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Access Analysis of Secondary Users with an Opportunistic perspective in Cognitive Radio Networks

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Abstract

This paper analyses opportunistic access of secondary user in cognitive radio network (CRN) using analytical M/D/1 queuing model. Through using spectrum sharing, secondary user opportunistically share primary user spectrum when it empty. Through spectrum sharing primary user generates revenue and secondary user maximizes its satisfaction with spectrum utilization. Number of secondary users and their packet waiting time in the system is calculated. Numerical illustration for the same is presented using utility function upper bound and lower bound of primary arrival was determined. By using upper bound and lower bound the optimal secondary arrival is calculated. Results indicate that optimal secondary arrival increases and decreases depending on reward and state of primary user's arrivals.

Keywords: Cognitive radio Networks, secondary user, priority based, deterministic, rewards, profit

INTRODUCTION

Spectrum scarcity is more growing nowadays due to invention of portable mobile devices like Smart phones and Tablets, which have attracted more number of wireless network users into the spectrum usage. Spectrum is limited and scarce resource as the available spectrum is already allocated to licensed users, which is not fully utilized in both time and location perspective. Spectrum allocation and usage happens to be the thrust area for the research, this becomes more complex when spectrum is apportioned for both primary and secondary users. This demands application of operation research techniques specifically queuing models.

The major functionality of the transceiver in the CRN's is to intelligently detecting which communication channels are in use and which are not, and instantly migrating the secondary users into a vacant channel while avoiding interference with occupied primary ones. This optimizes the use of available resources while maintaining the quality of transmitted packet.

Quality of service (QoS) is an important characteristic in designing the Cognitive Radio as it is chief criterion to select the best spectrum band. Spectrum management is addressed using four aspects which include spectrum sensing, spectrum decision, spectrum sharing and mobility. Spectrum management resources are reserved purposely to serve licensed user, using access method based on a fixed spectrum allocation known as Fixed Spectrum Access (FSA). These spectra allocated to licensed users are often underutilized in space and time as only small part of it is used at a certain time instant and location Salman A. AlQahtani and Hassan Ahmed [5]**.** In fixed spectrum much of its bands remain unused as licensed spectrum are exclusively owned operate by their network all the time whether under use it or not. The bands remain idle when not used by the licensed holder on the band.

The objective of the spectrum owner in CRNs who serves both Primary and Secondary users is to maximize the revenue while serving the regular Primary Users and attracting maximum number of Secondary Users. Through spectrum management, spectrum owner leases unused spectrum to unlicensed wireless networks (SU) through opportunistic spectrum access. Cognitive radio improves spectrum utilization and secondary market usage of spectrum and improves income of spectrum owner by striking the right balance between maximizing the spectrum allocation to SUs and protecting interest of the PUs Salman A. AlQahtani and Hassan Ahmed [5].

This paper mainly focuses on maximizing the spectrum allocation for the SUs which in turn maximizes the profit. To arrive this objective the CRN is considered as M/D/1 queuing model [1] where PUs and SUs are arrived with a Poisson process and their arrival rates are λ_p and λ_s. The common deterministic service rate for both is µ. The objective of the paper is twofold. (i) to derive the utility function for finding the optimum number of SUs for given λ_p, utilization factor Θ, reward for the completion of service of each SU is α and operational cost of SU is β (ii)To find the opportunistic λ_s^* for various values of λ_p, along with suitable numerical illustrations.

The remaining paper is organized as follows. In section II the review of the literature is presented. The assumed CRN model is depicted in section III. The derived expressions for the utility function and optimum λ_S^* are presented in section IV. The numerical illustration is given in Section V. Finally conclusions are drawn in Section VI and also given future scope of research.

REVIEW OF THE LITERATURE

Several studies have been conducted on opportunistic spectrum access in cognitive radio network, on how the SU user opportunistically access free channel to transmit their packets. However no study conducted on secondary user optimization in the given upper and lower bounds of primary arrivals.

Isameldin Suliman and Janne Lehtomaki [1] Analyzed opportunistic spectrum access in cognitive radio network. In their analysis primary user is considered to have higher priority user over secondary user and it does not care about the status of secondary user in its transmission. Through using time slotted system, secondary user senses a free time slot and start transmission and when it sense the presence of PU, it removes its packet form the channel. Waiting time in a queue for PU and SU is determined. Average waiting time was compared under different scenario to determine which has minimum delay. However how spectrum accessed opportunistically in given bound is not shown.

