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A Class of Log-Type Estimators for Population Mean Using Auxiliary Information on an Attribute and a Variable Using Double Sampling Technique

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Abstract

In this paper, a class of log-type estimator using the auxiliary information in form of attribute as well as variable is proposed under double sampling technique. The bias and mean square error has been obtained up to the first order of approximation. A class of estimators with estimated optimum values of the characterising scalars is also suggested. It has also been shown that under the estimated values of characterising scalars attains the same minimum MSE of the proposed classes. The proposed classes are compared to some commonly used estimators both theoretically as well as empirically.Also it has been shown both theoretically as well as empirically that the suggested classes of estimators perform better than the conventional estimators available in the literature.

Keywords: efficiency, double sampling, mean square error, characterizing scalars

INTRODUCTION

In sampling, auxiliary information proves to be fruitful way to increase the precision of the estimator**.** This auxiliary information may exists in both the forms: auxiliary variable and auxiliary attribute. For example- the heights for male and female are different which shows that sex is a helpful attribute while dealing with height and also height is related with the weight of the person concerned which is a variable. Similarly, the amount of milk produced by a cow depends upon the breed (attribute) as well as on the diet (variable) of the cow; etc. Naik and Gupta (1996) defined ratio estimator of population mean when the prior information of population proportion of attribute as auxiliary information is available. Bahl and Tuteja (1991) have proposed an exponential type estimator by using auxiliary variable. However, the fact that the known population proportion of an attribute also provides similar type of information has not drawn as much attention. Though some work has been done in this area like

Koyuncu (), Singh (), Bhushan(), etc. But there are many situations, where the auxiliary information about the auxiliary attribute and auxiliary variable can be easily available, we can take the advantages of both the forms of auxiliary information to increase the efficiency of the estimators in various estimation problems. But in cases where such auxiliary information is lacking, we can make use of double sampling or two-phase sampling technique provided that such information may be easily and economically obtained. Neyman (1938) was the first person to introduce the double sampling technique in the history of sampling. Thereafter many renowned authors have successfully used the technique for their work (Hidiroglou and Sarndal (1998), Bhushan (2013), etc.). However, in recent years, various authors (viz, Sisodia and Dwivedi (1981), Upadhayaya and Singh (1999), Kalidar and Cingi (2006), Singh et al (2004, 2008), etc) have made the use of prior knowledge about parameters of auxiliary variable like population mean along with coefficient of variation, covariance coefficient, coefficient of kurtosis, coefficient of skewness, standard deviation, etc for estimation of population mean of a variable of interest. In fact, such prior knowledge can also be very useful when a relation between the presence (or absence) of an attribute and the value of a variable, known as point biserial correlation, is observed. Let Y be the study variable, X be the auxiliary variable and ϕ be the auxiliary attribute. Consider a finite population of size N and let, $\bar{Y} = N^{-1}\sum_{i=1}^{N} Y_i$, $\bar{X} = N^{-1}\sum_{i=1}^{N} X_i$ $P = N^{-1}\sum_{i=1}^{N}\phi_i$ be the means of the study variable , auxiliary variable and auxiliary attribute respectively. And further let, $S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$, $S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(X - \bar{X}_i)^2$, $S_p^2 = (N-1)^{-1}\sum_{i=1}^{N}(\phi_i - P)^2$ be the respective population variances for the study variable, auxiliary variable and auxiliary attribute. Assuming that a simple random sample of size n ,further assume $\bar{y} = n^{-1}\sum_{i=1}^{n} Y_i$, $\bar{x} = n^{-1}\sum_{i=1}^{n} X_i$, $p = n^{-1}\sum_{i=1}^{n}\phi_i$, as the respective sample means for the study variable, auxiliary variable and auxiliary attribute and also denote $s_y^2 = (n-1)^{-1}\sum_{i=1}^{n}(Y_i - \bar{y})^2$, $s_x^2 = (n-1)^{-1}\sum_{i=1}^{n}(X_i - \bar{x})^2$ and $s_p^2 = (n-1)^{-1}\sum_{i=1}^{n}(\phi_i - p)^2$ as the sample variances for the study variable , auxiliary variable and auxiliary attribute respectively. The present study extends the work of Bhushan and Gupta (2015) where a class of log-type estimator using auxiliary information on both attribute and variable had been proposed for the estimation of the population mean.

