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A Genetic Algorithm Solution to The Direct Kinematics of Underconstrained Cable-Driven Parallel Robot

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Abstract

Underconstrained cable-driven robots are moved in their workspace by winding and unwinding the cables. But for specified lengths of cables, the equilibrium pose of the moving platform depends also on the forces of gravity and external forces. Therefore, solving the direct kinematics requires simultaneously solving geometric constraint equations and equilibrium equations. In using analytical methods, the existence of non-linearity imposes difficulty. In this study, genetic algorithm is successfully used to solve the direct kinematics of an underconstrained cable-driven parallel robot that is suspended and controlled using four cables. The position and orientation of the platform at equilibrium and also the forces in the cables are determined.

Keywords- Cable-Driven Parallel Robot (CDPR), Underconstrained Mechanism, Direct Kinematics, Genetic Algorithm.

I. INTRODUCTION

Parallel manipulators that are driven using cables are finding more and more applications. They are known by the names cable-driven robots, Cable Driven Parallel Robots (CDPRs), Cable Driven Parallel Manipulators (CDPMs), etc. The manipulator or mobile platform which carries the end effector, remains suspended using cables and is moved about in the workspace by winding or unwinding these cables. They are used in guiding camera in large stadiums for recording aerial views, transporting objects in factory floors and construction sites, guiding the concrete depositing head in additive manufacturing of structures and buildings, etc. [1-6]. Their use in assisting human exercises for patients requiring rehabilitation has also been demonstrated [7].

Each of the cable has one of its end attached to the mobile platform and the other end attached to a corner of the workspace. There will be a separate motor for winding/unwinding each of the cable and these motors will be usually mounted on the fixed base near the attachment points, thus reducing the weight of the mobile platform. There are some advantages that favour the use of CDPRs. Since they are driven using cables, they can have very large workspaces. Due to the avoidance of rigid links, they are lighter in weight and can be used in high acceleration applications [8]. They can be easily dismantled, transported and re-installed [1]. They can also be re-configured easily according to the requirements of a new workspace. CDPRs can be broadly classified into two types as shown in Fig. 1 (a) and (b): fully constrained CDPRs and under-constrained ones. In fully constrained cable-driven robots the position and orientation of the mobile platform is determined by the length of the cables. The number of cables required to fully constrain a manipulator is *n* + 1, where, *n* is the degrees of freedom of the manipulator [3]. A manipulator that has a 3-D workspace has 6 degrees of freedom and requires at least 7 cables to fully constrain it. An under-constrained cable robot, even when the cable lengths are specified, will have some freedom of movement. The equilibrium pose of an under-constrained cable robot is decided by the forces of gravity, external forces and inertia forces, apart from the forces in the cables.

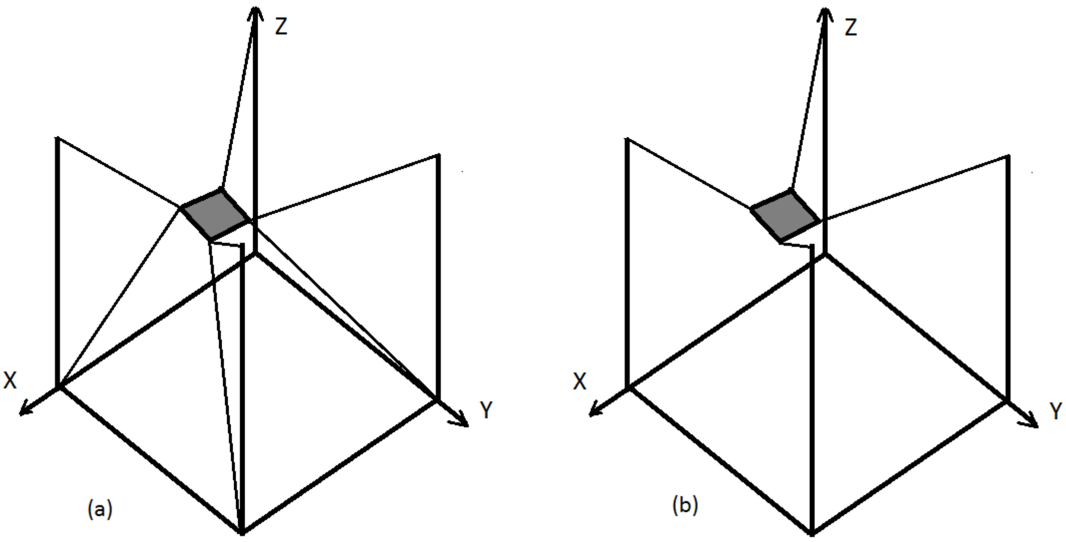

Figure. 1: (a) A fully constrained CDPR (b) An underconstrained CDPR driven by four cables

Compared to fully constrained ones, under-constrained cable robots are comparatively less stable, but they have one big advantage: The cable interference with objects in the workspace is much reduced. For example, in transporting objects in a factory floor, if the cables are attached to the upper corners of the workspace, the manipulator can pick and place objects without interference of cables.

