

Mathematical Analysis of the Heuristic Optimisation Mechanism of Evolutionary Programming

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Abstract: Evolutionary algorithms are robust and adaptive. They have found a wide variety of applications solving optimisation and search problems. As one of the main stream algorithms in evolutionary computation evolutionary programming (EP) mainly uses real values of parameters. This makes it very attractive for many engineering optimisation applications. In addition to the evolutionary characteristics more in depth dynamic optimisation mechanisms of EP is investigated in this paper. An optimisation model based on differential equations is developed to explore and exploit the inherent optimal operation process of EP. The proposed model is based on the characteristics of population evolution and uses two performance measures, (i) Population On-line Performance measure and (ii) Population Off-line Performance measure. These two measures are used to quantify the dynamic population optimisation of EP and to form the foundation for the construction of the differential equation based optimisation model. The model is proposed with strict theoretical and numerical analysis. A number of important conclusions and observations are presented in accordance with the analytical results.

Keywords: Evolutionary programming; Population on-line performance; Population off-line performance; Differential equation; Integral curve

I. Introduction

Inspired by the neo-Darwinian paradigm on evolutionary theory, evolutionary computation algorithms were proposed as early as the late 1950's. The main stream algorithms for evolutionary computation are genetic algorithms (GAs),

evolutionary programming (EP), evolution strategies (ES) and genetic programming (GP) [1-8]. These algorithms have been recognized as important mathematic tools in solving NP-hard combinatorial and continuous optimization problems. Evolutionary algorithms possess the following salient characteristics of the Darwinian evolution: (a) Genetic variation is largely a chance phenomenon and stochastic processes play a significant role in evolution; (b) A population of agents with non-deterministic recruitment is used; (c) Inherent parallel search mechanisms are used during evolution; (d) Evolution is a change in adaptation and diversity, not merely a change in gene frequencies; and (e) the evolutionary algorithms operate with a mechanism of competition-cooperation. These algorithms simulate the principle of evolution (a two step process of variation and selection), and maintain a population of potential solutions (individuals) through repeated application of some evolutionary operators such as mutation and crossover. They yield individuals with successively improved fitness, and converge, hopefully, to the fittest individuals representing the optimum solutions. The evolutionary algorithms can avoid premature entrapment in local optima because of the stochastic search mechanism.

GA and EP are among the two most widely used evolutionary computation algorithms. EP was developed initially different from the basic GA in two main aspects. (i) the authors of EP felt that their representations (whether real or binary) represented phenotypic behaviour whereas the authors of GA felt that their representations represented genotypic traits; and (ii) EP depends more on mutation and

selection operations while GA mainly relies on crossover. It is noted that, given the wide availability and development in encoding/decoding techniques for GA, the first difference between the two algorithms is diminishing. However, the inherent characteristics of EP have made it a widely practiced evolutionary computation algorithm in many applications; especially where the search diversity is a key concern in the optimisation process.

As one of branches of evolutionary algorithms, ES and EP share many common features, i.e. the real-valued representation of search points, emphasis on the utilization of normally distributed random mutations as main search operator, and most importantly, the concept of self-adaptation of strategy parameters on-line during the search. There exist, however, some striking differences, such as the specific representation of mutation, most notably the missing recombination operator in EP and the softer, probabilistic selection mechanism used in EP. And the combination of these properties seems to have some negative impact on the performance of EP. In this paper, we focus our research in the dynamic optimisation mechanisms of EP only.

As a powerful and general global optimization tool, EP seeks the optimal solution by evolving a population of candidate solutions over a number of generations or iterations. A new population is generated from an existing population through the use of a mutation operator. This operator perturbs each component of every solution in the population by a Gaussian random variable \mathbf{d} with zero mean and preselects variance \mathbf{s}^2 to produce new ones. A mutation operator with high efficiency should fully reflect the principle of organic evolution in nature, i.e. the lower fitness score is the higher the mutation possibility is; and vice versa. Through the use of a competition scheme, the individuals in each population compete with each other. The winning individuals will form a resultant population which is regarded as the next generation. For optimisation to occur, the competition scheme must ensure that the more optimal solutions have a greater chance of survival than the poorer solutions. Through this process, population is expected to evolve towards the global optimum.

It is known that there are further researches needed in the mathematical foundation for the EP or its variants with regard to its much experimental and empirical research. The state-of-the-art of EP mainly focuses on the application of solving optimization problems, especially for the application to real-valued function optimization [9-14]. So far, there is very little theoretical research available in explaining the mechanisms of the successful search capabilities of EP or its variants to the best of the authors' knowledge even though some convergence proofs [2,5,15] with certain assumptions for the EP have been carried out with varying degrees of success in the past few years.

We introduce a differential equation model in this paper. This model can be used to reveal the inherent operating mechanism of EP, to explore and exploit its dynamic

optimization process in essence, and to explain the efficiency and validity of such a stochastic search technique as a whole. The objective of this investigation is to provide theoretical foundations for efficiently controlling the optimisation process so to improve the operating effectiveness of EP. As prior work, two performance measures termed as Population On-line Performance and Population Off-line Performance measures are firstly defined in this paper. Based on these two measurements, two differential equations representing the evolution of the population on-line and off-line performances in a finite population size are developed according to the fusion of a large number of numerical case studies and the performance measures defined. Ultimately, we can obtain the analytical solution via solving the proposed differential equation model. The corresponding analysis and discussion with respect to the integral curve are also presented based on the mathematical theory. The correctness of the developed model is verified through a typical mathematical test function case study. Some suggestions and comments can be made according to the analysis results.