Opportunistic spectrum access channel determined according to targeted QoS before making decision on which channel to use at the beginning of each slot on a channel. A CR user desire to request information regarding available channel characteristics from the base station based on information sensed on channel. The spectrum is selected according to the QoS requirement and the spectrum characteristics. Data rate and bandwidth are determined according to decision rule and appropriate spectrum band chosen Aylin Turhan, Murat Alanyali, and David Starobinski [6].

Primary users have higher priority spectrum access and they don't care about the state of SU and interruption can occur at any time even a time SU start transmission. SU leases a channel when it is free however it is preempted for a certain period on PU arrival and resume when PU finishes transmission. But during the channel handoff between PU and SU, the Primary User (PU) may face interference for a certain time of delay on rejoining the channel Isameldin Suliman and Janne Lehtomaki [1]. How the secondary user optimizes the channel in case it has almost to finish transmission, primary user interfere transmission, instead of tolerating its short until SU finish transmission.

Narvada Khedun and Vandana Bassoo [2] modelled the system based on the M/D/1 systems with priority queuing. Pre-emptive and non pre-emptive approach to determine which use minimum time. The M/D/1 was extended to M/D/S model, and then the performance of M/D/s non pre-emptive and pre-emptive priority queuing model was accessed. Waiting time using M/D/s modal was compared to waiting time of M/D/1

Yun Zhang, Tao Jiang, Lei Zhang, Daiming Qu, and We Peng [3] employed the pre-emptive resume priority (PRP) M/M/1 queuing model to analyze the transmission delay of priority based secondary users (SUs) in the opportunistic spectrum access (OSA) based cognitive radio network. The interrupted packet transmission time was reduced by assigning of interrupted user to higher priority than the newly arrived SUs. The expression of transmission delay was derived basing on the PRP M/M/1 queuing modal. An analytical result was compared with simulation results to determine SUs minimum delay.

Mina Fahima and Abdorasoul Ghasemi [4] formulated PRP M/G/1 model to investigate delivery time of SU in cognitive radio network. The rate of channel handoff was investigated using pre-emptive resume priority M/G/1 model on multiple handoff and data delivery time for SU. Stability region for arrival rate of secondary user was determined.

Salman A. AlQahtani and Hassan Ahmed [5] proposed and investigated efficient admission with eviction control for CRN to improve spectrum utilization efficiency. They considered a tolerance waiting time for the PU and enhanced eviction police. They proposed scheme which provides better balance between SU QoS and primary user's protection. They compared call admission control (CAC) and delayed primary user (DPU) to determine the impact on transmission and profit maximization on admission control.

Aylin Turhan, Murat Alanyali, and David Starobinski [6] determined optimal admission control of secondary users (SUs) in cognitive radio (CR) considering as primary user arrives to the system and finds all channel occupied. SU user is pre-empted except if all channels are occupied by PU. They have applied admission control on SU to determine the number of admitted SU. Through using dynamic programming, optimal admission control policy maximized long run average profit. They found optimal admission control police, and they compared optimal admission policy with total number of user in the system and they determined the effect optimal admission control policy of SU to the dynamic programming to determine which is more efficient.

SYSTEM MODEL

The system model is adopted from M/D/1 preemptive priority model as suggested [2], wherein PU will have higher priority with an ability to preempt SU. The queuing model is presented in the following section.

M/D/1 Queuing Model

This paper considers the Model as suggested [1] and opportunistic spectrum access in which there is one primary user and one Secondary user on the channel. The secondary user accesses the channel at the beginning and transmits packet when channel is free. It is assumed that the channel has perfect sensing (noise and error free) available for transmission.

The spectrum allocation is based on the FCFS scheduling mechanism, giving the higher priority to PU. Time spent by the packet in the system is defined using M/D/1 priority queuing scheme, in which packet are transmitted according to Poisson distribution process and service time is deterministic.

The waiting time of each packet in the system has three components, the time until the beginning of the next slot, time spent in a queue waiting for the service to begin followed by the average service time (transmission time).

