$
$$+\frac{4\sqrt{\tau_0}}{3P}[y_0(Py_0+\tau_0)^{\frac{3}{2}} - y_0(P(h-\varepsilon)+\tau_0)^{\frac{3}{2}}]$$
$$-2y_0\sqrt{\tau_0}\beta(P(h-\varepsilon)+\tau_0)^{\frac{1}{2}} \tag{10}$$

The volume flux through each cross section in the wave frame is given by

$$q = \int_0^{y_0} u_p dy + \int_{y_0}^{h-\varepsilon} u dy$$
$$= \frac{P}{3}[(h-\varepsilon)^3 - y_0^3 + 3(h-\varepsilon)^2\beta] + \tau_0[(h-\varepsilon)^2 - y_0^2 + \frac{(h-\varepsilon)}{2}\beta]$$
$$+\frac{4\sqrt{\tau_0}}{3P}\{y_0(Py_0+\tau_0)^{\frac{3}{2}} - (h-\varepsilon)(P(h-\varepsilon)+\tau_0)^{\frac{3}{2}}$$
$$+\frac{2}{5P}[(P(h-\varepsilon)+\tau_0)^{\frac{5}{2}} - (Py_0+\tau_0)^{\frac{5}{2}}]\}$$
$$-2\tau_0(h-\varepsilon)(P(h-\varepsilon)+\tau_0)^{\frac{1}{2}} - (h-\varepsilon) \tag{11}$$

The instantaneous volume flow rate Q(X, t) in the laboratory frame betweenthe centre line and the wall is

$$Q(X,t)=\int_0^H U(X,y<t)dy$$
$$=\frac{P}{3}[(h-\varepsilon)^3-y_0^3+3(h-\varepsilon)^2\beta]+\tau_0[(h-\varepsilon)^2-y_0^2+\frac{(h-\varepsilon)}{2}\beta]$$
$$+\frac{4\sqrt{\tau_0}}{3P}\{y_0(Py_0+\tau_0)^{\frac{3}{2}}-(h-\varepsilon)(P(h-\varepsilon)+\tau_0)^{\frac{3}{2}} \tag{12}$$
$$+\frac{2}{5P}[(P(h-\varepsilon)+\tau_0)^{\frac{5}{2}}-(Py_0+\tau_0)^{\frac{5}{2}}]\}-2\tau_0(h-\varepsilon)(P(h-\varepsilon)+\tau_0)^{\frac{1}{2}}$$

From equation (11) we have to fix P in order to calculate the pressure riseΔP. Since right hand side of equation (11) is non-linear, we assume that thepowers of τ_0 , which are greater than 1 are negligible. In view of this approximation we obtain the expression for $\frac{dp}{dx}$ from equation (11) as

$$\frac{dp}{dx}=\frac{3[q+(h-\varepsilon)-\tau_0((h-\varepsilon)^2+\beta\frac{(h-\varepsilon)}{2}-2(h-\varepsilon)\beta]}{[(h-\varepsilon)^3+3(h-\varepsilon)^2\beta]} \tag{13}$$

The dimensionless average volume flow rate $\overline{Q}$ over one wavelength is obtained as

$$\overline{Q}=\frac{1}{T}\int_0^T Qdt=q+1 \tag{14}$$

IV. THE PUMPING CHARACTERISTICS

Integrating the equation (13) with respect to x over one wavelength, we get thepressure rise (drop) over one cycle of the wave as

$$\Delta P=\int_0^1 -\frac{3[\overline{Q}-1+(h-\varepsilon)-\tau_0((h-\varepsilon)^2+\beta\frac{(h-\varepsilon)}{2}-2(h-\varepsilon)\beta]}{[(h-\varepsilon)^3+3(h-\varepsilon)^2\beta]}dx \tag{15}$$

The pressure rise required to produce zero average flow rate is denoted by ΔP_0. Hence ΔP_0 is given by

$$\Delta P_0 = \int_0^1 -\frac{3[-1+(h-\varepsilon)-\tau_0((h-\varepsilon)^2+\beta\frac{(h-\varepsilon)}{2}-2(h-\varepsilon)\beta]}{[(h-\varepsilon)^3+3(h-\varepsilon)^2\beta]}dx \tag{16}$$