The paper is organized as follows. Section II gives a short introduction into the key ideas of EP. In Section III, we give the definition of Population On-line Performance and Population Off-line Performance measures in detail. The construction and solution of the evolving population differential equation model are presented in Section IV. Section V provides the corresponding case study. Conclusions are given in Section VI before the References section.

II. Fundamental Principles of Evolutionary Programming

EP was first proposed by Fogel, Owens and Walsh [6]. EP is similar to GAs in principle. It works on a population of trial solutions, imposes random changes to those solutions to create offspring and incorporates the use of selection to determine which solutions to maintain into future generations and which to remove from the pool of trials. However in contrast to GAs, the individual component of a trial solution in EP is viewed as a behavioural trait, not as a gene. In other words, EP emphasizes the behavioural link between parents and offspring rather than the genetic link. It is assumed whatever genetic transformations occur, the resulting change in each behavioural trait will follow a Gaussian distribution with zero mean difference and some standard deviation. When applied to a minimization process, EP can be described as follows.

(1) Represent the problem variables to be determined as a n -dimensional trial vector \mathbf{S} , where each vector \mathbf{S} is an individual of the population to be evolved. Mathematically, it can be expressed as

$$\mathbf{S} = [s_1, s_2, \dots, s_k, \dots, s_n], \quad \underline{s}_k \leq s_k \leq \bar{s}_k$$

where s_k is the k th component in vector \mathbf{S} ; \underline{s}_k and \bar{s}_k are the lower and upper limits of the k th component s_k .

(2) Randomly choose an initial population of parent vectors \mathbf{S}_i for $i = 1, 2, \dots, P$, where P is the population size, from a feasible range in each dimension. The distribution of these initial parent vectors is typically uniform.

(3) Each \mathbf{S}_i for $i = 1, 2, \dots, P$, is assigned a fitness score f_i . $f_i = F(\mathbf{S}_i)$, where $F: \mathbf{S}_i \rightarrow \mathfrak{R}$. F can be as complex as required and is usually regarded as an objective function.

(4) Mutation operation to generate an offspring vector \mathbf{S}_{i+P} , $i = 1, 2, \dots, P$, from each parent vector \mathbf{S}_i by adding a Gaussian random variable with zero mean and preselected variance to each component of vector \mathbf{S}_i . The population size becomes $2P$ after mutation operation. This process is described as follows.

$$\mathbf{S}_{i+P,k} = \mathbf{S}_{i,k} + N(0, \mathbf{s}^2) \quad \forall k=1, 2, \dots, n$$

where $N(0, \mathbf{s}^2)$ represents a Gaussian random variable with zero mean and variance \mathbf{s}^2 .

Evaluate function $f(\mathbf{S}_{i+P})$ for $i = 1, 2, \dots, P$.

(5) Based on the mutated population with the size of $2P$, a competition is conducted to reproduce the offspring. For each \mathbf{S}_i , $i=1, 2, \dots, 2P$, a value w_i is assigned to weight the individual according to the following two equations:

$$w_i = \sum_{c=1}^C w_c^*$$

$$w_c^* = \begin{cases} 1, & \text{if } f_i \leq f_r \\ 0, & \text{otherwise} \end{cases}$$

where C is the number of competitors; $r = \text{int}(2Pu_1+1)$, $r \neq i$, where $\text{int}(x)$ denotes the greatest integer less than x , and $u_1 \sim U(0,1)$.

(6) The individuals \mathbf{S}_i , $i = 1, 2, \dots, 2P$, are ranked in descending order of their corresponding value w_i . The first P individuals are transcribed along with their corresponding fitnesses f_i to form the basis of the next generation.

(7) Return to step (4) and carry out the process repeatedly until the given termination conditions are satisfied.

It is noticed that as mentioned earlier, EP mimics the survival-of-the-fittest principle of nature to conduct a search process. Therefore, EP is naturally suited for solving maximization problems. Minimization problems are usually transformed into maximization problems by a suitable transformation. There are many ways to convert a minimization problem to a maximization problem and vice-versa, such as (i) *C-objective*; (ii) *1/objective*; (iii) *-objective*. As for the fitness function, in general, it is first derived from the objective function and used in successive genetic operations. There are a lot of means to construct the fitness function as well. For the maximization problem, the fitness is simply equal to the value of objective function. For the minimization problem, the fitness can be the reciprocal of the value of objective function.

The implementation procedure of an EP is described briefly in Pascal as follows:

Procedure Evolutionary Programming

Begin

 Initialize;

 Set $k := 0$; (k is the number of individuals)

 Set $r := 0$; (r is the generations)

Repeat

$k := k+1$;

 Mutation;

 Competition;

 Selection;

 Until ($k == P$) or ($r == \text{maxConverg}$);

• End

III. Population On-line and Population Off-line Performances

De Jong proposed two performance measures to quantitatively evaluate GAs in his Ph.D. dissertation in 1975 [16]. The two performance measures are termed as standard online and offline performances as shown in Eqs. (1) and (2).

$$\text{online}(T) = \frac{1}{T} \sum_{t=1}^T \text{mean}(t) \quad (1)$$

$$\text{offline}(T) = \frac{1}{T} \sum_{t=1}^T \text{best}(t) \quad (2)$$

where $\text{mean}(t)$ is the mean fitness of the population at generation t , and $\text{best}(t)$ is the fitness of the fittest individual of the population at generation t . In other words, the online performance is the average fitness of all trials up to the current trial inclusively and the offline is an average of the fitness of the best individuals from that strategy up to and including the current trial. De Jong devised these two standard performance measures for the first time [16]. The offline performance is to gauge convergence and the online one is to gauge ongoing performance.