Consider the following estimators of the study variable

(i). General estimator of population mean in case of SRSWOR

$\bar{Y}_1 = \bar{y} =$ sample mean

with $MSE\left(\bar{Y}_1\right) = f_n \bar{Y}^2 C_y^2$ (1.1) (ii). Double sampling ratio estimator using auxiliary variable

$$\overline{Y}_2 = \frac{\bar{y}}{\bar{x}}\bar{x}'$$

with $MSE\left(\overline{Y}_2\right) = \overline{Y}^2\left[f_n C_y^2 + f_{nn'}\left(C_x^2 - 2\rho_{yx}C_yC_x\right)\right]$ (1.2)

(iii). Naik and Gupta (1996) ratio estimator using auxiliary attribute

$$\overline{Y}_3 = \frac{\bar{y}}{p}p'$$

with $MSE\left(\overline{Y}_3\right) = \overline{Y}^2\left[f_n C_y^2 + f_{nn'}\left(C_p^2 - 2\rho_{yp}C_yC_p\right)\right]$ (1.3)

(iv). Double sampling product estimator using auxiliary variable

$$\overline{Y}_4 = \frac{\bar{y}}{\bar{x}'}\bar{x}$$

with $MSE\left(\overline{Y}_4\right) = \overline{Y}^2\left[f_n C_y^2 + f_{nn'}\left(C_x^2 + 2\rho_{yx}C_yC_x\right)\right]$ (1.4)

(v). Naik and Gupta (1996) product estimator using auxiliary attribute

$$\overline{Y}_5 = \frac{\bar{y}}{p'}p$$

with $MSE\left(\overline{Y}_5\right) = \overline{Y}^2\left[f_n C_y^2 + f_{nn'}\left(C_p^2 + 2\rho_{yp}C_yC_p\right)\right]$ (1.5)

(vi). Bahl and Tuteja (1991) double sampling exponential ratio estimator using auxiliary variable

$$\overline{Y}_6 = \bar{y}\exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right)$$

with $MSE\left(\overline{Y}_6\right) = \overline{Y}^2\left[f_n C_y^2 + f_{nn'}\left(\frac{1}{4}C_x^2 - \rho_{yx}C_yC_x\right)\right]$ (1.6)

(vii). Following Bahl and Tuteja (1991) and Sawan, N (2010), exponential ratio type estimator using auxiliary attribute

$$\overline{Y}_7 = \bar{y}\exp\left(\frac{p' - p}{p' + p}\right)$$

with $MSE\left(\overline{Y}_7\right) = \overline{Y}^2\left[f_n C_y^2 + f_{nn'}\left(\frac{1}{4}C_p^2 - \rho_{yp}C_yC_p\right)\right]$ (1.7)

(viii). Bahl and Tuteja (1991) double sampling exponential ratio estimator using auxiliary variable

$$\overline{Y}_8 = \bar{y}\exp\left(\frac{\bar{x}' + \bar{x}}{\bar{x}' - \bar{x}}\right)$$

with $MSE\left(\overline{Y}_8\right) = \overline{Y}^2\left[f_n C_y^2 + f_{nn'}\left(\frac{1}{4}C_x^2 + \rho_{yx}C_yC_x\right)\right]$ (1.8)

(ix). Following Bahl and Tuteja (1991) and Sawan, N (2010), exponential product type estimator using auxiliary attribute

$$\overline{Y_9} = \bar{y}\exp\left(\frac{p' + p}{p' - p}\right)$$

with $$MSE\left(\overline{Y_9}\right) = \bar{Y}^2\left[f_n C_y^2 + f_{nn'}\left(\frac{1}{4}C_p^2 + \rho_{yp}C_yC_p\right)\right] \quad (1.9)$$

2. THE SUGGESTED CLASS OF ESTIMATORS

In this section, the following class of estimators for population mean using auxiliary information in form of both, attribute as well as variable has been proposed using the double sampling technique.

$$T_R = \bar{y}\left[1+\log\left(\frac{\bar{x}^*}{\bar{x}'^*}\right)\right]^{\alpha}\left[1+\log\left(\frac{p^*}{p'^*}\right)\right]^{\beta} \quad (2.1)$$

where α and β are the characterizing scalars.