Reward is collected from each successful SU transmission, the successful transmission time of the SU packets depends on the three factors the packet arrival time and the number of packets both primary and secondary waiting in the system for transmission and the time each secondary packet spends in the queue waiting for free channel is determined. By using utility function optimum secondary arrival is determined in which opportunistic lower bounds for primary users when optimum access of secondary users for given primary users earnings and SU operation cost found and through using upper bounds for primary users when opportunistic access for SU given PU reward and operation cost.

Assumptions

- The arrival of the packets follows the Poisson process where arrival occurs at the rate λ

- The Service rate is deterministic (constant) for all arrivals (primary and secondary users), the channel is divided into fixed time slots of the equal size, in which when a packet arrives it occupies one slot.
- The queue service discipline is FCFS
- The system is assumed to have an infinite buffer size
- Error free

Notations

λ_P	Primary user arrival rate
λ_S	Secondary user arrival rate
μ	Service time
$\overline{X} = E[X] = \frac{1}{\mu}$	Average service time
N^P	Average number of packets in the queue of primary user
N^S	Average packet in the queue for the secondary user
W^P	Waiting time in a queue for primary user
W_Q^S	Waiting time in a queue for secondary user
D^P	Total time spent by the primary user in the system
D^S	Total time spent by secondary user in the system
T_D	Waiting time till the beginning of a new slot
$\rho^P = \frac{\lambda_P}{\mu}$	Utilization factor for the primary queue
$\rho^S = \frac{\lambda_S}{\mu}$	Utilization factor for secondary queue
α	Reward collected on successful Secondary Transmission
β	Operational Cost pertaining to the channel when the same is offered to the SU.
θ	Utilization is a quantified measure of the channel usage over a period of time.

In this model the optimal number of secondary users is calculated analytically based on the M/D/1 model [1] as follows. The service times of PU and SU are same i.e. $\mu_p = \mu_s = \mu$

The waiting time for any new primary user, includes T_D and transmission time of the existing primary users.

$$W^P = T_D + \frac{1}{\mu} N^P \tag{1}$$

$$W^P = T_D + \frac{1}{\mu} \lambda_P W^P = T_D + \rho^P W^P \quad \text{Since } \because N^P = \lambda_P W^P$$

$$W^P = \frac{T_D}{1-\rho^P} \tag{2}$$

Waiting time for SU in queue is sum of waiting time until the beginning of a slot, the time which primary user spends in the queue, the time during which the existing secondary user transmits their respective packet, and the subsequent arrived PU service time.

$$W^S = T_D + \frac{1}{\mu} N^P + \frac{1}{\mu} N^S + \frac{1}{\mu} \lambda_P W^S \tag{3}$$

Apply little's formula and substitute (1) into (3) then waiting time of secondary user in a queue is

$$\begin{aligned} W^S &= T_D + \frac{1}{\mu} \lambda_P W^P + \frac{1}{\mu} \lambda_S W^S + \frac{1}{\mu} \lambda_P W^S \\ &= T_D + \rho^P W^P + \rho^S W^S + \rho^P W^S \end{aligned} \tag{4}$$

$$W^S = \frac{T_D \rho^P W^P}{1-\rho^P - \rho^s} \tag{5}$$

Substitute (2) into (5) then

$$W^S = \frac{T_D}{(1-\rho^P)(1-\rho^P-\rho^{S)}} \tag{6}$$

Average delay time/packet of the primary user.

$$D^P = \overline{X} + \frac{T_D}{1-\rho^P} \tag{7}$$

Average delay time/packet of the secondary user.

$$D^S = \overline{X} + \frac{T_D}{(1-\rho^P)(1-\rho^P-\rho^{S)}} \tag{8}$$

From little's formula, the expected number of packets of the secondary user in the system is given as

$$N_S = \lambda_s W_S = \lambda_s(\overline{X} + \frac{T_D}{(1-\rho^P)(1-\rho^S-\rho^P)}) \tag{9}$$

Optimal number of Secondary Users

In this section profit to consider for PU is used to derive the optimum number of secondary user on the channel. The utility function depends on reward (α) paid by SU for using the spectrum and operation cost (β) for maintaining SU packets in the system.

$$P = \alpha N_S - \beta W_S = W_S(\lambda_S \alpha - \beta) \tag{10}$$

αN_s, is total reward charged on successful secondary transmission. βW_s, is total operation cost for packet waiting in the system. The profit of spectrum owner on successful secondary user transmission is total revenue minus operation cost as shown below

