International Journal of Computational Intelligence Research.
ISSN 0973-1873 Vol.3, No. 1 (2007), pp. 50-54
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Exchange Options Pricing with Evolutionary Neural-based Fuzzy Inference Systems

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***Abstract*: The options markets and earlier studies take the Black-Scholes Generalized Model (BSG) as the practical model and develop more prospering. However, BSG is also based on many assumptions and constrains such that derivatives valuation with this model shows miss-pricing seriously, especially while compared with the market prices in foreign exchange options market. In order to overcome the drawbacks derived from BSG, we employ the proposed options pricing model through enhanced neural-fuzzy-based inference systems (ENFIS) in options pricing and then compared with the BSG. The evidence from empirical studies is using the euro foreign exchange options listed on the Chicago Mercantile Exchange (CME). The performance valuating comparisons were focused in the research period from 2002 to 2005. The results show that the ENFIS framework is superior to the BSG no matter in error degree or in the interpretation capability.**

***Keywords*:** Black-Scholes generalized Model, enhanced neural-fuzzy-based inference systems, Genetic Algorithms, euro foreign exchange options, interpretation capability**.**

I. Introduction

The financial contracts of option that provide the investors the rights to buy (call option) or sell (put option) an asset for a specified price, strike price, on or before a specified time. Option trading also allows investors to reduce the financial risk and to bet on future events. But, what the contract is worth is anything but trivial. Thus, option-pricing model constantly is the concern focused no matter in academy or in empirical field. Before 1970, since there is not any proper probability distribution to describe the objects pricing behavior, therefore, no satisfactory pricing model could be developed.

Until 1973, Black and Scholes found that the Geometric Brownian Motion could describe the object-pricing behavior. By this primary assumption, they derived the call option pricing model using stochastic calculus so-call the BSM. It provided an adequate method for participates to evaluate the option prices. Among these six assumptions, the one under active debate is the dynamic process of the underlying asset price. For example, the geometric Brownian motion assumption has been challenged by empirical evidence. It is not surprising that the BSM has been shown demonstrating systematic biases as in a lot of empirical researches [1]. To avoid the empirical biases of the BSM, parametirc-free pricing methods, which do not rely on restrictive parametric assumption are involved and techniques derived from computational intelligence are developed.

Parameter-free pricing methods are highly data-driven, requiring large quantities of historical prices to obtain a sufficiently well trained networks or rule-bases. According to the data used, the literature can be classified into two kinds. The first kind assumes that the BSM is the true model and uses the artificial data generated by the BSM to train and to establish a parametric-free model. Barucci et al. [2] are this type of applications. However, as mentioned above, when the assumptions behind the BSM no longer hold, it does no make too much sense to establish the parametric-free model. In this case, the second kind of applications, which is based on real data, seems to be more appropriate. Lajbcygier et al. [3] are among the few of this camp.

Base on the point of view mentioned above, this paper propose parametric-free model called ENFIS to overcome the existence of pricing biases for the BSM, such as time-horizon and option maturities included in BSM. And another important choice between historical volatility and implied volatility for BSM would both be compared with the proposed ENFIS model to show the superior in euro foreign exchange options of *CME*.

II. Black-Scholes Generalized foreign exchange options model

The BSM has led to many insights into the valuation of derivative securities. The basic variables included in the formula are (1) the current market price of the currency (S); (2) the strike price of the option (K); (3) the volatility of the stock price (σ); (4) the time of maturity (τ), and (5) the instantaneous risk-free interest rate (r_f). However, the most challenging issue lies in the choice of the mathematical closed form that can integrate these variables well together. By making six assumptions, Black and Scholes [4] provided the following formula for option pricing.

$$C_{BSG} = S\Psi(d_1) - \mathrm{K}e^{-r_f\tau}\Psi(d_2)\ ,$$

$$d_1 = \frac{\ln(S/\mathrm{K}) + \gamma_f\tau}{\sigma\sqrt{\tau}} + \frac{1}{2}\sigma\sqrt{\tau}\ ,\ d_2 = d_1 - \sigma\sqrt{\tau} \qquad (1)$$

where, C_{BSG} is the fair value of call price, and $\Psi(.)$ is the cumulative distribution function for the standard normal distribution. This formula can apply to the case of European-style call option. By its derivations is violated. The extended Black-Scholes model employed here is named Black-Scholes Generalized (BSG) [5] foreign exchange options model and we apply the FiNCAD^{TM} developer by Financialcad Corp. to implement the empirical study framework. FiNCAD^{TM} Analytics are used by over 25,000 professionals in 72 countries (http://www.fincad.com).