Inspired by the work described above under certain conditions, two performance measures based on EP algorithm to evaluate the dynamic optimization process in different senses are defined in this paper. They are termed as Population On-line and Off-line Performances.

Definition 3.1. Let a given environment be the fitness function f , the Population On-line performance $U_f(n)$ based on EP algorithm is defined as

$$U_f(n) = \frac{1}{P} \sum_{p=1}^P u_f^n(p) \quad n = 0, 1, \dots, T-1 \quad (3)$$

where P is the population size; T is the number of the total generations; $u_f^n(p)$ is the fitness value of the p -th individual at generation n .

Eq. (3) states that the Population On-line performance of a strategy on a given environment such as fitness function is the average fitness of the current population. Population On-line performance is a measure designed to determine the ability of a genetic system (such as EP) to perform well in

on-line applications. The major requirement of on-line applications is that the adaptive system is expected to find a 'satisfactory' solution rapidly. When a solution is found, the system may (or may not) continue its search for superior solutions. The main characteristic of an on-line application is that learning is conducted 'on-line'. In other words, the system is trained dynamically on the job. It represents the on-going status for the job.

Definition 3.2. Let a given environment be the fitness function f , the Population Off-line performance $U_f^*(n)$ based on EP algorithm is defined as

$$U_f^*(n) = \max \{u_f^n(1), u_f^n(2), \dots, u_f^n(p), \dots, u_f^n(P)\} \quad (4)$$

$$n = 0, 1, \dots, T-1$$

From Eq. (4), the Population Off-line performance represents the fitness of the best individual of the current population. Population Off-line performance is a measure of convergence. It is intended to indicate the expected performance of a genetic system's ability when applied in off-line applications. As the name implies, in off-line learning there is no expectation of the system to learn rapidly. Rather, the system conducts its learning until reaching some termination criterion, independent of the quality of found solutions. In an off-line learning environment, the system is afforded the luxury of saving the best solutions found currently, so that they can be produced later upon satisfaction of its termination criterion.

IV. Evolving Population Differential Equation Model

A. Population Difference Equations

As mentioned earlier, mutation is the primary performance operator in EP. It is known that the evolving process for each individual \mathbf{S} can be described by Eq. (5).

$$\mathbf{S}_i(t+1) = \mathbf{S}_i(t) + \mathbf{d} \quad (\mathbf{d} \sim N(0, \mathbf{S}^2))$$

$$i = 1, 2, \dots, P; t = 0, 1, \dots, T-1 \quad (5)$$

where the \mathbf{S}^2 variance indicates that the range of the offspring is created around the parent trial solution.

In this paper, our research in EP mainly focuses on the population or species behaviour. The two performance measures: Population On-line and Off-line performances will be the highlight of our specific concern. As to mutation, competition and reproduction operations used in EP, we just consider their comprehensive influences on the evolving population holistically. In order to easily understand and analyse the model to be constructed, some reasonable simplifications and assumptions are required. Seeing that Population On-line and Off-line performances are discrete during evolution of population, an approximate method treating the discrete variables as continuous with respect to time t is introduced in this paper. It is noted that this

assumption is valid given that many optimisation problems require a considerable number of iterations with EP; and the differences in these two performance measures are small enough in between two consecutive iterations. Consequently the Population On-line and Off-line performances can be represented as the function of time t . Four assumptions are presented as follows accordingly.

(i) Transfer the constrained optimization problem to the non-constrained optimization problem using appropriate methodology;

(ii) Let the preselected fitness function be positive, i.e. the fitness score is greater than zero during generations via appropriate transformation;

(iii) Elitist selection strategy [1,17] is applied to keep the best individual in each generation during evolution;

(iv) The final solution to a certain optimization problem using EP is not always a global optimum.

According to Eq. (5) and the meanings inspired by the defined Population On-line and Off-line performances as well as the statistics analysis from a large number of numerical simulations (the approximate optimization curves are shown in Fig. 1), we propose the evolving population first-order difference equations in Eqs (6) and (7). These equations can be used to simulate the evolution of the Population On-line and Off-line performances based on a finite population size. The model is derived with the premise of certain assumptions mentioned earlier in this section.

$$U_f(t + \Delta t) = U_f(t) + U_f(t) \cdot \mathbf{M} \cdot \left(1 - \frac{U_f(t)}{U_f^*(t)}\right) \cdot \Delta t \quad (6)$$

$$U_f^*(t + \Delta t) = U_f^*(t) + U_f^*(t) \cdot \mathbf{M}^* \cdot \left(1 - \frac{U_f^*(t)}{U_f^0}\right) \cdot \Delta t \quad (7)$$

where $U_f(t)$ and $U_f^*(t)$ are Population On-line and Off-line performances at time t , and $0 < U_f(t) \leq U_f^*(t)$; U_f^0 is the best fitness. \mathbf{M} ($0 < \mathbf{M} < 1$) and \mathbf{M}^* ($0 < \mathbf{M}^* < 1$) are constant coefficients represented the comprehensive influences on Population On-line and Off-line performances due to the operations of mutation and selection, and $\mathbf{M} < \mathbf{M}^*$ (to be explained in more detail in the later sections of the paper).