Substitute (9) into (10) and applying little's formula then profit function as follows

$$P = \left[\frac{1}{\mu} + \frac{\mu^2 T_D}{(\mu-\lambda_p)(\mu-\lambda_p-\lambda_s}\right](\lambda_s\alpha-\beta) \tag{11}$$

Analysis by obtain optimum number of secondary users is given in index I

$$\lambda_s = (\mu-\lambda_p) + \sqrt{\mu T_D(\frac{u^2}{(\mu-\lambda_p)}\frac{\beta}{\alpha} - \frac{\mu^2}{(\mu-\lambda_p)}(\mu-\lambda_p))} \tag{23}$$

By simplification to get the boundary equation

$$\lambda_S^* = (\mu-\lambda_P) + \mu\sqrt{\mu T_D\left(\frac{\beta}{(\mu-\lambda_P)\alpha} - 1\right)}$$

From the above equation, only λ_P is varying and the remains are constant. So the bounds for λ_P is as follows:

The upper bound for λ_P is

1. $\mu - \lambda_P \succ 0 \quad \therefore \lambda_P \prec \mu$ (24)

The lower bound of the λ_P satisfy the following conditions

2. $\dfrac{\beta}{(\mu - \lambda_P)\alpha} - 1 \succ 0 \quad \therefore \mu - \dfrac{\beta}{\alpha} \prec \lambda_P$ (25)

The stability region for the primary arrivals

3. $\mu - \dfrac{\beta}{\alpha} < \lambda_p < \mu$ (26)

THE NUMERICAL ILLUSTRATION

For fixed values of α, β, μ to study boundaries in IV and to calculate λ_S^* for various λ_P

Four scenarios are being considered for admitting secondary user.

1. λ_P increasing
2. λ_P and α increasing
3. α increasing
4. α decreasing

The service rate (μ) is 4, Maximum bound reward (α) is 5 and operation cost (β) is 1. Service rate and operation cost are constant. The lower and upper bound for primary user arrival as in the following range $4 - \dfrac{1}{5} < \lambda_p < 4 \Rightarrow 3.8 < \lambda_p < 4$, Values per scenario are as shown in tables below

λ_P increasing	
λ_P	λ_S^*
3.81	2.79
3.83	4.92
3.85	6.68
3.87	8.43
3.89	10.34
3.91	12.60
3.93	15.49
3.95	19.65
3.97	26.96

Table1

λ_P and α increasing		
λ_P	α	λ_S^*
3.82	0.56	34.12
3.84	0.63	34.10
3.86	0.71	34.08
3.88	0.83	34.06
3.9	1.00	34.04
3.92	1.25	34.02
3.94	1.67	34.00
3.96	2.50	33.98
3.98	5.00	33.96

Table2

α increasing	
α	λ_S^*
0.56	151.38
0.63	142.67
0.71	133.40
0.83	123.43
0.9	118.73
1	112.58
1.25	100.57
2.50	70.66
5	49.33

Table3

α decreasing	
α	λ_S^*
5	49.33
2.5	70.66
1.25	100.57
1	112.58
0.9	118.73
0.83	123.43
0.71	133.40
0.63	142.67
0.56	151.38