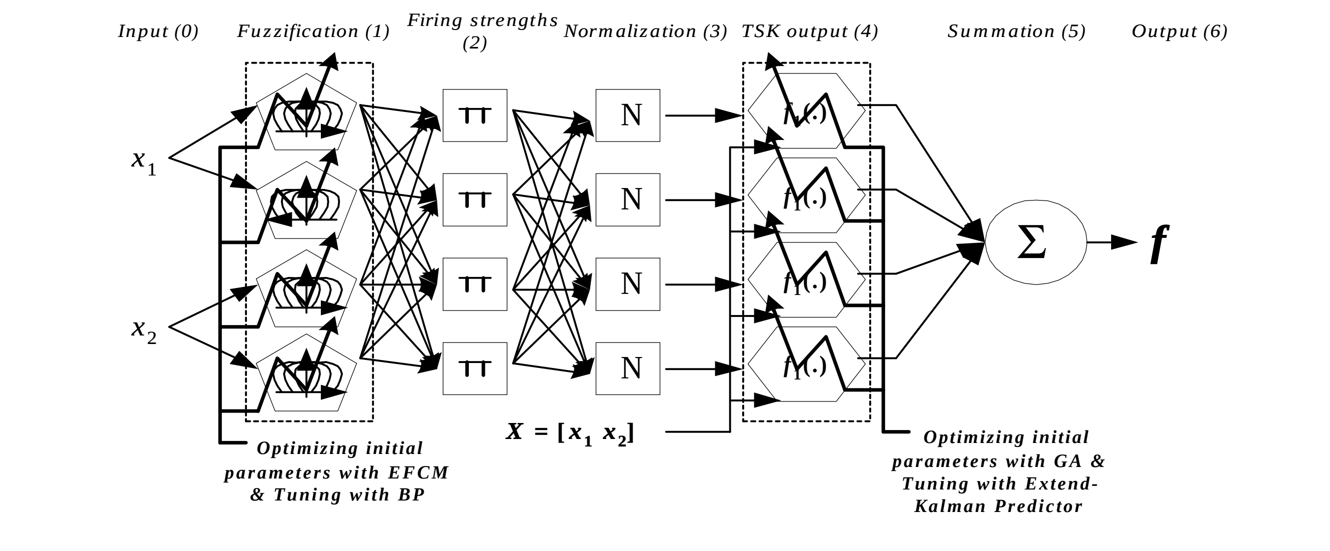

Figure 1. A 2-input, 4-rule, and 1-output ENFIS. The dash parts include premise and consequent parameters

III. Valuation with Evolutionary Neural-Fuzzy-Based Inference Systems

Over the past years, many researches in computational intelligence areas reveal that the artificial neural networks (ANNs) pertain excellent learning, high speed computing capabilities, fault-tolerance abilities and the capability of processing non-linear problems. The features of ANNs are employed to deal with the complex trading behavior phenomenon, information uncertainty and the systematic risk (unsystematic risk) that resident in the cross-country markets in this work. Base on the hierarchy of the neural networks, fuzzy inference systems under consideration in uncertainty market environment is integrated, called enhanced neural-fuzzy-based inference systems, whose initial parameters of premise universe can be adjusted systematically by enhanced fuzzy c-means clustering method (EFCM) [6] and programming initially consequence universe with genetic algorithms (GAs). ENFIS is a fist-order Sugeno model. The i^{th} IF-THEN rule of Sugeno model is:

$$R_i\text{: If } x_1 \text{ is } \tilde{A}_{i1}\ \textit{and}\ldots\textit{and}\ x_n \text{ is } \tilde{A}_{in}\ ,$$

$$\text{then } f_i = c_{i0} + c_{i1}x_1 + \ldots + c_{in}x_n . (2)$$

In *Eq*. (2), $\tilde{A}_{ij}$ is a fuzzy set and f_i is the i^{th} first-order consequent eqution. Example of two input and single output ENFIS is showed in *Fig*. 1. The fine-tune procedures of ENFIS include applying recursive least-squares estimator and steepest descent algorithms for calibrating both premise and consequent parameters iteratively. The two-phase learning starts from the consequent parameters. The updating formula for estimating consequent parameters derived from *Extented-Kalman predictor* is:

$$P^{(k+1)} = P^{(k)} - \frac{P^{(k)}a^{(k+1)}(a^{(k+1)})^T P^{(k)}}{1 + (a^{(k+1)})^T P^{(k)} a^{(k+1)}}\ ,$$

$c^{(k+1)} = c^{(k)} + P^{(k+1)}a^{(k+1)}[t^{(k+1)} - (a^{(k+1)})^T c^{(k)}]$. In last both equations, vector $\boldsymbol{c}$ contains the estimated consequent parameters, elements of vector $\boldsymbol{a}$ are the normalized firing strength of each rule multiplies its corresponding inputs, and $t^{(k+1)}$ is the target value for the $(k+1)^{th}$ training pattern. The initial conditions for this iterative process are $\boldsymbol{c}(0) = 0$ and $\boldsymbol{P}(0) = \bullet \boldsymbol{I}$, where $\boldsymbol{I}$ is an identify matrix and $\bullet$ is a large positive value.

The second stage of learning involves the reviewing premise parameters. Define the sum of squared errors for the k^{th} training-pattern as $E^{(k)} = (t^{(k)} - O_5^{(k)})^2$ and $O_5^{(k)}$ is the actual output produced by the presentation of the k^{th} pattern. For the internal node j at layer L, the error rate can be derived by chain rule: $\frac{\partial E^{(k)}}{\partial O_{L,j}} = \sum_{i=1}^{\#(L+1)} \frac{\partial E^{(k)}}{\partial O_{L+1,j}} \frac{\partial O_{L+1,j}}{\partial O_{L,j}}$, where $\#(L+1)$ is the number of nodes in layer $(L+1)$ and $O_{L,j}$ is the node output in node j of layer L. The error rate of an internal node is expressed as a linear combination of the error rates of the nodes in the next layer. If • is a parameter of the given MF, We have: $\frac{\partial E^{(k)}}{\partial \alpha} = \sum \frac{\partial E^{(k)}}{\partial O^*} \frac{\partial O^*}{\partial \alpha}$. In last equation, O is the set of nodes whose outputs are related to •. Then the derivative of the overall error measure $E = \sum_{k=1}^{P} E^{(k)}$ with respect to • is $\frac{\partial E}{\partial \alpha} = \sum_{k=1}^{P} \frac{\partial E^{(k)}}{\partial \alpha}$, where ***P*** is the number of data pairs [7].

We conclude the operations of GAs as a pseudo code:

Procedure GeNe: Begin; $e = 0$; initial population $Pc(e)$; fitness $Pc(e)$; While (termination criterion not reached); $e = e + 1$; Select $Pc(e)$ from $Pc(e\text{-}1)$; Crossover $Pc(e)$; Mutate $Pc(e)$; Fitness $Pc(t)$; End. [8]

IV. Key Factors for the inference of fair value

In this study, we apply the factors analysis to select the 19 key successful pricing factors from 42 candidate variables to compensate BSG for the lack of mathematical equations. The key factors here are classified according to three phases by their attributes, they are: *market phase*, *risk phase*, and *value phase*. These phases of premise part in Table 1 include the *buy/sell trading volume ratio*, *bid/ask price ratio*, *moneyness*, discount factor curve (primary & secondary), foreign *exchange rate*, *country risk*, *political risk*, *economic risk*, *financial risk*, *implied volatility*, *BS volatility*, *time value of options* and *intrinsic value of options*. The consequence part is the *estimated fair value* of euro foreign exchange options of CME.