For guiding a CDPR in its workspace, the cable lengths corresponding to every pose of the platform along the desired path has to be calculated. The motors are actuated to wind or unwind each cable precisely to attain the pose. Finding the cable lengths, which are the joint parameters for cable-driven robots, for a given pose, constitutes the inverse kinematics. The cable forces are also determined by solving the inverse kinematic equations. Finding the resulting pose of the mobile platform when cables are set to specified lengths constitutes the direct kinematic problem. The direct and inverse kinematics and the workspaces of fully constrained CDPRs have been widely researched [9-14]. For an underconstrained CDPR, solving the direct problem is difficult since it involves simultaneously solving geometric constraint equations and equilibrium equations [15,16]. Some analytical methods [17,18], methods based on neural networks [19] and direct kinematic solution when incremental changes are made in cable lengths [20], can be found in literature. In this paper, a method of solving the direct kinematics of a 4-cable underconstrained CDPR using genetic algorithm is presented.

II. FOUR-CABLE UNDERCONSTRAINED CDPR

The CDPR used in this study, has the manipulator or mobile platform suspended and driven using four cables. The mobile platform is a square lamina of sides 0.3 metres each and with negligible thickness. It has a mass m of 0.3 kg. The cable attachment points are at the four corners of the lamina and the centre of gravity is assumed to be at the geometric centre of the lamina. The workspace in which the CDPR moves, is assumed as a cuboid of length, breadth and height each equal to 3 metres. The other ends of the cables are attached to the top four corners of the workspace. A co-ordinate system X-Y-Z is fixed to the base with origin O at one bottom corner of the workspace and Z-axis pointing upwards. The workspace lies on the positive sides of X and Y axes. The cable attachment points on the corners of workspace are P1, P2, P3 and P4 with column vectors representing the position, $\mathbf{P_1} = [0,3,3]^T$, $\mathbf{P_2} = [3,3,3]^T$, $\mathbf{P_3} = [3,0,3]^T$ and $\mathbf{P_4} = [0,0,3]^T$. The position of the moving platform is defined as the position of its centre of gravity G with respect to X-Y-Z co-ordinate system. The column vector $\mathbf{X} = [X,Y,Z]^T$ represents the position of G. A co-ordinate system x-y-z is fixed to the mobile platform with origin at G, and x and y axes parallel to the edges of platform. The cable attachment points on the platform are B_1, B_2, B_3 and B_4, with column vectors for position with respect to x-y-z frame, $\mathbf{B_1} = [-0.15, 0.15, 0]^T$meters, $\mathbf{B_2} = [0.15, 0.15, 0]^T$ meters, $\mathbf{B_3} = [0.15, -0.15, 0]^T$ meters and $\mathbf{B_4} = [-0.15, -0.15, 0]^T$meters. $\mathbf{L_i}$ is the vector from point B_i to P_i expressed in frame X-Y-Z. The orientation of the platform is defined by the vector of Euler angles, $\boldsymbol{\theta} = [\theta, \varphi, \psi]^T$ radians, by which it successively rotates – first by θ about fixed X-axis, then by φ about y-axis of moving co-ordinate system x-y-z and then by ψ about z-axis of x-y-z. The equilibrium of the platform depends on the forces of gravity, external forces and inertia forces. Since the cables can only exert pull and not push, the forces in the cable have to be non-negative for a pose to be feasible. As a simplifying assumption, external forces other than gravity and inertia forces are not considered in this study.

III. GENETIC ALGORITHM TO SOLVE DIRECT KINEMATICS

A solution to the direct kinematic problem finds the position and orientation of the platform when the lengths of the four cables are specified. It also finds the tension in the cables. A chromosome with 10 genes, is constructed to represent a possible solution. The first three genes represent the X, Y and Z co-ordinates of the position of the platform in metres. The next three genes represent the Euler angles defining the orientation of the platform. The final four genes represent the forces in the four cables in Newtons.