In Eqs. (6) and (7), we let $\overline{\mathbf{M}} = \mathbf{M} \cdot \left(1 - \frac{U_f(t)}{U_f^*(t)}\right)$ and $\overline{\mathbf{M}}^* = \mathbf{M}^* \cdot \left(1 - \frac{U_f^*(t)}{U_f^0}\right)$; then $\overline{\mathbf{M}}$ and $\overline{\mathbf{M}}^*$ can be regarded as the dynamic comprehensive influence coefficients. These coefficients are called dynamic coefficients because $\overline{\mathbf{M}}$ and $\overline{\mathbf{M}}^*$ are functions of time t . Especially, when $U_f(t) \rightarrow U_f^*(t)$ and $U_f^*(t) \rightarrow U_f^0$, we have $\overline{\mathbf{M}} \rightarrow 0$

and $\overline{M}^* \rightarrow 0$. It shows that the influences exerted by mutation and selection operations on the dynamic optimization performance will gradually decrease with increasing of generation numbers. This process tallies with the principle of organic evolution in nature as we have mentioned earlier.

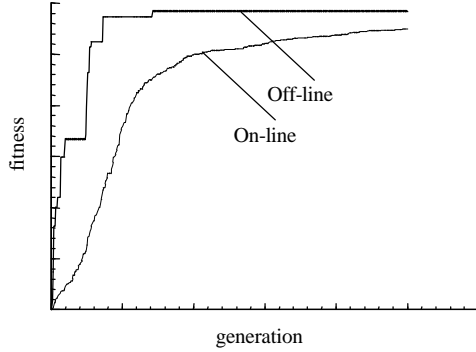


Figure 1. Optimisation curves depicting the population on-line and population off-line performance measures

B. Evolving Population Differential Equation Model

According to the theory of limits, for Δt , $\Delta U_f(t)$ and $\Delta U_f^*(t)$ in Eqs. (6) and (7), we have

$$\Delta t \rightarrow dt$$

$$\Delta U_f(t) \rightarrow dU_f(t)$$

$$\Delta U_f^*(t) \rightarrow dU_f^*(t)$$

Subsequently, the evolving population differential equation model can be constructed as follows.

$$\left\{ \begin{array}{l} \frac{dU_f(t)}{dt} = U_f(t) \cdot M \cdot \left(1 - \frac{U_f(t)}{U_f^*(t)} \right) \end{array} \right. \quad (8.a)$$

$$\left\{ \begin{array}{l} \frac{dU_f^*(t)}{dt} = U_f^*(t) \cdot M^* \cdot \left(1 - \frac{U_f^*(t)}{U_f^0} \right) \end{array} \right. \quad (8.b)$$

with the following initial conditions

$$\left\{ \begin{array}{l} 0 < M < M^* < 1 \\ 0 < U_f(t) \leq U_f^*(t) \\ U_f(t)|_{t=0} = U_f(0) \\ U_f^*(t)|_{t=0} = U_f^*(0) \end{array} \right. \quad (9)$$

where $U_f(0)$ and $U_f^*(0)$ are random initial values.

The differential equations in (8.a) and (8.b) are the first-order nonlinear Ordinary Differential Equations (ODEs). In order to obtain the solution of the differential equations, we can transfer the nonlinear ODEs to the form of Bernoulli Equation [18]. The nonlinear ODEs can then be solved using Bernoulli's method. The Eq. (8.b) would be solved first to get the solution curve of Population Off-line performance $U_f^*(t)$. Thereafter the solution curve of Population On-line performance $U_f(t)$ can be obtained by solving Eq. (8.a) based on the solution of Eq. (8.b).

The analytical solution of the ODEs (8.a) and (8.b) are given in Eqs. (10) – (11).

$$U_f^*(t) = \frac{U_f^0}{1 + \left(\frac{U_f^0}{U_f^*(0)} - 1 \right) \cdot e^{-M^* \cdot t}} \quad (10)$$

$$U_f(t) = \left\{ \begin{array}{l} \left[\frac{1}{U_f(0)} - \frac{M \cdot U_f^0 - M^* \cdot U_f^*(0)}{U_f^0 \cdot U_f^*(0) \cdot (M - M^*)} \right] \cdot e^{-M \cdot t} \\ + \frac{M \cdot (U_f^0 - U_f^*(0))}{U_f^0 \cdot U_f^*(0) \cdot (M - M^*)} \cdot e^{-M^* \cdot t} + \frac{1}{U_f^0} \end{array} \right\}^{-1} \quad (11)$$

C. Discussion of Solution

From Eqs. (8.a), (8.b), (10) and (11), some analyses and discussions are presented as follows.

(1) According to the solution in Eqs. (10) and (11), when $t \rightarrow T$, the $U_f(t)$ and $U_f^*(t)$ will hold that $U_f(t) \rightarrow U_f^0$ and $U_f^*(t) \rightarrow U_f^0$. Therefore we can draw a conclusion that no matter what initial values, the search using EP will converge to the global or local optimum within certain generations.