V. Empirical Research Architecture

To illustrate the validity of the proposed method, some examples here are tested. The empirical models of this research contain two parts: *experimental-sets* and *comparative-sets*. The *experimental-sets* structures the option pricing model through ENFIS using various time horizon strategies and the *comparative-sets* structures the option pricing model by BSG with considering the historical volatility and implied volatility. The research data are selected from the transaction of euro foreign exchange options of the *CME* provided by Financial Futures Institution, and the time interval is from 1/1/2002 to 12/31/2005. Among the time interval, data from 1/1/2002 to 12/31/2003 should be selected to calculate the historical volatility.

ENFIS		
Premise		***Consequence***
Market Phase (9 factors): Phase I		*Fair Value* C_{ENFIS}
buy/sell trading volume ratio	*Long position/Short position*	
bid/ask price ratio	*Bid price/ask price*	
Moneyness	*S/K*	
discount factor curve	*primary*	
discount factor curve	*secondary*	
FX rate	*Euro/USD*	
domestic no-risk rate	r_d	
external no-risk rate	r_f	
strike price	K	
Risk Phase (6 factors) : Phase 2		
Country Risk	[9]	
Political Risk,	[10]	
Economic Risk	[11]	
Financial Risk	[12]	
Imp_Vol.	σ	
BS_Vol.	$C\times(0.398\times S/\sqrt{K})^{-1}$	
Value Phase (4 factors) : Phase 3		
Time_Val.	$C(S, T, K) - Max(0, S - K)$	
Intrinsic_Val.	$Max(0, S - K)$	
Time and Variant	***Chg_S, Chg_*** r_d, ***Chg_*** r_f.	
Moving average statistics	***Avg_S, Avg_*** r_d, ***Avg_*** r_f.	

Table 1. The valuation factors of ENFIS framework

In the Black-Scholes pricing model, the volatility is difficult to be estimated. Even the historical volatility could be calculated by the statistical analyze, but how many data should be involved still have not a determined answer. This research applies the suggestions of Hull [13] and takes 106 days to calculate the historical volatility. Another method to estimate the volatility is referenced the forecast of the future volatility in contracts of the markets. Because the rational volatility should be reflected in the newly contracts, therefore the reasonable volatility could be calculated by others conditions when they are considered to be known. The volatility detected by this way is called implied to be the input factors of ENFIS.

In *comparative-sets*, one is applying the historical volatility (BSG_{his}) and the other is applying the implied volatility (BSG_{imp}). The option price at the money should belong to a time series data. However, the data obtained from the experiment belong to irregular band distribution and they are not a linear time series data. In order to pertain the time series characters of the original data, this research transform the original data and describe as Table 2. On the other hand, BSG ignores that: (1) the data near to the learning period should produce better forecasting results, therefore properly separating the period should improve the overall experiment results; (2) thee-phases consideration that reflects the practical environment. Therefore, in this research, the *experimental-sets* could be divided to four subsets: module consists of phase I & phase III (*Model-I*), module consists of phase I & phase II (*Model-II*), module consists of phase II & phase III (*Model-III*) and module consists of phase I ~ phase

III (*Model-IV*). The input factors of various models for ENFIS are shown in *Table* 2, and *Fig*. 2 is the differences of time horizon strategies between *Model-II* and *Model-III*.

Original data: include call value, ***C***; exchange present price, ***S***; strike price, ***K***; domestic no-risk rate, r_d; external no-risk rate, r_f; and contract period, ***T***.

Stock price/strike price: represent the degrees of in-the–money and out-the-money.

Time and Variant: Consider time-delay data as input item and use the variation to verify the time relationship including present exchange price, ***Chg_S***; domestic instant no-risk rate, ***Chg_r***$_d$, and external instant no-risk rate, ***Chg_r***$_f$.