$\bar{x}'^* = a_1\bar{x}' + b_1$; $\bar{x}^* = a_1\bar{x} + b_1$; $p'^* = a_2 p' + b_2$; $p^* = a_2 p + b_2$

such that $a_1(\neq 0), b_1$ and $a_2(\neq 0), b_2$ which are either known values or functions of the known parameters of the auxiliary variables x and auxiliary attribute ϕ such as the standard deviations S_x, S_p ,coefficient of variation C_x, C_p ,coefficient of kurtosis $\beta_2(x), \beta_2(p)$ and correlation coefficient ρ of the population.

It can be easily observed that, if $\alpha = \beta = 0$, then the proposed estimator becomes the usual per unit mean estimator $\bar{y}$ .If $\alpha = \beta = +1$, then the proposed class of estimators become a product-product type estimator and when $\alpha = \beta = -1$, then the proposed class of estimators become a ratio-ratio type estimator. Similarly, if $\alpha = 1; \beta = -1$, then the proposed class of estimators become product-ratio type estimator and if $\alpha = -1; \beta = 1$,then the proposed class of estimators become ratio-product type estimator.

3. PROPERTIES OF THE PROPOSED CLASS OF ESTIMATORS

Theorem 3.1

The bias and MSE of the proposed class of log type estimators are given as follows

$$Bias(T_R) = f_{nn'}\bar{Y}\left[\begin{array}{l}\alpha\left(\upsilon_1\rho_{yx}C_yC_x - \frac{\upsilon_1^2C_x^2}{2}\right) + \beta\left(\upsilon_2\rho_{yp}C_yC_p - \frac{\upsilon_2^2C_p^2}{2}\right)\\ +\alpha\beta\upsilon_1\upsilon_2\rho_{xp}C_xC_p + \frac{\alpha(\alpha-1)\upsilon_1^2}{2}C_x^2 + \frac{\beta(\beta-1)\upsilon_2^2}{2}C_p^2\end{array}\right] \quad (3.1)$$

$$MSE(T_R) = \bar{Y}^2\left[\begin{array}{l}f_nC_y^2 + \alpha^2 f_{nn'}\upsilon_1^2C_x^2 + \beta^2 f_{nn'}\upsilon_2^2C_p^2 + 2\alpha f_{nn'}\upsilon_1\rho_{yx}C_yC_x\\ +2\beta f_{nn'}\upsilon_2\rho_{yp}C_yC_p + 2\alpha\beta\upsilon_1\upsilon_2 f_{nn'}\rho_{xp}C_xC_p\end{array}\right] \quad (3.2)$$

where $$\upsilon_1 = \frac{a_1\bar{X}}{a_1\bar{X} + b_1} \;;\; \upsilon_2 = \frac{a_2P}{a_2P + b_2} \quad (3.3)$$

Corollary 3.2

The mean square error of the proposed class of estimator T_R will be minimum for the optimum value of the characterising parameters, given by

$$\alpha_{(opt)} = \frac{\left(\rho_{xp}\rho_{yp} - \rho_{yx}\right)}{\upsilon_1\left(1-\rho_{xp}^2\right)}\frac{C_y}{C_x} \tag{3.4}$$

$$\beta_{(opt)} = \frac{\left(\rho_{xp}\rho_{yx} - \rho_{yp}\right)}{\upsilon_2\left(1-\rho_{xp}^2\right)}\frac{C_y}{C_p} \tag{3.5}$$

and the minimum value of the mean square error within the proposed class of estimator is

$$MSE_{\min}\left(T_R\right) = \bar{Y}^2 C_y^2\left(f_n - f_{nn'} R_{y.xp}^2\right)(\text{say}) \tag{3.6}$$

where $R_{y.xp}^2$ is the multiple correlation coefficient of y on x and ϕ.