(2) For the Population On-line and Off-line performances $U_f(t)$ and $U_f^*(t)$, the following holds s.t. $0 < M < M^* < 1$:

$$U_f(t) < U_f^*(t)$$

Proof: In accordance with the conclusion $U_f(t) < U_f^*(t)$, we can deduce the following inequality

$$\frac{U_f(0) \cdot (M \cdot U_f^0 - M^* \cdot U_f^*(0)) - U_f^0 \cdot U_f^*(0) \cdot (M - M^*)}{U_f(0) \cdot (M - M^*) \cdot U_f^0 \cdot U_f^*(0)} \cdot e^{-M \cdot t} < \frac{U_f(0) \cdot (U_f^0 - U_f^*(0)) \cdot M^*}{U_f(0) \cdot (M - M^*) \cdot U_f^0 \cdot U_f^*(0)} \cdot e^{-M^* \cdot t} \quad (12)$$

Accordingly, the proof to (2) is transferred to the proof to Eq.(12). The corresponding procedures are as follows.

Consider the following given conditions:

$$U_f^0 > U_f^*(0) > U_f(0) > 0 \text{ and } 0 < M < M^* < 1$$

we have

$$U_f(0) \cdot U_f^0 (M - M^*) > U_f^0 \cdot U_f^*(0) (M - M^*)$$

Re-arranging the equation above, we can get

$$U_f(0) \cdot U_f^0 \cdot M - U_f^0 \cdot U_f^*(0) (M - M^*) > U_f(0) \cdot U_f^0 \cdot M^*$$

Subtracting equation above by $U_f(0)M^*U_f^*(0)$

throughout, we have

$$\begin{aligned} &U_f(0) (M \cdot U_f^0 - M^* \cdot U_f^*(0)) - U_f^0 \cdot U_f^*(0) (M - M^*) \\ &> U_f(0) (U_f^0 - U_f^*(0)) M^* \end{aligned}$$

Multiplying equation above by e^{-Mt} throughout, we have

$$\begin{aligned} &\left[U_f(0) (M \cdot U_f^0 - M^* \cdot U_f^*(0)) - U_f^0 \cdot U_f^*(0) (M - M^*) \right] \cdot e^{-Mt} \\ &> U_f(0) (U_f^0 - U_f^*(0)) M^* \cdot e^{-Mt} \end{aligned}$$

Since $M < M^*$, the following holds:

$$\begin{aligned} &U_f(0) (U_f^0 - U_f^*(0)) M^* \cdot e^{-Mt} \\ &> U_f(0) (U_f^0 - U_f^*(0)) M^* \cdot e^{-M^*t} \end{aligned}$$

Finally, we have

$$\begin{aligned} &\frac{U_f(0) \cdot (M \cdot U_f^0 - M^* \cdot U_f^*(0)) - U_f^0 \cdot U_f^*(0) \cdot (M - M^*)}{U_f(0) \cdot (M - M^*) \cdot U_f^0 \cdot U_f^*(0)} \cdot e^{-Mt} \\ &< \frac{U_f(0) \cdot (U_f^0 - U_f^*(0)) \cdot M^*}{U_f(0) \cdot (M - M^*) \cdot U_f^0 \cdot U_f^*(0)} \cdot e^{-M^*t} \end{aligned}$$

i.e.

$$U_f^*(t) > U_f(t)$$

According to the meanings of Population On-line and Off-line performances, the conclusion is reasonable and in accordance with the characteristics of a real evolution process.

(3) According to Eqs. (10) and (11), when $0 < M < M^* < 1$ and $0 < U_f(t) \leq U_f^*(t)$, we have

$$\frac{dU_f^*(t)}{dt} = (U_f^*(t))^2 \cdot M^* \cdot \left(\frac{1}{U_f^*(t)} - \frac{1}{U_f^0} \right) > 0 \quad (13)$$

$$\frac{dU_f(t)}{dt} = (U_f(t))^2 \cdot M \cdot \left(\frac{1}{U_f(t)} - \frac{1}{U_f^*(t)} \right) > 0 \quad (14)$$

From Eqs. (13) and (14), it can be seen that $U_f(t)$ and $U_f^*(t)$ are all monotonically increasing functions of time t . On the one hand, it is in accordance with the characteristics of the real evolution process. On the other hand, it can guarantee a good searching behaviour.

(4) For the Population Off-line performance $U_f^*(t)$, we take the second derivative with respect to time t as in Eq. (13).

$$\begin{aligned} \frac{d^2U_f^*(t)}{dt^2} &= M^* \cdot \left(\frac{U_f^0 - U_f^*(t)}{U_f^0} \right) \cdot \frac{dU_f^*(t)}{dt} \\ &+ \frac{M^* \cdot U_f^*(t)}{U_f^0} \cdot \left(-\frac{dU_f^*(t)}{dt} \right) \\ &= \frac{dU_f^*(t)}{dt} \cdot \left(M^* - \frac{2M^* \cdot U_f^*(t)}{U_f^0} \right) \end{aligned} \quad (13)$$