The dimensionless frictional force F at the wall across one wavelength is given by

$$F = \int_0^1 h(-\frac{dp}{dx})dx \tag{17}$$

V. RESULTS AND DISCUSSION

From equation (15), we have calculated the pressure difference as a function of $\overline{Q}$. Figure 2 shows that the pressure rise Δp verses flux $\overline{Q}$ for different amplitude ratio ϕ with fixed β=0.005, τ=0.02, ε= 0.02. We also observed that Δp decreases as $\overline{Q}$ Increases. Figure 3 shows that the pressure rise Δp verses flux $\overline{Q}$ for different τ Values with fixed ϕ, β, ε. We also observed that Δp decreases as $\overline{Q}$ Increases. The pressure rise Δp verses flux $\overline{Q}$ for different ε Values with fixed ϕ, β, τ gives in Figure 4 and it is clear that Δp decreases as increase of $\overline{Q}$. One can observe that the larger β values the greater the pressure rise against which the pump works for a given flux $\overline{Q}$ in figure5, It is noticed that the pressure difference Δp increases with increasing $\overline{Q}$.

From equation (17), we have calculated the Frictional force F as a function of $\overline{Q}$. Figure 6 shows that the frictional force Fverses flux $\overline{Q}$ for different amplitude ratio ϕ with fixed β=0.005, τ=0.02, ε= 0.02. We also observed that F increases as $\overline{Q}$ Increases. Figure 7 shows that Fverses flux $\overline{Q}$ for different τ Values with fixed ϕ, β, ε. We also observed that F increases as $\overline{Q}$ Increases. The Frictional force F verses flux $\overline{Q}$ for different ε Values with fixed ϕ, β, τ gives in Figure 8 and it is clear that Fdecreases as increase of $\overline{Q}$. One can observe that the larger β values the greater the Frictional force F against which the pump works for a given flux $\overline{Q}$ in figure9, the Frictional force F increases with increasing $\overline{Q}$.

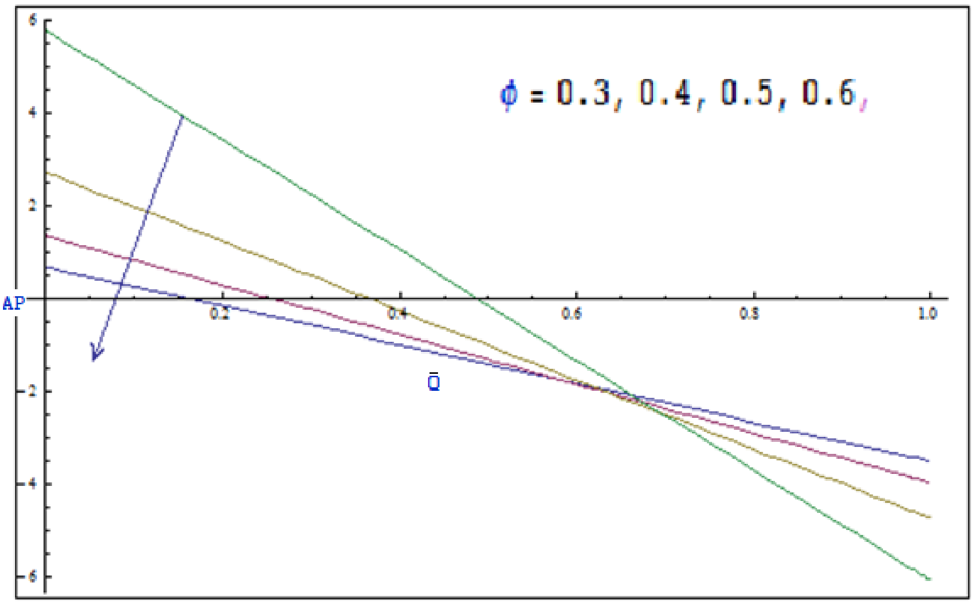

Figure: 2: The variation of Δp with $\overline{Q}$ for different values of ϕ with fixed values of β =0.005, τ =0.02, ε = 0.02.