4. SOME MEMBERS OF THE CLASS OF THE ESTIMATORS T_R

It can be easily seen that the proposed class T_R is a generalized form of class of estimators for the constants $a_1(\neq 0), b_1$ and $a_2(\neq 0), b_2$ which are either real numbers or functions of the known parameters of the auxiliary variables x and auxiliary attribute ϕ such as the standard deviations S_x, S_p, coefficient of variation C_x, C_p, coefficient of kurtosis $\beta_2(x), \beta_2(p)$ and correlation coefficient ρ of the population. Some of them are listed below:

Table 1

Some Generalized members of the proposed class of estimators T_R

Double sampling based Log-type estimators		
	a	b
$T_{R_1} = \bar{y}\left[1+\log\left(\frac{\bar{x}}{\bar{x}'}\right)\right]^\alpha\left[1+\log\left(\frac{p}{p'}\right)\right]^\beta$	1	0
$T_{R_2} = \bar{y}\left[1+\log\left(\frac{\bar{x}+C_x}{\bar{x}'+C_x}\right)\right]^\alpha\left[1+\log\left(\frac{p+C_p}{p'+C_p}\right)\right]^\beta$	1	C_x
$T_{R_3} = \bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+C_x}{\beta_2(x).\bar{x}'+C_x}\right)\right]^\alpha\left[1+\log\left(\frac{\beta_2(p).p+C_p}{\beta_2(p).p'+C_p}\right)\right]^\beta$	$\beta_2(x)$	C_x
$T_{R_4} = \bar{y}\left[1+\log\left(\frac{C_x.\bar{x}+\beta_2(x)}{C_x.\bar{x}'+\beta_2(x)}\right)\right]^\alpha\left[1+\log\left(\frac{C_p.p+\beta_2(p)}{C_p.p'+\beta_2(p)}\right)\right]^\beta$	C_x	$\beta_2(x)$
$T_{R_5} = \bar{y}\left[1+\log\left(\frac{\bar{x}+S_x}{\bar{x}'+S_x}\right)\right]^\alpha\left[1+\log\left(\frac{p+S_p}{p'+S_p}\right)\right]^\beta$	1	S_x
$T_{R_6} = \bar{y}\left[1+\log\left(\frac{\beta_1(x).\bar{x}+S_x}{\beta_1(x).\bar{x}'+S_x}\right)\right]^\alpha\left[1+\log\left(\frac{\beta_1(p).p+S_p}{\beta_1(p).p'+S_p}\right)\right]^\beta$	$\beta_1(x)$	S_x
$T_{R_7} = \bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+S_x}{\beta_2(x).\bar{x}'+S_x}\right)\right]^\alpha\left[1+\log\left(\frac{\beta_2(p).p+S_p}{\beta_2(p).p'+S_p}\right)\right]^\beta$	$\beta_2(x)$	S_x
$T_{R_8} = \bar{y}\left[1+\log\left(\frac{\bar{x}+\rho}{\bar{x}'+\rho}\right)\right]^\alpha\left[1+\log\left(\frac{p+\rho}{p'+\rho}\right)\right]^\beta$	1	ρ

$T_{R_9}=\bar{y}\left[1+\log\left(\frac{\bar{x}+\beta_2(x)}{\bar{x}'+\beta_2(x)}\right)\right]^{\alpha}\left[1+\log\left(\frac{p+\beta_2(p)}{p'+\beta_2(p)}\right)\right]^{\beta}$	1	$\beta_2(x)$
$T_{R_{10}}=\bar{y}\left[1+\log\left(\frac{C_x\bar{x}+\rho}{C_x\bar{x}'+\rho}\right)\right]^{\alpha}\left[1+\log\left(\frac{C_p p+\rho}{C_p p'+\rho}\right)\right]^{\beta}$	C_x	ρ
$T_{R_{11}}=\bar{y}\left[1+\log\left(\frac{\rho\bar{x}+C_x}{\rho\bar{x}'+C_x}\right)\right]^{\alpha}\left[1+\log\left(\frac{\rho p+C_p}{\rho p'+C_p}\right)\right]^{\beta}$	ρ	C_x
$T_{R_{12}}=\bar{y}\left[1+\log\left(\frac{\beta_2(x).\bar{x}+\rho}{\beta_2(x).\bar{x}'+\rho}\right)\right]^{\alpha}\left[1+\log\left(\frac{\beta_2(p).p+\rho}{\beta_2(p).p'+\rho}\right)\right]^{\beta}$	$\beta_2(x)$	ρ
$T_{R_{13}}=\bar{y}\left[1+\log\left(\frac{\rho\bar{x}+\beta_2(x)}{\rho\bar{x}'+\beta_2(x)}\right)\right]^{\alpha}\left[1+\log\left(\frac{\rho p+\beta_2(p)}{\rho p'+\beta_2(p)}\right)\right]^{\beta}$	ρ	$\beta_2(x)$