Moving average statistics: represent the data changed by the time scale. In this research, 3-day moving average is taken including present exchange price, ***Avg_S***; domestic instant no-risk rate, ***Avg_r***$_d$, and external instant no-risk rate, ***Avg_r***$_f$.

Contracts: To calculate the same contracts in different constrictions by historical volatility and implied volatility at the same time.

Table 2. Data pre-process stages.

We also make comparisons among these models in Table. 3 to emphasize the styles of different models. The source data come from two resources. One is from the financial markets

Model-I (Phase I& III)	***Model-II*** (Phase I& II)	***Model-III*** (Phase II & III)	***Model-IV*** (Phase I ~ III)
Long /Short position	*Long /Short position*	*Country Risk*	*Long /Short position*
Bid /ask price	*Bid /ask price*	*Political Risk,*	*Bid /ask price*
S/K	*S/K*	*Economic Risk*	*S/K*
primary	*primary*	*Financial Risk*	*primary*
secondary	*secondary*	σ	*secondary*
Euro/USD	*Euro/USD*	$C\times(0.398\times S/\sqrt{K})^{-1}$	*Euro/USD*
r_d	r_d	$C(S, T, K)$– $Max(0, S-K)$	r_d
r_f	r_f	$Max(0, S-K)$	r_f
K	K	***Chg_S***	K
$C(S, T, K)$– $Max(0, S-K)$	*Country Risk*	***Chg_r***$_d$	*Country Risk*
$Max(0, S-K)$	*Political Risk,*	***Chg_r***$_f$	*Political Risk,*
Chg_S	*Economic Risk*	***Avg_S***	*Economic Risk*
Chg_r$_d$	*Financial Risk*	***Avg_r***$_d$	*Financial Risk*
Chg_r$_f$	σ	***Avg_r***$_f$	σ
Avg_S	$C\times(0.398\times S/\sqrt{K})^{-1}$		$C\times(0.398\times S/\sqrt{K})^{-1}$
Avg_r$_d$			$C(S, T, K)$– $Max(0, S-K)$
Avg_r$_f$			$Max(0, S-K)$
			Chg_S
			Chg_r$_d$
			Chg_r$_f$
			Avg_S
			Avg_r$_d$
			Avg_r$_f$

Table 3. Input factors for *Model-I ~ Model-IV* fed into the ENFIS framework.

and the other is from the contracts. When the data are extracted, they should be pre-processed before they are sent to the models. There pre-process contains the calculations of factors in Table 2 & Table 3 and their moving averages. Except that, the historical volatility and implied volatility of currencies come from the BSG should be obtained as well. All of these data are the input data that feed in pairs to the ENFIS framework. When the ENFIS are well learned, they could be applied to estimate the options prices and hence to provide the simulated trading decision-making on-line to the traders mark-to-market.

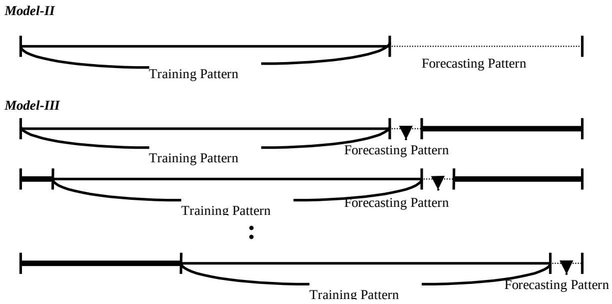

Figure 2. Time variation between *Model-II* and *Model-III*

For evaluate the performance of the pricing models, The *MAE* (mean absolute errors) criterion is selected here. The *MAE* could evaluate the error degree between estimated price and the practical price as well. $MAE = \frac{1}{n}\sum_{i=1}^{n}|S_i - R_i|$, where S_i is the actual score and R_i is the predicted score. If the *MAE* close to 0, it means the estimated price is closer to the practical (market) price.