Table4

Scenario 1: λ_P increasing

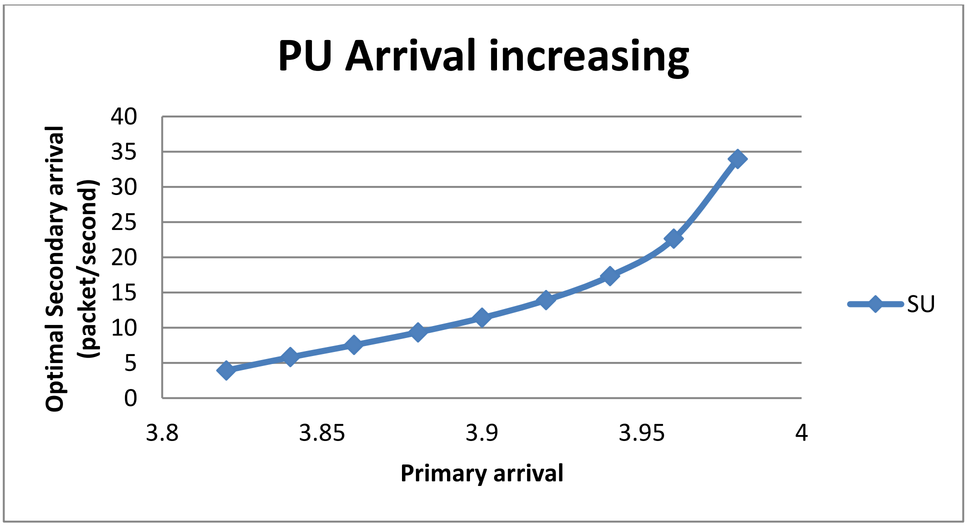

Figure 1: λ_S^* for given value μ , α , β , T_d are 4, 5,1, and 2 respectively for various values of λ_P ,which is increasing

For given values of μ, α, β, T_d as λ_P increases from lower bound, free spectrum decreases, decreasing of free spectrum causes λ_S^* looking for free spectrum to increase. This idea is the same as decreasing free spectrum space so as to increase spectrum demand. From economic theory of demand and supply when the supply decrease the price increases and when supply increases price decreases.

Scenario 2: λ_P and α increasing

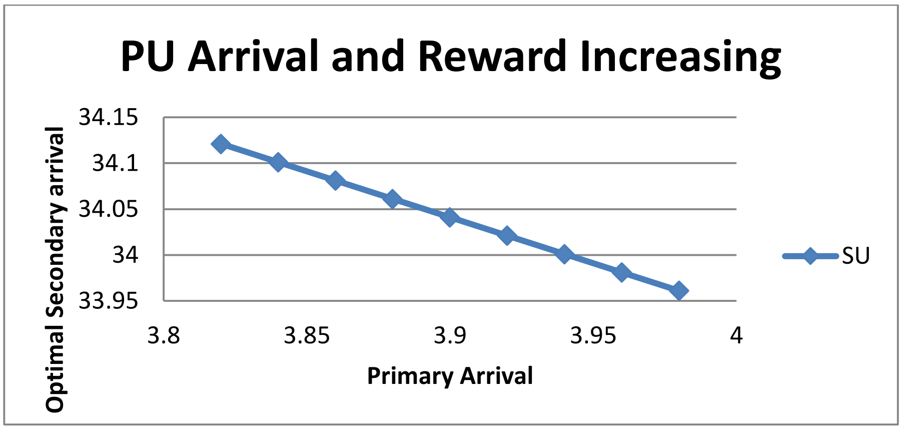

Figure 2: λ_S^* for given value μ, β, T_d are 4,1, and 2 respectively for various λ_P and α increasing

Increasing λ_P decreases free spectrum. The decrement of Spectrum forces the market to increase α, these enable spectrum owner to get high profit due to increased α for the same service. However increasing α reduces demand as the secondary users are not willing to pay extra for the same service.

Scenarios 3: α is increasing

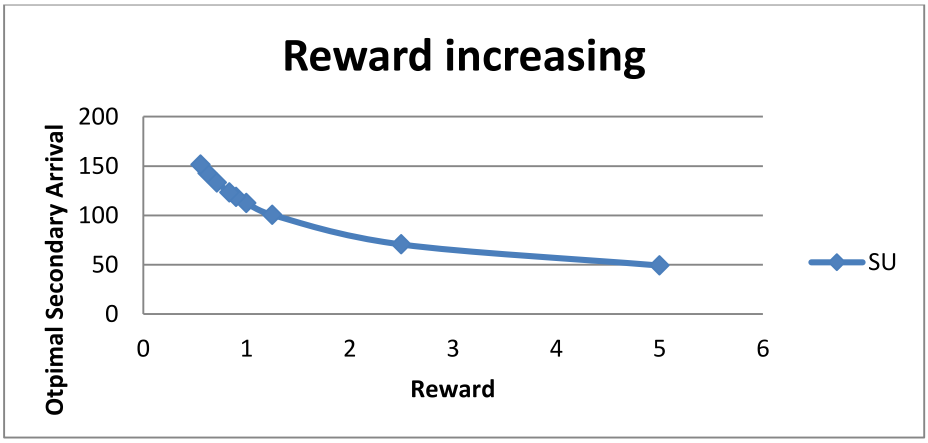

Figure3: λ_S^* for given value λ_P, μ, β, T_d are 3.9, 4,1, and 2 respectively for variation α_i increasing

For given value λ_P, μ, β, T_d as α increase from low to upper bound, λ_S^* decreases. This is the same as the economics theory in which when price is increased product demand decreases. Low α increases λ_S^* demand however spectrum owner gets low profit. High α decreases λ_S^* demand as users are not willing to pay extra for same services but spectrum owners profit increases as extra is charged for same service.