Let $\frac{d^2U_f^*(t)}{dt^2} = 0$. Consider $\frac{dU_f^*(t)}{dt} \neq 0$, the following holds:

$$M^* - \frac{2M^* \cdot U_f^*(t)}{U_f^0} = 0$$

then we have

$$U_f^*(t) = \frac{U_f^0}{2} \quad (14)$$

Substituting $U_f^*(t)$ into Eq. (10) yields

$$U_f^*(t) = \frac{U_f^0}{2} = \frac{U_f^0}{1 + \left(\frac{U_f^0}{U_f^*(0)} - 1 \right) \cdot e^{-M^*t}} \quad (15)$$

$$t = -\frac{1}{M^*} \cdot \ln \left(\frac{U_f^*(0)}{U_f^0 - U_f^*(0)} \right) \quad (16)$$

It can be seen that when $U_f^*(t) = \frac{U_f^0}{2}$ and $t = -\frac{1}{M^*} \cdot \ln\left(\frac{U_f^*(0)}{U_f^0 - U_f^*(0)}\right)$, there exists a point of inflexion on the solution curve of Population Off-line performance. Especially, when $U_f^*(t) < \frac{U_f^0}{2}$, the convergence rate of the EP represented by Population Off-line performance is faster. When $U_f^*(t) > \frac{U_f^0}{2}$, the convergence rate, however, becomes slower. The main reason for such situation is that the probability of mutation becomes lower due to the stronger adaptive capacity of the population to the current environment at that time. Accordingly, we can draw a conclusion that the convergence rate will be improved with a good selection of initial population.

(5) The random initial values $U_f(0)$ and $U_f^*(0)$ have a certain influence on the convergence rate and global optimization capability avoiding search from being trapped into the local optimum.

The corresponding solution curves are shown in Fig. 2. Furthermore, the influences exerted by the comprehensive influence factors M and M^* on the Population On-line and Off-line performances are shown in Fig. 2 as well. It can be seen from Fig. 2 that the solution curves will go from flat to steep as the factors M and M^* increase. The probability of converging to the local optimum is a little bit higher even though it helps to accelerate the process of convergence. The following case study can illustrate the issues described above.

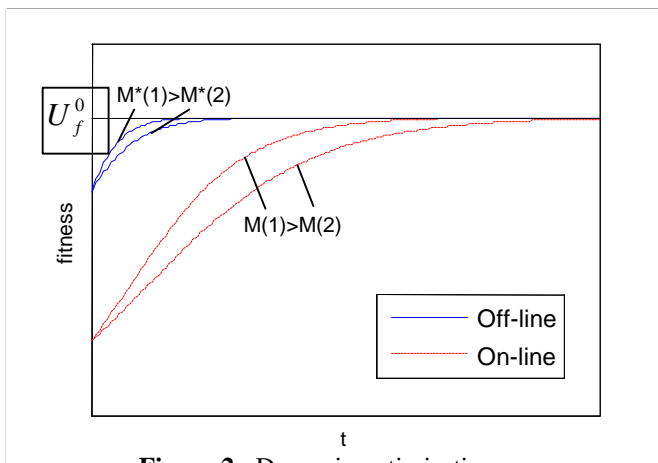


Figure 2. Dynamic optimisation curves

V. Experimental Study

We have applied EP to a series of optimization problems. For convenience of illustration, the following problem [19] is

selected as a bench mark example to report the discoveries from the experimental results.

$$\text{global max}_{x \in [0.01, 1]} F(x) = 10 + \frac{\sin\left(\frac{1}{x}\right)}{(x - 0.16)^2 + 0.1}$$

As shown in Fig. 3, $F(x)$ has a unique global optimal point $x^* = 0.1275$ (the corresponding optimal value is 19.8949) and infinite number of local maxima in $(0, 1]$. We notice that the local maximum $x = 0.4621$ has the largest attractive basin.

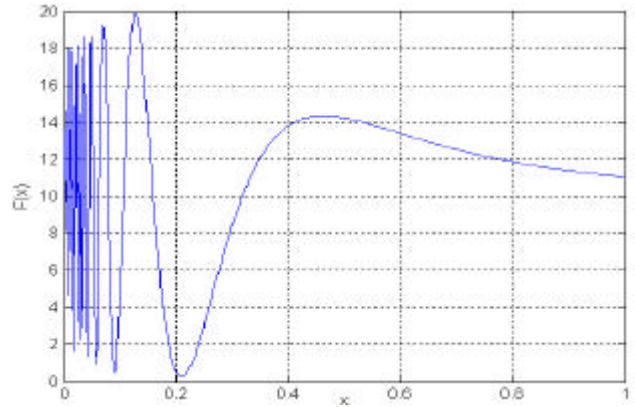


Figure 3. Functional curve

The parameters of EP used are chosen to be:

- Population size: $P = 20$;
- Total generations: $T = 30$;
- Selection method: Competition;
- Fitness function: $f(F(x)) = F(x)$;

The variance s^2 in mutation: $s^2 = b \cdot \frac{f_i(F(x))}{f_\Sigma}$, where

b is a constant and $0 < b < 1$; f_Σ is the summation of fitness scores of all individuals in the current population. Parameter b represents the influences on the Population On-line and Off-line performances exerted by M and M^* .

To illustrate the reliability and stability of EP, 100 trials were performed based on the MATLAB[®] [19] environment applied to the test system. The simulation results are shown in Fig. 4, Fig.5 and Fig.6. Fig. 4 shows the Population On-line and Off-line performances with different \hat{a} (0.1, 0.55 and 0.85) with respect to the typical three different initial populations. Furthermore, we find the global optimum under the three different initial populations.