5. CLASS OF ESTIMATORS BASED ON THE ESTIMATED VALUES OF THE CHARACTERISING SCALARS

In practice, it is rarely to know the optimum values of the characterizing scalars, hence they may be estimated by estimators based on the sample values. The optimizing values of the characterizing scalars can be written as

$$\alpha_{(opt)}=\frac{\left(\rho_{xp}\rho_{yp}-\rho_{yx}\right)}{\upsilon_1\left(1-\rho_{xp}^2\right)}\frac{C_y}{C_x}$$
$$=\frac{\left(\mu_{011}\mu_{101}-\mu_{110}\mu_{002}\right)}{\upsilon_1\left(\mu_{020}\mu_{002}-\mu_{011}^2\right)}\frac{\bar{X}}{\bar{Y}} \quad (5.1)$$

$$\beta_{(opt)}=\frac{\left(\rho_{xp}\rho_{yx}-\rho_{yp}\right)}{\upsilon_2\left(1-\rho_{xp}^2\right)}\frac{C_y}{C_p}$$
$$=\frac{\left(\mu_{011}\mu_{110}-\mu_{101}\mu_{020}\right)}{\upsilon_2\left(\mu_{020}\mu_{002}-\mu_{011}^2\right)}\frac{P}{\bar{Y}} \quad (5.2)$$

where $\mu_{abc}=\frac{1}{N}\sum_{i=1}^{N}\left(Y_i-\bar{Y}\right)^a\left(X_i-\bar{X}\right)^b\left(\phi_i-P\right)^c$

We may also take α and β as the unbiased estimator of $opt(\alpha)$ and $opt(\beta)$ such that

$$\alpha=\frac{\left(\mu_{011}\mu_{101}-\mu_{110}\mu_{002}\right)}{\upsilon_1\left(\mu_{020}\mu_{002}-\mu_{011}^2\right)}\frac{\bar{x}}{\bar{y}} \quad (5.3)$$

$$\beta=\frac{\left(\mu_{011}\mu_{110}-\mu_{101}\mu_{020}\right)}{\upsilon_2\left(\mu_{020}\mu_{002}-\mu_{011}^2\right)}\frac{p}{\bar{y}} \quad (5.4)$$

where $\hat{\mu}_{110} = m_{110}$; $\hat{\mu}_{011} = m_{011}$; $\hat{\mu}_{101} = m_{101}$; $\hat{\mu}_{200} = m_{200}$; $\hat{\mu}_{020} = m_{020}$ and $\hat{\mu}_{002} = m_{002}$ are the estimators of the population parameters μ_{110}; μ_{011}; μ_{101}; μ_{200}; μ_{020} and μ_{002} respectively such that $m_{abc} = \frac{1}{n-1}\sum_{i=1}^{n}\left(Y_i - \bar{y}\right)^a \left(X_i - \bar{x}\right)^b \left(\phi_i - p\right)^c$

Let us take

$$\hat{\mu}_{110} = \mu_{110}\left(1+e_3\right)$$

$$\hat{\mu}_{011} = \mu_{011}\left(1+e_4\right)$$

$$\hat{\mu}_{101} = \mu_{101}\left(1+e_5\right)$$

$$\hat{\mu}_{020} = \mu_{020}\left(1+e_6\right)$$

$$\hat{\mu}_{002} = \mu_{002}\left(1+e_7\right)$$

with $E\left(e_k\right) = 0, \quad \forall$ k=3,4,5,6,7 (5.5)

then, under the estimated values of characterizing scalars α and β, the proposed classes of log-type estimators becomes

$$T_R^* = \bar{y}\left[1+\log\left(\frac{\bar{x}^*}{\bar{x}'^*}\right)\right]^{\alpha}\left[1+\log\left(\frac{p^*}{p'^*}\right)\right]^{\beta} \tag{5.6}$$