The chromosomes in the initial population are randomly generated. The values of genes X, Y and Z are limited to be between 0 and 3 metres, so that the mobile platform is within the workspace. The underconstrained CDPRs can have only a limited range of feasible orientations. A mapping of the range of orientations can be found in [21]. The values of genes θ, φ and ψ are limited between –1.0472 to +1.0472 radians, i.e., -60 to +60 degrees. The values of genes f_1, f_2, f_3 and f_4, which represent the forces in the four cables, are limited between 0 and 10 Newtons, which is practically the maximum possible cable tension for the given configuration.

For testing the convergence of the algorithm, the set of cable lengths required for a selected pose of the platform is obtained using the indirect kinematics method described in [21]. A pose that is sufficiently away from the planes of symmetry in the workspace is selected with position of G, $\mathbf{X} = [0.5, 1.0, 1.5]^T$ *m* and orientation $\boldsymbol{\theta} = [-0.3489, 0.9071, 0.1279]^T$ *rad*. The set of lengths and forces in the cables obtained using the indirect kinematics solution is [2.3523, 3.4806, 2.9890, 1.6611] *m* and [1.3337, 0.3700, 0.5998, 2.0293] *N* respectively.

A. Evaluation function

The goodness of a solution is evaluated by how well it adheres to the geometric constraints and equilibrium conditions. The geometric constraints of the CDPR are given by the equations,

$\boldsymbol{X} + R\boldsymbol{B_i} + \boldsymbol{L_i} - \boldsymbol{P_i} = 0$ for i = 1, 2, 3, 4.

and $\boldsymbol{L_i} = \boldsymbol{P_i} - \boldsymbol{X} - R\boldsymbol{B_i}$ for i = 1, 2, 3, 4.

where R is the rotation matrix defining the orientation of frame x-y-z with respect to frame X-Y-Z and $R\boldsymbol{B_i}$ represents the position vector $\overrightarrow{G\,B_i}$ in X_Y_Z.

The length of each cable, l_i , implied by the position and orientation represented by a solution is,

$l_i = {L_i}^T L_i$ for i = 1, 2, 3, 4.

The deviation from the geometric constraint is given by,

$l_i' = l_i - l_{i-g}$ for i = 1, 2, 3, 4.

where, l_{i-g} is the specified length for the Cable-i and l_i' is the difference in length.

The equations of equilibrium for the platform can be stated as,

Equations for force balance: $\frac{\boldsymbol{L_1}}{l_1} f_1 + \frac{\boldsymbol{L_2}}{l_2} f_2 + \frac{\boldsymbol{L_3}}{l_3} f_3 + \frac{\boldsymbol{L_4}}{l_4} f_4 - \boldsymbol{G} = \boldsymbol{0}$

where, $\boldsymbol{G} = [0, 0, -9.8\, m]^T$ Newtons is the vector representing force of gravity.

Equations for moment balance: $\sum_{\boldsymbol{i=1}}^{\boldsymbol{4}} R\boldsymbol{B}_{\boldsymbol{i}} \times \boldsymbol{L}_{\boldsymbol{i}}\frac{f_i}{l_i} = \boldsymbol{0}$

The resultant force and moment corresponding to a chromosome can be represented as

$\boldsymbol{W} = \begin{bmatrix}\boldsymbol{W_F}\\ \boldsymbol{W_T}\end{bmatrix}$, where, $\frac{\boldsymbol{L_1}}{l_1}f_1 + \frac{\boldsymbol{L_2}}{l_2}f_2 + \frac{\boldsymbol{L_3}}{l_3}f_3 + \frac{\boldsymbol{L_4}}{l_4}f_4 - \boldsymbol{G} = \boldsymbol{W_F}$ and $\sum_{\boldsymbol{i=1}}^{\boldsymbol{4}} R\boldsymbol{B}_{\boldsymbol{i}} \times \boldsymbol{L}_{\boldsymbol{i}}\frac{f_i}{l_i} = \boldsymbol{W_T}$.

A measure of deviation from geometric constraints is given by $\sum_{i=1}^{4}(l_i{}')^2$ and a measure of deviation from equilibrium conditions is given by $(\mathbf{W})^{\mathrm{T}}\mathbf{W}$. A measure of the total deviation is given by, $\Delta = \sum_{i=1}^{4}(l_i{}')^2 + (\boldsymbol{W}^T\boldsymbol{W})$. The fitness of a chromosome is expressed using the function, $\mathcal{F} = \frac{1}{\Delta}$. The goal of the genetic algorithm is to maximize $\mathcal{F}$.