Fig. 5 shows the histogram of the location of the solutions with different \hat{a} (0.1, 0.55 and 0.85) under the 100 trials. Fig. 6 shows the dynamic optimization curves while obtaining the global optimum and one of the local optima ($x = 0.4621$) as \hat{a} is equal to 0.1.

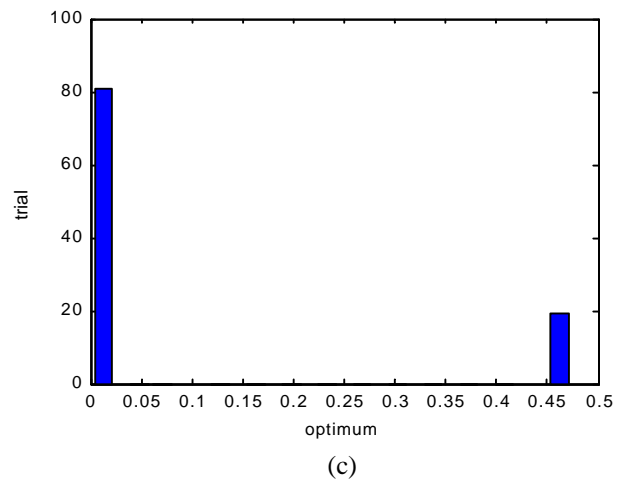
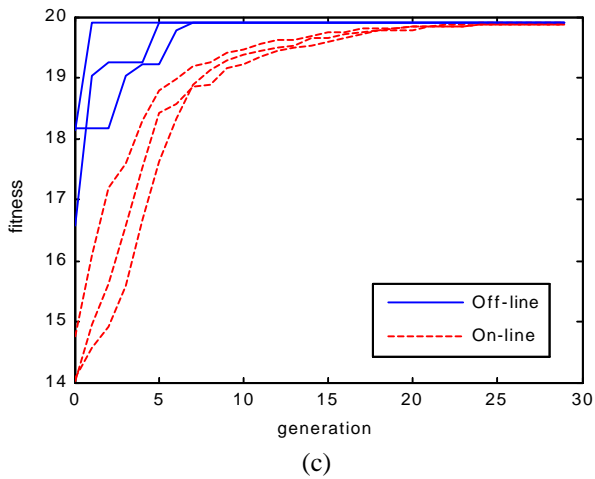
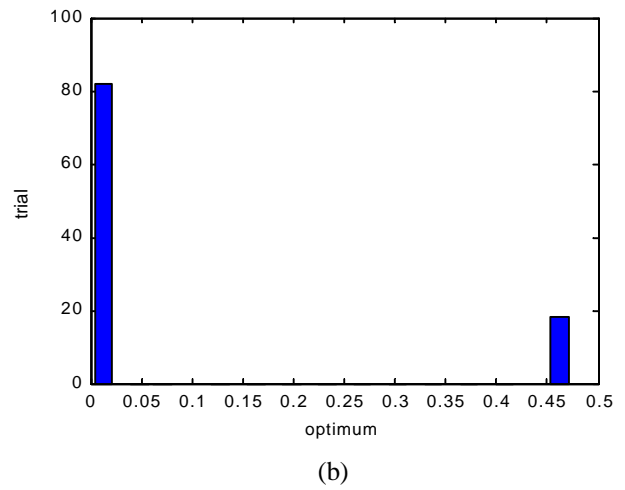
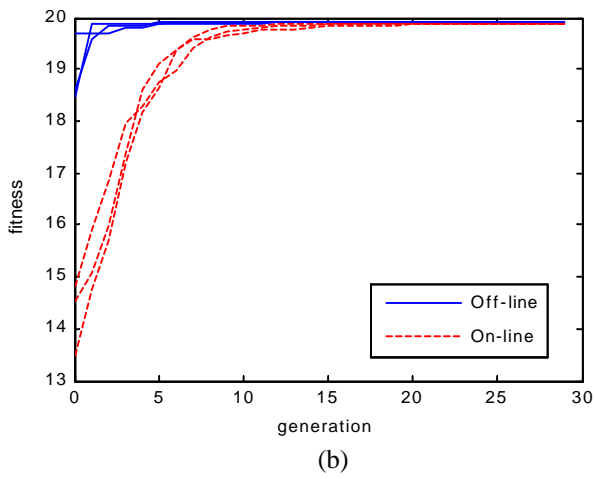
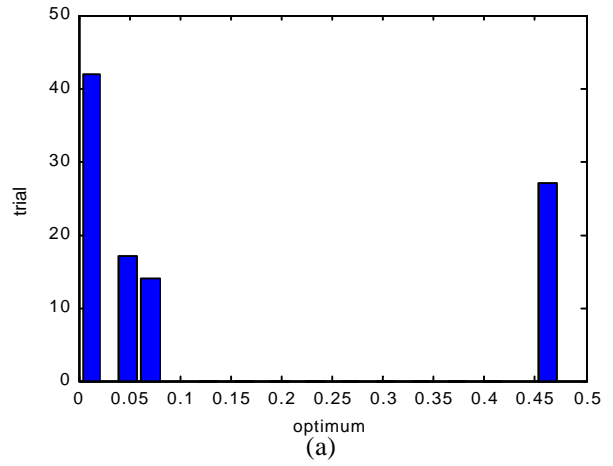
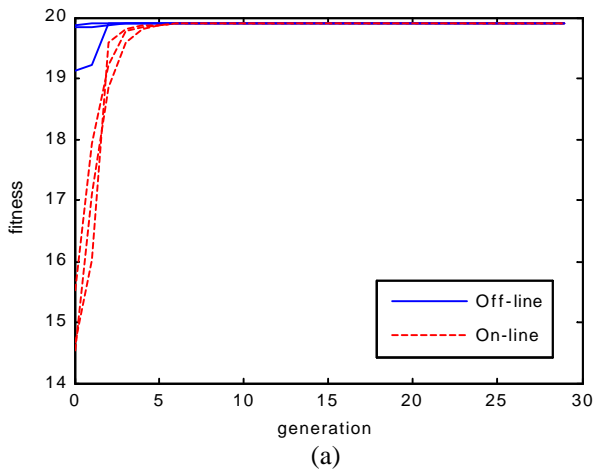