Scenario 4: α is decreasing

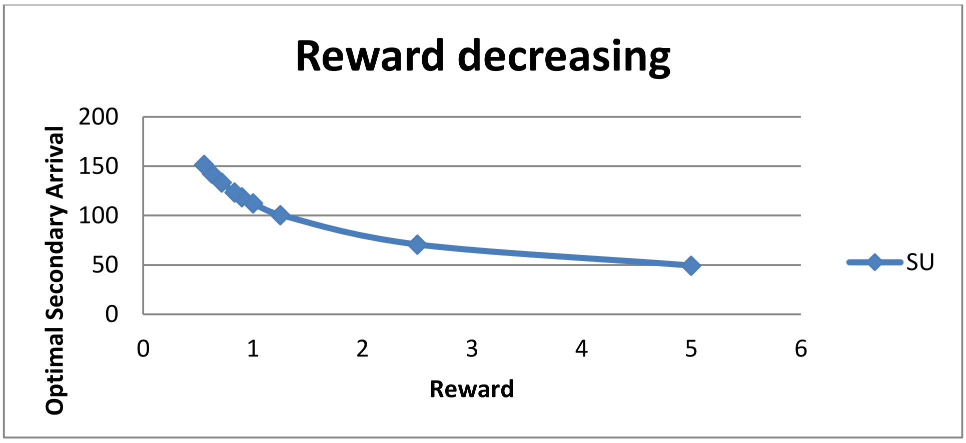

Figure4: λ_S^* for given value λ_P, μ, β, T_d are 3.9, 4,1, and 2 respectively for various α_i decreasing

For given λ_P, μ, β, T_d, as α decreases from upper bound to lower bound the λ_S^* increases. From the economic theory when the price decreases the demand increases. Increasing λ_S^* decreases the spectrum owner's profits. If few λ_S^* pays high α which enables spectrum owner to get super profit, as few λ_S^* pays more for same μ and β before and after increasing α, until reaches at point of maximum satisfaction where few λ_S^* afford proposed α. This factor forces spectrum owner to reduce α so as to attract more λ_S^*.

CONCLUSION

This paper has determined λ_S^* which maximizes single channel utilization through using utility function. Number of packets admitted into the system and their waiting time was calculated using M/D/1 model in which successful transmission of packets, the spectrum owner gets rewarded. Through utility function, upper and lower bound of λ_P in which the optimal number of secondary users admitted is calculated. λ_P and α have effect on λ_S^* admitted. Increasing λ_P with constant α increases λ_S^*. Increasing

λ_P and α decreases λ_S^*. Keeping λ_P constant and decreasing the reward increases λ_S^* .The study is important for maximizing the reward of primary users and also the maximum utilization of spectrum. The future scope of this work lies in determining the equilibrium point where spectrum supply and demand gets satisfaction for both the sides.

REFERENCES

[1] Isamalden. Suliman,Jane Lehtomaki, "Queueing Analysis of Opportunistic Access in Cognitive Radio", Centre for wireless communication (CWC), IEEE computer society, pp 4583 - 4587, 2009

[2] Narvada Khadun, Vandana Bassoo, "Analysis of Priority Queuing with Multichannel in Cognitive Radio Network", IEEE Journal v1, pp 8569-2,2015.

[3] Yun Zhang, Tao Jiang, Lei Zhang, Daiming Qu, Wei Peng, "Analysis on the Transmission Delay of Priority – Based Secondary User in Cognitive Radio Networks", IEEE Journal vol1 pp 308 – 315, 2013.

[4] Mina Fahami, Abdorasoul Ghasemi, "Analysis of the PRP M/G/1 Queuing System for Cognitive Radio Networks with Handoff Management", IEEE Journal vol1 pp 4409 – 4414, 2014.

[5] Salman A. AlQahtani, Hassan Ahmed "An Admission Control Scheme for Secondary Users in Cognitive Radio Networks", IEEE Journal vol1 pp 3106 – 3116, 2014.