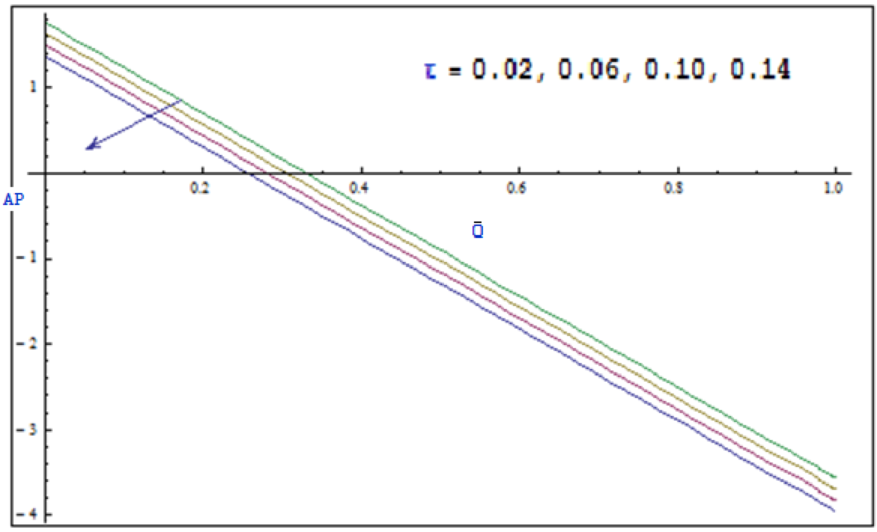

Figure 3: The variation of Δp with $\overline{Q}$ for different values of τ with fixed values of ϕ =0.4, β =0.005, ε =0.02

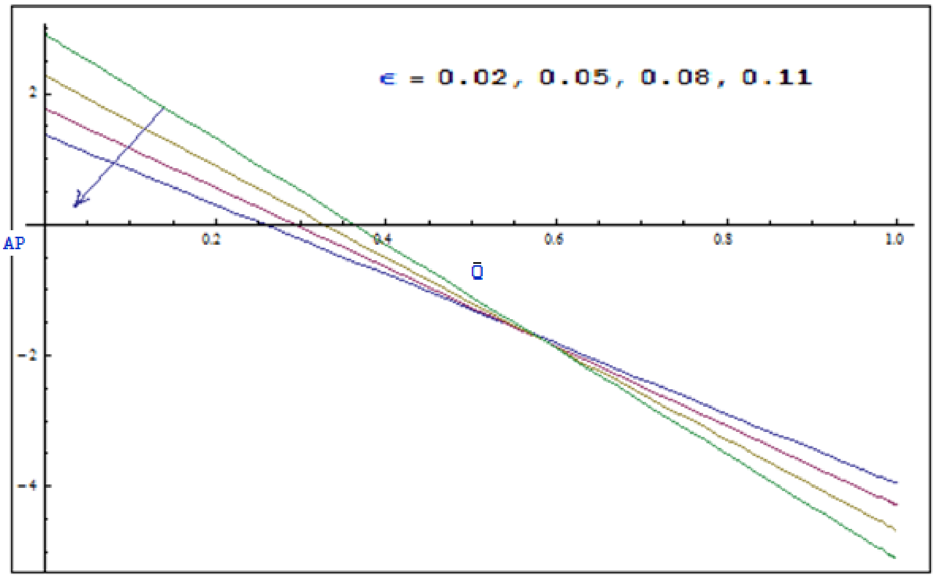

Figure 4: The variation of Δp with $\overline{Q}$ for different values of ε with fixed values of ϕ =0.4, β =0.005, τ =0.02.