VI. Results and Analysis

In the learning periods, the *MAE* of the *experimental-sets* is smaller then the *comparative-sets*, however, for the *Model-I ~ Model III* in *experimental-sets*, the *MAE* in forecast periods is greater than the *comparative-sets*. This phenomenon can be explained by the reason that ENFIS could not forecast precisely under part of the three-phases and also the longer periods if no suitable time horizon and rolling windows for data pre-processing (see table 4).

When the data were processed by the *experimental-sets* are both better than the *comparative-sets*. After comparing the *MAE* of different sets, it can be concluded that the *Model-IV* contains the minimum errors.

The consistent conclusion has shown in *Table*. 4 and *Table*. 5 through *MAE*. It takes 56 seconds of time costs to accomplish 1,000 epochs for each model including the learning and forecasting procedures via modern workstation architecture. The framework is coding with C language.

From the results shown in tables, whatever the evaluation parameters are taken, and the *experimental-sets* of the time separate ENFIS are the beat. Otherwise, from the empirical results, the evaluated error and the resultant variation are smaller than the pricing results from the BSG as shown in *Table*. 6.

	Comparative-Sets		***Experimental-Sets***			
	BSG_{imp}	BSG_{his}	*Model-I*	*Model-II*	*Model-III*	*Model-IV*
Learning	0.014096423	0.017474888	0.013802741	0.013473371	0.010170889	**0.00913889**
Forecasting	0.014634049	0.018071073	0.041860203	0.03510969	0.013955258	**0.01265101**

Boldface shows the minimal MAE in Table.

Table 4. MAE of models

	Model-III		***Model-IV***	
Experimental Modules	**Pattern**	***MAE***	**Pattern**	***MAE***
1	Learning	0.013802741	Learning	0.013473371
	Forecasting	0.014078862	Forecasting	0.013752437
2	Learning	0.009539333	Learning	0.007972654
	Forecasting	0.017065111	Forecasting	0.016394644
3	Learning	0.008306988	Learning	0.008178019
	Forecasting	0.015006574	Forecasting	0.01392611
4	Learning	0.011420172	Learning	0.008784754
	Forecasting	0.017573104	Forecasting	0.013517771
5	Learning	0.008359823	Learning	0.00827376
	Forecasting	0.008224631	Forecasting	0.008191653
6	Learning	0.009596288	Learning	0.008150765
	Forecasting	0.011783296	Forecasting	0.010123405

Learning: in-sample; Forecasting: out-sample

Table 5. MAE of *Model-III* and *Model-IV*

	BSG_{imp}	BSG_{his}	*Model-IV*
Learning	0.014096	0.017475	**0.010769**
Forecasting	0.014634	0.018071	**0.012822**

Boldface shows the minimal MAE in Table.

Table 6. Comparisons of *MAE* between *Experimental -sets* and *Comparative-sets*

VII. Conclusions

In this paper, we have presented ENFIS pricing model that integrates ANNs, EFCM, Extended-Kalman predictor, GAs and time horizon techniques based data pre-processing techniques. Since the rolling windows reflects the real time information through time horizon strategies and the optimized ENFIS are determined, the more precise pricing model would be obtained through structure identification skills.

From the empirical study of this research, it can be concluded that: (1) The ENFIS is more easier to extend the pricing variables; (2) The ENFIS pricing model is superior to the Black-Scholes pricing model through time horizon rolling windows based on three phases considerations and the data pre-processing strategies • (3) From the aspect of the volatility, it can be found that the historical volatility is more stable than the implied volatility in different models • (4) The initial programming for the ENFIS could optimize the structures and improve the performance for pricing problems • (5) The pre-process of the input data and suitable time horizon of rolling windows for ENFIS would effect the predict results • (6) Since the forecast data is closer to the learning period, the ENFIS pricing model would product good performance, it shows that the real time information and on-line learning is quite necessary to tune the pricing model precisely, including in-the-money, at-the-money, and out-the money. For more details about this work, please refer to [14].

Acknowledgment

This research was part supported by National Science Council, (NSC 95-2516-S-018-016-), Taiwan, R.O.C.

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