[6] Aylin Turhan, Murat Alanyali, David Starobinski, "Optimal Admission Control of Secondary Users in Preemptive Cognitive Radio Networks", Boston, 2012

[7] K.L. Muruganantha Prasad, B.Ush, "A comparison between M/M/1 and M/D/1 queuing models to vehicular traffic at Kanyakumari district", IOSR-JM vol11 pp 13 – 15

[8] Shunfu Jin, Yuan Zhao, Wuyi Yue, Zsolt Saffer, "Performance Analysis and Optimization of an Adaptive Admission Control Scheme in Cognitive Radio Networks, Hindawi Publishing Corporation, Volume 2013.

[9] Shung Li, Zizhan Zheng, Eylem Ekici, Ness Shroff, "Maximizing System Throughput by Cooperative Sensing in Cognitive Radio Networks",

[10] Rahul Urgaonkar, Michael J. Neely, "Opportunistic Scheduling with Releiability Gurantees in Cognitive Radio Networks", IEEE INFOCOM, 2008.

[11] Li – Chun Wang, Chung – Wei Wang, "Spectrum Handoff for Cognitive Radio Networks: Reactive – Sensing or Proactive – Sensing?, IEEE vol1 3367 – 3371, 2008.

[12] Li – Chun Wang, Chung – Wei Wang, Chung – Ju Chang, "Modelling aand Analysis for Spectrum Handoff in Cognitive Radio Networks", IEEE Transaction on mobile computing, vol11, pp 1499 – 1513, 2012

[13] Li – Chun Wang, Chung – Wei Wang, Fumiyuki Adachi, "Load – Balancing Spectrum Decision for Cognitive radio Networks", IEEE Selected area in communication, vol29 No. 4, pp 757 – 769, 2011.

[14] Li – Chun Wang, Chung – Wei Wang, Fumiyuki Adachi, "Modelling and Analysis for reactive – decision Spectrum Handoff in Cognitive Radio Networks", IEEE Globecom, pp 5638 – 5641, 2010.

[15] Li – Chun Wang, Chung – Wei Wang, Fumiyuki Adachi, "Modeling and Analysis for Proactive – decision Spectrum Handoff in Cognitive Radio networks", IEEE, 209.

[16] Ekendeke Ndiomo Maurice, Bin Guo, "Research on Lifetime Estimation of Secondary connection Using the PRP M/G/1 Queering Network Model, ISCCA, 2013

[17] Long Bao Le, Ekram Hossain, "Resource Allocation for Spectrum Underlay in in Cognitive Radio Networks", IEEE Transaction on wireless communication, vol7, 1276 – 1284, 2008.

[18] Xin – Lin Huang, Xiang Wu, Jun Wu, Yuan Xu, Min Wang, "Queuing Theory Based Spectrum Allocation in Cognitive Radio Networks", International Conference on Wireless Communication, 7339 – 7341, 2014.

[19] Said Lakhal, Abdellah Idrissi, "Queues Management of Secondary Users in a Cognitive Radio Network", Fifth International on NGNS,2014.

[20] Diego Pacheco – Paramo, Vicent Pla and Jorge Martinez – Bauset, "Optimal Admission Control in Cognitive Radio Networks", 2010.

[21] Ahmed M. Alaa, "Stable Throughput Region of Cognitive – Relay Network with Imperfect Sensing and Finate Relaying Buffer", conference paper 2014.

[22] Gaofei Sun, Xiaohua Tian, Youyun Xu, Xinbing Wang, "Profit maximization for secondary users in dynamic spectrum auction of cognitive radio networks", Journal of wireless communications and mobile computing, pp 1331 – 1341, 2015.

Index.1

For easy simplification we let $X = \frac{\mu^2}{\mu - \lambda_p}$ and $Y = \mu - \lambda_s$

$$P = (\left[1 + \frac{X}{(Y - \lambda_s)}\right](\lambda_s \alpha - \beta)) \tag{12}$$

Expand the terms

To determine optimal profit gain on secondary admission we differentiate with respect to λ_s