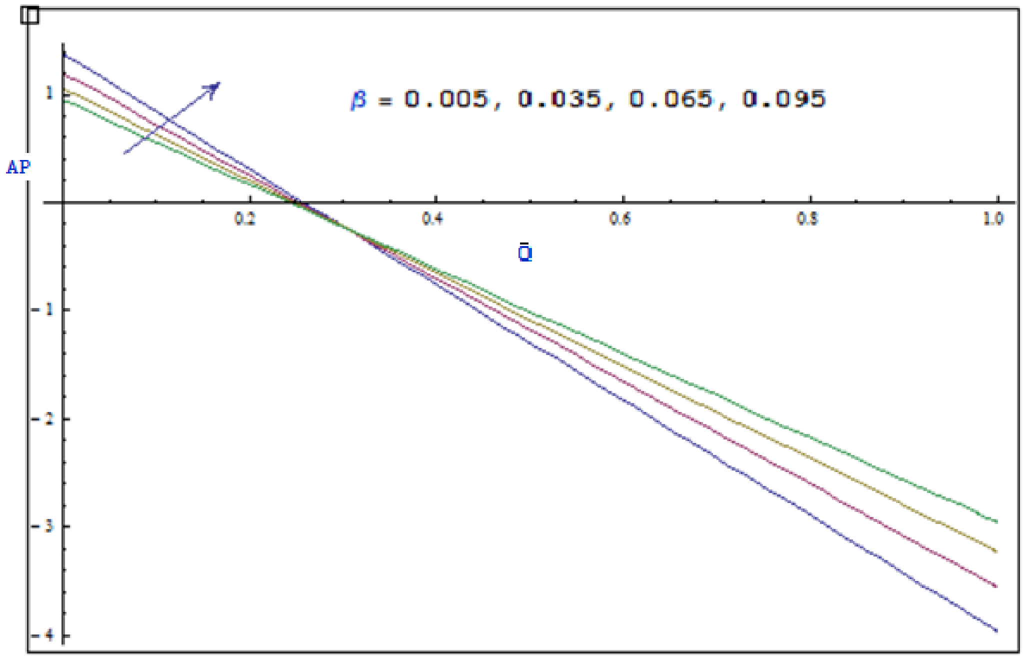

Figure 5: The variation of Δp with $\overline{Q}$ for different values of β with fixed values of ϕ =0.4, ε =0.02 , τ =0.02

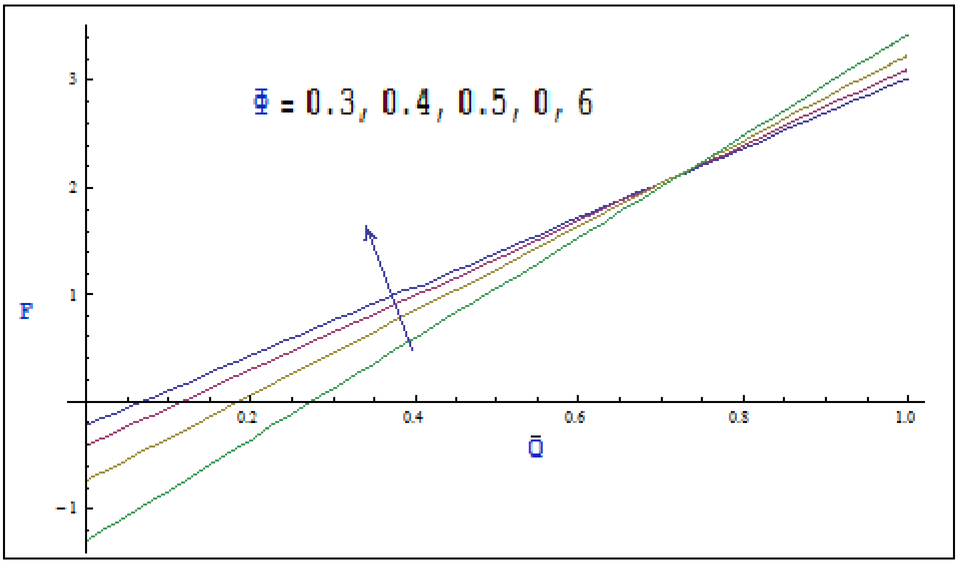

Figure: 6: The variation of F with $\overline{Q}$ for different values of ϕ with fixed values of β =0.005, τ =0.02, ε = 0.02

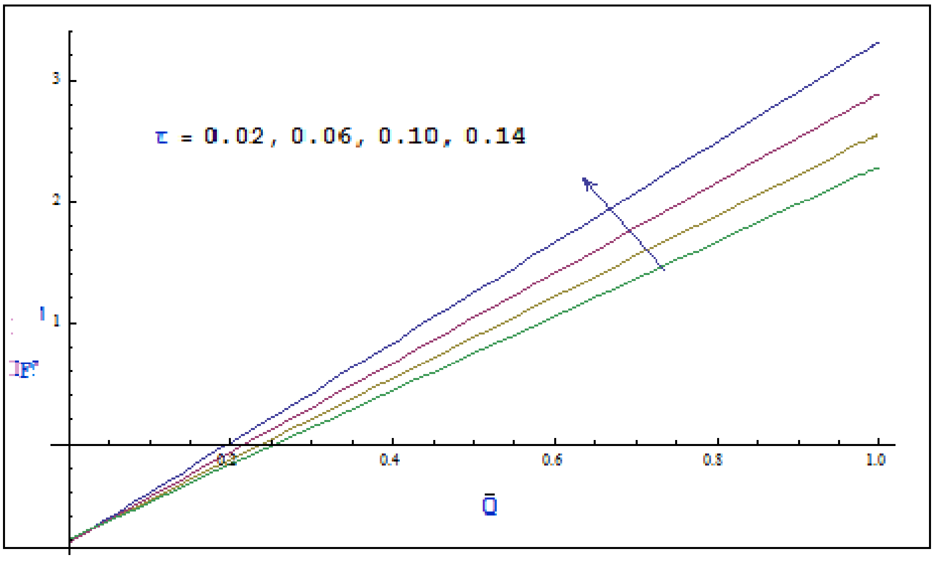

Figure 7: The variation of F with $\overline{Q}$ for different values of τ with fixed values of ϕ =0.4, β =0.005, ε =0.02

