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Total strong (weak) domination in bipolar fuzzy graph

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Abstract

In this paper, the new kind of parameter strong (weak) domination number in a bipolar fuzzy graph is defined and established the parametric conditions. Another new kind of parameter a totalstrong (weak) bipolar domination number is defined and established the parametric conditions. The properties of strong (weak) bipolar domination number and totalstrong (weak) bipolar domination numbersare discussed.

Keywords: Bipolar set, Dominating bipolar set, strong (weak) bipolar dominating set, total strong (weak) bipolar dominatingset, Bipolar domination number, strong (weak) bipolar domination number, totalstrong (weak) bipolar dominationnumber.

Introduction

The concept of fuzzy graph was proposed by kaufmann, from the fuzzy relations introduced by Zadeh. Although, In 1975, Rosenfeld introduced another elaborated concept, including fuzzy vertex and fuzzy edges and several fuzzy analogues of graph theoretic concepts such as paths, cycles, connectedness and etc. In the year 1998, the concept of domination in fuzzy graphs was investigated by A. Somasundaram, S.Somasundaram. In the year, 2004 A.Somasundaram investigated the concepts of domination in fuzzy graph - II . In the year 2003, A.NagoorGani and M. BasheerAhamed investigated Order and Size in fuzzy graph.In 2010, C.Natarajan and S.K. Ayyasamy introduced On strong (weak) domination in fuzzy graph. In 2011, Muhammad Akram introduced Bipolar fuzzy graphs. In the year 2012, Muhammad Akram was proposed regular bipolar fuzzy graphs. In 2012, P.J.Jayalakshmi et.al introduced total strong (weak) domination in fuzzy graph.

Mathematical Classification: 03E72, 05C69,05C72, 05C76.

I Basic definitions

In this section, Some definitions are discussed.

1.1 Defintion

A fuzzy subset μ on a set X is a map μ:X →[0,1]. A map v:X × X → [0,1] is called a **fuzzy relation** on X if v(x,y) ≤ min(μ(x),μ(y)) for all x,y∈X. A fuzzy relation v is symmetric if v(x,y) = v(y,x) for x,y∈ X.

1.2 Definition

Let X be a non-empty set. A **bipolar fuzzy set** B in X is an object having the form $B=\{(x,\mu_B^P(x),\mu_N^P(x))/x\in X\}$ where $\mu_B^P : X \to [0,1]$ *and* $\mu_B^N : X \to [-1,0]$ are mappings.

1.3 Definition

By a **bipolar fuzzy graph**, we mean a pair G = (A,B) where $A=(\mu_A^P,\mu_A^N)$ is a bipolar fuzzy set in V and $B=(\mu_B^P,\mu_B^N)$ is a bipolar relation on V such that $\mu_B^P(\{x,y\})\le \min(\mu_A^P(x),\mu_A^P(y))$ and $\mu_B^N(\{x,y\})\ge \max(\mu_A^N(x),\mu_A^N(y))$ for all {x,y} ∈ E. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E, respectively.

1.4 Definition

Let G = (A,B) be a bipolar fuzzy graph where $A=(\mu_A^P,\mu_A^N)$ and $B=(\mu_B^P,\mu_B^N)$ be two bipolar fuzzy sets on a non-empty finite set V and E⊆V×V respectively. The **positive degree of a vertex** μ_A^P ∈G is $d(\mu_A^P(x)) = \sum_{xy\in E}\mu_B^P(xy)$. Similarly, the **negative degree of a vertex** μ_A^N ∈G is $d(\mu_A^N(x)) = \sum_{xy\in E}\mu_B^N(xy)$. The **degree of a vertex μ** is d(μ) = (d^p(μ), d^N(μ)).

1.5 Definition

Let G = (A,B) be a bipolar fuzzy graph. The **order of a bipolar fuzzy graph G** is $O(G)=\left(\sum_{x\in V}\mu_A^P(x),\sum_{x\in V}\mu_A^N(x)\right)$. The **size of a bipolar fuzzy graph G** is $S(G)=\left(\sum_{xy\in V}\mu_A^P(xy),\sum_{xy\in V}\mu_A^N(xy)\right)$.

1.6 Definition

Let G = (A,B) be a bipolar fuzzy graph. If each vertex of G has same closed neighbourhood degree, then G is called a totally bipolar fuzzy graph. The closed neighbourhood degree of a vertex x is defined by $\deg[x] = (\deg^P[x] + \deg^N[x])$, where $\deg^P[x] = (\deg^P[x] + \mu_A^P[x])$, $\deg^N[x] = (\deg^N[x] + \mu_A^N[x])$.