The chromosomes are selected to the mating pool by Roulette wheel selection, where the probability of being selected is proportional to the fitness value. A probability p_c is assigned, and each pair of chromosomes selected for mating are changed to new ones by cross-over, if a random number generated is within this probability value; otherwise they are passed to the new generation as such. A probability for mutation p_m is also assigned. A convergence criterion that the measure for total deviation, $\Delta < 0.0001$, is used for stopping the iterations.

IV. RESULTS AND DISCUSSIONS

The results of the GA procedure and the effects of the algorithm parameters on convergence are discussed as follows:

A typical solution obtained is represented by the chromosome,

[0.4929, 1.0046, 1.5033, -0.3584, 0.8916, 0.1363, 1.3476, 0.3606, 0.5894, 2.0322]

which gives the solution as $\mathbf{X} = [0.4929, 1.0046, 1.5033]^{\mathrm{T}}$ *m*, orientation $\boldsymbol{\theta} = [-0.3584, 0.8916, 0.1363]^{\mathrm{T}}$ *rad* and forces in the cables $f_1 = 1.3476$, $f_2 = 0.3606$, $f_3 = 0.5894$ and $f_4 = 2.0322$ Newtons. A population size of 200, probability for cross-over $p_c = 0.7$ and probability for mutation $p_m = 0.1$ was used. A two position cross-over by swapping genes 1 to 3 and 7 to 10 of one chromosome with the genes 4 to 6 of the mating pair was used. Mutation was effected on every gene that met the probability, with the mutated value limited to be within the originally set range. The algorithm converged in 11849 iterations.

Another solution, abtained with mutation probability 0.2 and all other parameters remaining same is represented by the chromosome:

[0.4963, 1.0006, 1.5010, -0.3532, 0.8935, 0.1300, 1.3353, 0.3697, 0.5891, 2.0347]

Figure 2 shows the improving values of highest fitness and the average fitness of the population with number of iterations.

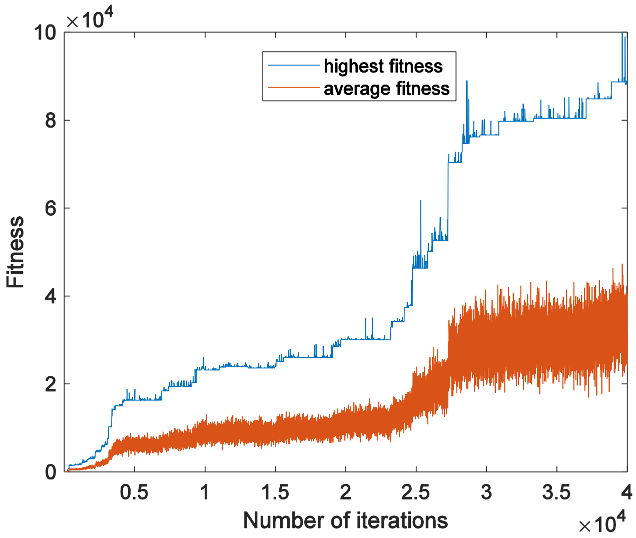

Figure. 2: The highest and average fitness improving with iterations

A. Effect of population size on convergence

The number of chromosomes in the population has an effect on the speed of convergence. The genetic algorithm was executed with the size of population varying from 20 to 2000. For a low number of chromosomes such as 20, the algorithm failed to converge. A population size of 200 produced good solutions. Further increase in size resulted in low speed of convergence.

B. Effect of cross-over site

A cross-over scheme in which the mating chromosomes swapped the first six genes of one chromosome with the last four genes of the other one was also tested. But it was observed that the population lost variety and the algorithm failed to converge to good solution. Typical values for highest and average fitness of the population, as obtained after 4000 iterations in one trial, are 450.6 and 393.8. The closeness of average fitness of the population and the highest fitness points to the lack of variety in chromosomes.

C. Effect of probability for mutation

A value of 0.2 for probability for mutation p_m led to better solution compared to p_m equal to 0.1, but number of iterations for convergence increased. With higher values for p_m algorithm failed to converge.

V. CONCLUSIONS

The direct kinematics of underconstrained cable-driven robots imposes difficulty due to the involvement of geometric and equilibrium constraints which need to be solved simultaneously. It is shown that genetic algorithm can be successfully employed to solve this direct kinematics. The solution procedure for an underconstrained CDPR driven using four cables has been demonstrated. The position and orientation of the mobile platform at equilibrium and also the forces in the cables can be determined when the lengths of the four cables are specified. Optimum GA parameters lead to fast convergence.

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