Figure 4. Dynamic optimization curves with different \hat{a} corresponding to three different initial population. (a) $\hat{a}=0.1$. (b) $\hat{a}=0.55$. (c) $\hat{a}=0.85$.

Figure 5. The histogram of the location of the best individual with different \hat{a} . (a) $\hat{a}=0.1$. (b) $\hat{a}=0.55$. (c) $\hat{a}=0.85$.

Let \bar{n} be the convergence rate of EP. Along points of the dynamic optimization curves, it represents the degree of flatness and steepness for the Population On-line performance curve. The flatter the curve is, the slower the convergence; the steeper the curve is, the faster the convergence, and vice versa. From the figures, we can see that $\bar{n}_{b=0.1} > \bar{n}_{b=0.55} > \bar{n}_{b=0.85}$.

It indicates the influences on the computational efficiency based on the parameter \hat{a} . On the other hand, it implies the comprehensive influences on the Population On-line and Off-line performances due to the operations of selection and mutation. From Fig. 5, it can be seen that about 40% solutions are optimal as \hat{a} is equal to 0.1. When \hat{a} is equal to 0.8 or 0.85, about 80% solutions are optimal. It shows that parameter \hat{a} has great influence on the global convergence of EP. From Fig.6, obviously the initial Population On-line performance with respect to the local optimum is lower than that of with respect to the global optimum. In some senses, it represents the influence on the global optimization capability based on the selection of initial population.

According to the simulations, it can be seen clearly that the mathematical analysis for heuristic optimization mechanism of EP closely resembles the real dynamic optimization process in EP. The choices of initial population, selection and mutation operations have great influences on the convergence rate, the global convergence capability, and avoiding premature convergence. It should be noted that the mutation operator will greatly affect the dynamic search process of EP to converge to the global optimum.

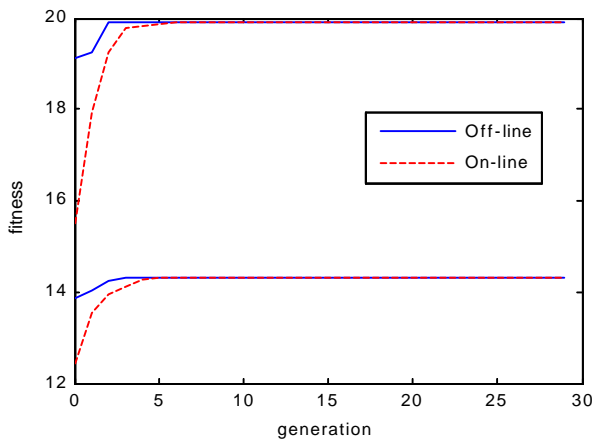


Figure 6. Dynamic optimisation curves corresponding to global and local optimum

VI. Conclusions

Evolutionary programming is one of the main lines of evolutionary computation algorithms which have a long history of research and applications. However there are limited mathematical researches on the dynamic

characteristics of the EP optimisation process. The development of mathematical foundations for EP with respect to its applications and improvements are of significant interests in evolutionary computation research. In this paper, two performance measures: Population On-line and Off-line performances are proposed by the authors to form foundation of a dynamic model for EP optimisation process. The former measure indicates the dynamic optimization performance of EP. The latter is the convergence performance measure of EP. A differential equation based model which can be used to reveal the dynamic optimization mechanism of EP is proposed in the paper. The idea of the proposed model is based on observations for the dynamic optimization process based on a large number of simulations and the definition as well as physical meanings of the two measures as well. A case study is employed to illustrate the dynamic optimization process with the effects of choice of initial population, selection and mutation operations. It also demonstrates the correctness and effectiveness of the proposed model.

In some senses, the presentation of the model gives the corresponding theoretical foundations for how to apply EP to solve the optimization problem with higher efficiency and to avoid improving EP in a random way. It also provides a feasible way to systematically explore and exploit the inherent optimization essence of EP. Some meaningful conclusions and comments are presented in the paper. It is noted that there is not a single model which can provide a complete description of a true physical system. Every model has its pros and cons including the proposed model in this paper. The model presented in this paper fulfils the objectives of developing such a model which is to reveal the physical meanings of optimization process in EP. Future work is under way by the authors to further enhance the model proposed.

Acknowledgment

General support from the Department of Electrical Engineering, Hong Kong Polytechnic University is gratefully acknowledged from the 2nd author.

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