1.7 Definition

A bipolar fuzzy graph G = (A,B) is called **strong bipolar fuzzy graph**, if
$\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) = \min(\mu_A^N(x), \mu_A^N(y))$ for all xy$\in$ E.

1.8 Definition

Let G be a bipolar fuzzy graph. Let A and B be any two vertices. Then **A totally strong dominates B (B totally weak dominates A)** if

i. $\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) = \min(\mu_A^N(x), \mu_A^N(y))$ for all xy$\in$ E.

ii. $d_N(A) \geq d_N(B)$ and

iii. every vertex in G dominates A.

1.9 Definition

Let G be a bipolar Fuzzy Graph. T_b is said to be **total strong (weak) dominating bipolar set of G** if

i. $\mu(A,B) \geq \mu^\infty(A,B)$ for all A,B$\in$ V(G)

ii. $d_N(A) \geq d_N(B)$ for all A$\in$ T_b, B$\in$ V - T_b and

iii. $\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) = \min(\mu_A^N(x), \mu_A^N(y))$ for all xy$\in$ E.

iv. T_b is the total dominating bipolar set.

1.10 Definition

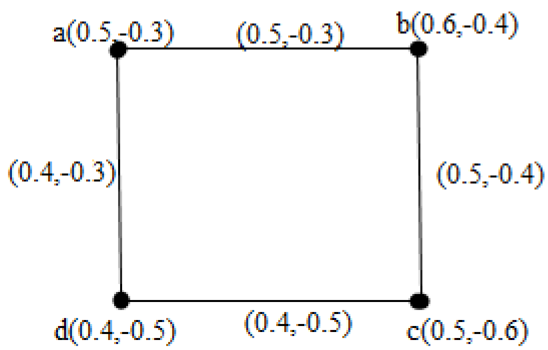
A total strong (weak) dominating bipolar set T_b of a fuzzy graph G is called **minimal total strong (weak) dominating bipolar set** of G, if there does not exist any total strong (weak) dominating bipolar set of G, whose cardinality is less than the cardinality of T_b.

1.11 Definition

The minimum fuzzy cardinality among all minimal total strong (weak) dominating bipolar set G is called **total strong (weak) dominating bipolar set of G** and its total strong (weak) domination bipolar number is denoted by $\gamma_{T_b}(G)$.

1.12 Example

Let G be a bipolar fuzzy graph.



G

Total strong (weak) dominating bipolar set, $T_b = \{a,b\}$.

Total strong (weak) bipolar domination number, $\gamma_{T_b}(\mu_A^P, \mu_A^N) = (1.1, -0.7)$.

$\deg(\mu_a^P, \mu_a^N) = (0.9, -0.6)$, $\deg(\mu_b^P, \mu_b^N) = (1, -0.7)$, $\deg(\mu_c^P, \mu_c^N) = (0.9, -0.9)$, $\deg(\mu_d^P, \mu_d^N) = (0.8, -0.8)$.

Order of bipolar fuzzy graph, p = O(G) = (2,-1.8)

Size of bipolar fuzzy graph, q = S(G) = (1.8,-1.5)

Weight of bipolar fuzzy graph, W(G) = (1.1,-0.7)

1.13Theorem

Let G be a bipolar fuzzy graph. Let T_b be a minimal total strong (weak) dominating set of a bipolar fuzzy graph G. Then for each B $\in T_b$, one of the following holds:

i. No vertex in T_b strongly dominates B.

ii. There exists B $\in V - T_b$ such that v is the only vertex in T_b which strongly dominates A.

iii. T_b is the total dominating set of bipolar fuzzy graph.

Proof

Assume that T_b is a minimal total strong (weak)dominating set of a bipolar fuzzy graph G. Then for every vertex $B \in T_b$, $T_b - \{B\}$ is not a total strong (weak)dominating set in G. There exists $A \in V - T_b$, which is not strongly dominated by any vertex in $T_b - \{B\}$. But T_b is a total strongdominating set of a bipolar fuzzy graph. Therefore B is the only vertex which strongly dominates B. Which is satisfies the condition ii.

Conversely, assume that T_b is a total strong (weak) dominating set of a bipolar fuzzy graph and for each vertex $B \in T_b$, one of the following two conditions holds.

i. If T_b is not a minimal total strong (weak)dominating set of a bipolar fuzzy graph, then there exists a vertex $B \in T_b$, T_b - {B} is the total strong dominating set of a bipolar fuzzy graph. Hence B is strongly dominated by atleast one vertex in T_b - {B}, which is a contradiction using the condition i.

ii. If T_b - {B} is a total strong (weak) dominating set of a bipolar fuzzy graph, then every vertex in V - T_b is totally strong (weak) dominated by atleast one vertex in T_b - {B}, the second condition does not hold. Therefore, T_b is a minimal total strong (weak)dominating set of a bipolar fuzzy graph.