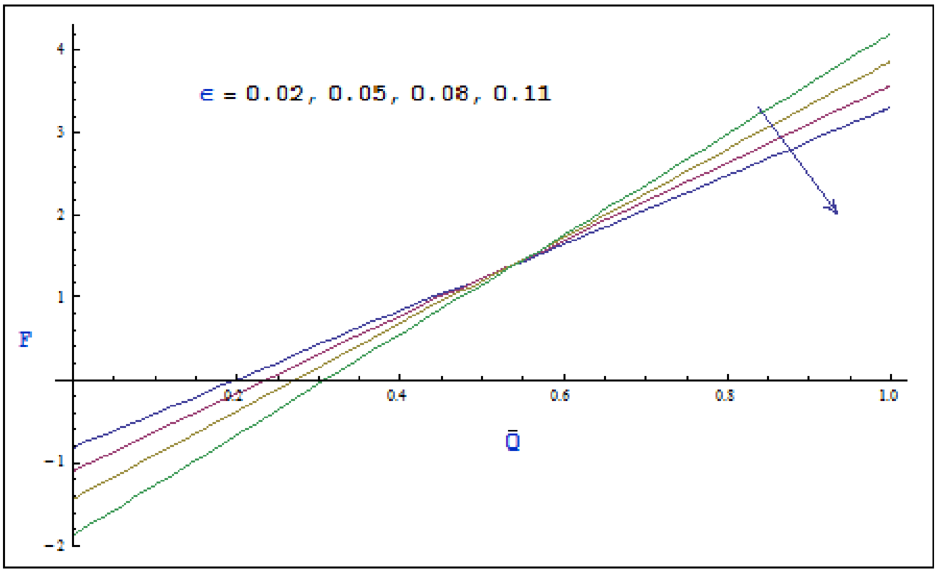

Figure 8: The variation of F with $\overline{Q}$ for different values of ε with fixed values of ϕ =0.4, β=0.005, τ=0.02.

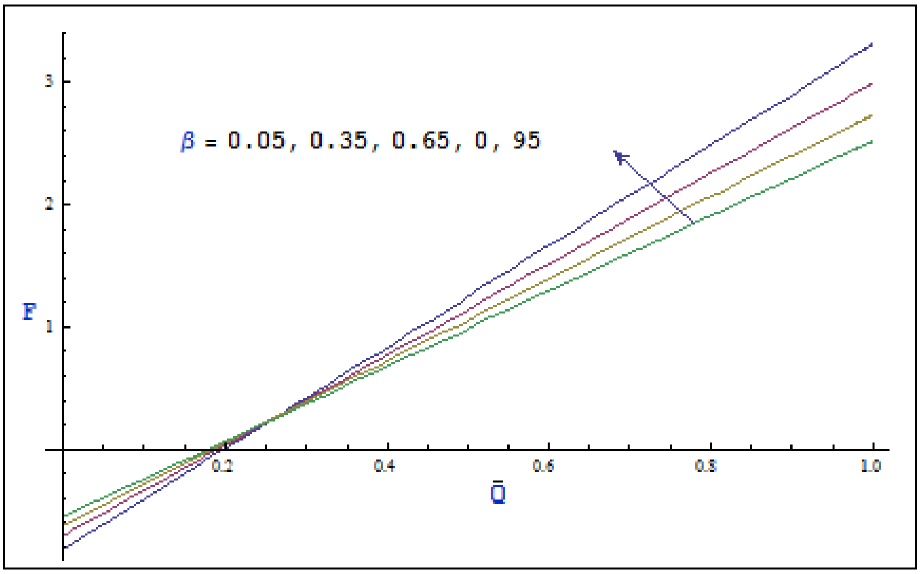

Figure 9: The variation of F with $\overline{Q}$ for different values of β with fixed values of ϕ =0.4, ε=0.02 , τ=0.02

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