Apply different ion by part

From $UV = \frac{V\partial U - U\partial V}{V^2}$

$$\frac{\partial P}{\partial \lambda_s} = \frac{(Y - \lambda_s)\partial(Y\alpha - \alpha\lambda_s^2 + \beta\lambda_s + X\alpha\lambda_s - Y\beta - X\beta) - (Y\alpha - \alpha\lambda_s^2 + \beta\lambda_s + X\alpha\lambda_s - Y\beta - X\beta\partial(Y - \lambda_s)}{(Y - \lambda_s)^2} \tag{13}$$

$$\frac{\partial P}{\partial \lambda_s} = \frac{(Y - \lambda_s)(Y\alpha - 2\alpha\lambda_s + \beta + X\alpha) - (Y\alpha\lambda_s - \alpha\lambda_s^2 + \beta\lambda_s - Y\beta - X\beta)(-1)}{(Y - \lambda_s)^2} = 0 \tag{14}$$

$$Y^2\alpha - 2Y\alpha\lambda_s + Y\beta + YX\alpha - Y\alpha\lambda_s + 2\alpha\lambda_s^2 - \beta\lambda_s - X\alpha\lambda_s + Y\alpha\lambda_s - \alpha\lambda_s^2 + \beta\lambda_s - Y\beta - X\beta + X\alpha\lambda_s = 0 \tag{15}$$

After Simplification

$$\alpha\lambda_s^2 - 2Y\alpha\lambda_s + YX\alpha + Y^2\alpha - X\beta = 0 \tag{16}$$

Secondary Arrival SU derivation

Solve for λ_s using quadratic formula and SU arrival rate must be positive and negative value is neglected.

From

$$\lambda s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{17}$$

$$a = \alpha$$

$$b = -2Y\alpha$$

$$c = YX\alpha + Y^2\alpha - X\beta$$

Substitutes in the above formula

$$\lambda_s = \frac{-(-2Y\alpha) \pm \sqrt{(-2Y\alpha)^2 - 4\alpha(YX\alpha + Y^2\alpha - X\beta)}}{2\alpha} \tag{18}$$

$$\lambda_s = \frac{2Y\alpha \pm \sqrt{4Y^2\alpha^2 - 4\alpha^2 YX\alpha - 4Y^2\alpha^2 + 4X\alpha\beta)}}{2\alpha} \tag{19}$$

$$\lambda_S = \frac{Y\alpha \pm \sqrt{X\alpha\beta - \alpha^2 YX}}{\alpha} \Rightarrow Y \pm \sqrt{\frac{X\alpha\beta - \alpha_1^2 YX}{\alpha^2}} \tag{20}$$

$$\lambda_S = Y \pm \sqrt{X\frac{\beta}{\alpha} - YX} \Rightarrow Y \pm \sqrt{X(\frac{\beta}{\alpha} - Y)} \tag{21}$$

Substitute the value of Y and X in the equation above

$$\begin{aligned} \lambda_s &= (\mu - \lambda_p) \pm \sqrt{\frac{u^2}{(\mu - \lambda_p)}\frac{\beta}{\alpha} - \frac{\mu^2}{(\mu - \lambda_p)}(\mu - \lambda_p)} \\ \lambda_s &= (\mu - \lambda_p) \pm \mu\sqrt{\frac{\beta}{(\mu - \lambda_p)\alpha} - 1} \end{aligned} \tag{22}$$

Index 2

Second derivative proof

$$\frac{\partial^2 P}{\partial \lambda_S^2}(\frac{\partial P}{\partial \lambda_s} = \frac{(Y - \lambda_s)\partial(Y - \alpha\lambda_s^2 + \beta\lambda_s + X\alpha\lambda_s - Y\beta - X\beta) - (Y\alpha - \alpha\lambda_s^2 + \beta\lambda_s + X\alpha\lambda_s - Y\beta - X\beta\partial(Y - \lambda_s))}{(Y - \lambda_s)^2}) \tag{28}$$

Using by part differentiation, we get

$$\frac{\partial^2 P}{\partial \lambda_S^2} = \frac{(Y-\lambda_S)^2(2\alpha\lambda_S - 2Y\alpha) + 2(\alpha\lambda_S^2 - 2\alpha Y\lambda_S - X\beta + Y^2\alpha)(Y-\lambda_S)}{(Y-\lambda_S)^4} \tag{29}$$

When we simplify the above equation we get

$$\frac{\partial^2 P}{\partial \lambda_s^2} = \frac{2X(Y\alpha - \beta)}{(Y-\lambda_S)^3} \tag{30}$$

Substitute the value of X and Y in the above equation, we get equation (31).