1.14 Theorem

Every complete bipolar fuzzy graph is total strong (weak) domination in bipolar fuzzy graph.

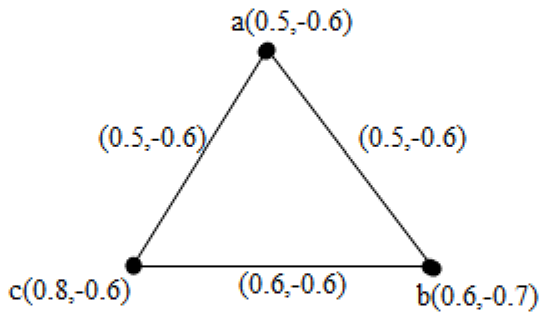
Proof

Since G is complete bipolar fuzzy graph. All edge in G are total strong(weak) dominating set and all vertices are joined together. Obviously, G is a total strong (weak) dominating set of a bipolar fuzzy graph.

Note: Converse need not be true.

1.15 Example

Let G be a bipolar fuzzy graph.

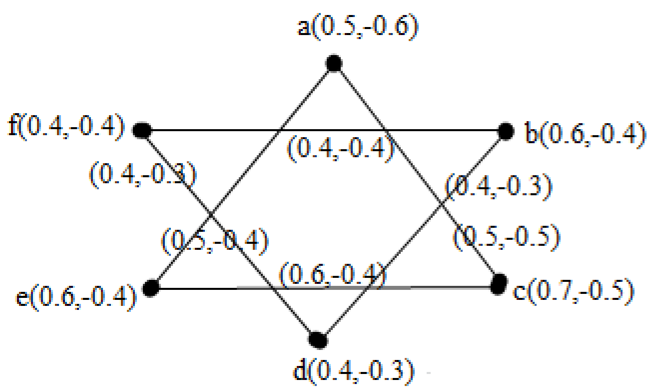


G

$T_b = \{a\}$, $\gamma_{T_b}(G) = (0.5,-0.6)$, $p = (1.9,-1.9)$, $q = (1.6,-1.8)$,
$d_N[a] = [1.9,-1.9] = d_N[b] = d_N[c]$, $d_E(a) = (1.5,-1.8)$, $d_E(b) = (1.7,-1.9)$, $d_E(c) = (1.9,-1.8)$

1.16 Example

Let G be a bipolar fuzzy graph.



G

$T_b = \{a, c, d\}$, $\gamma_{T_b}(G) = (1.6,-1.4)$, $p = (3.2,-2.6)$, $q = (2.8,-2.3)$,

Properties:

For a bipolar fuzzy graph,

1. $p - \Delta_E^+ \leq p - \delta_E^+$.
2. $p - \Delta_N^+ \leq p - \delta_N^+$.
3. ${\gamma_{T_b}}^+(G) \leq p - \Delta_E^+ \leq p - \delta_E^+$
4. ${\gamma_{T_b}}^+(G) \leq p - \Delta_N^+ \leq p - \delta_N^+$

References:

[1]. Arumugam. S and Natarajan. C, Strong (weak) domination in fuzzy graphs, International Journal of Computational and Mathematical Science 2010.

[2]. NagoorGani. A and BasheerAhamed. M, Order and Size in fuzzy graph, Bulletin of Pure and Applied Sciences, Vol.22.(2003) 145-148.

[3]. Nagoorgani. A and Chandrasekaran. V.T, Domination in fuzzy graph,Advances in fuzzy sets and system, 1, (2006) 17-26.

[4]. Somasundaram. A and Somasundaram.S, Domination in fuzzy graphs, Pattern Recognit. Lett.19 (9) (1998) 787-791.

[5]. Jayalakshmi.P.J., Harinarayanan.C.V.R, Total strong (weak) domination in fuzzy graph, International Journal of Computational engineering and Research, 2012.

[6]. Muhammad Akram, Bipolar fuzzy graphs, Information Sciences 181 (2011)5548-5564.

[7]. Muhammad Akram, Wieslaw A. Dudek, Regular bipolar fuzzy graphs, Neural Computational and Application (2012) 21 (Suppl I): S197-S205.

[8]. SovanSamanta, Madhumangal Pal, Irregular Bipolar fuzzy graphs, International Journal of Applications of Fuzzy Sets(ISSN 2241 – 12240) Vol.2 (2012), 91-102.