Theorem 5.1

The mean square error of the class of estimators is given by

$$MSE\left(T_R^*\right) = E\left(T_R^* - \bar{Y}\right)^2$$

$$= \bar{Y}^2 C_y^2 \left(f_n - f_{nn'} R_{y.xp}^2\right)$$

$$= MSE_{\min}\left(T_R\right) \tag{5.7}$$

$$= M \text{ (say)}$$

6. Comparative study

Let us consider the following estimators for population mean

(i). Sample mean $\bar{Y}_1 = \bar{y}$ vs T_R (or T_R^*)

$$MSE\left(\bar{Y}_1\right) - M = f_{nn'}\bar{Y}^2 R_{y.xp}^2 C_y^2 \geq 0 \tag{6.1}$$

(ii). The usual ratio estimator using auxiliary variables

$$\bar{Y}_2 = \frac{\bar{y}}{\bar{x}}\bar{x}' \text{ vs } T_R \text{ (or } T_R^* \text{)}$$

$$MSE\left(\bar{Y}_2\right) - M = f_{nn'}\bar{Y}^2\left[\left(C_x - \rho C_y\right)^2 + \left(R_{y.xp}^2 - \rho_{yx}^2\right)C_y^2\right] \geq 0 \tag{6.2}$$

(iii). Naik and Gupta (1996) ratio estimator using auxiliary attribute

$$\bar{Y}_3 = \frac{\bar{y}}{p}p' \text{ vs } T_R \text{ (or } T_R^* \text{)}$$

$$MSE\left(\bar{Y}_3\right) - M = f_{nn'}\bar{Y}^2\left[\left(C_p - \rho C_y\right)^2 + \left(R_{y.xp}^2 - \rho_{yp}^2\right)C_y^2\right] \geq 0 \tag{6.3}$$

(iv). The usual product estimator using auxiliary variables

$$\overline{Y_4} = \frac{\bar{y}}{\bar{x}'}\bar{x} \text{ vs } T_R \text{ (or } T_R^* \text{)}$$

$$MSE\left(\overline{Y_4}\right) - M = f_{nn'}\bar{Y}^2\left[\left(C_x + \rho C_y\right)^2 + \left(R_{y.xp}^2 - \rho_{yx}^2\right)C_y^2\right] \geq 0 \quad (6.4)$$

(v). Naik and Gupta (1996) product estimator using auxiliary attribute

$$\overline{Y_5} = \frac{\bar{y}}{p'}p \text{ vs } T_R \text{ (or } T_R^* \text{)}$$

$$MSE\left(\overline{Y_5}\right) - M = f_{nn'}\bar{Y}^2\left[\left(C_p + \rho C_y\right)^2 + \left(R_{y.xp}^2 - \rho_{yp}^2\right)C_y^2\right] \geq 0 \quad (6.5)$$

(vi). Bahl and Tuteja (1991) exponential ratio estimator using auxiliary variable

$$\overline{Y_6} = \bar{y}\exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right) \text{ vs } T_R \text{ (or } T_R^* \text{)}$$

$$MSE\left(\overline{Y_6}\right) - M = f_{nn'}\bar{Y}^2\left[\left(\frac{C_x}{2} - \rho_{yx}C_y\right)^2 + \left(R_{y.xp}^2 - \rho_{yx}^2\right)C_y^2\right] \geq 0 \quad (6.6)$$

(vii). Following Bahl and Tuteja (1991) and Sawan, N (2010), exponential ratio type estimator using auxiliary attribute

$$\overline{Y_7} = \bar{y}\exp\left(\frac{p' - p}{p' + p}\right) \text{ vs } T_R \text{ (or } T_R^* \text{)}$$

$$MSE\left(\overline{Y_7}\right) - M = f_{nn'}\bar{Y}^2\left[\left(\frac{C_p}{2} - \rho_{yp}C_y\right)^2 + \left(R_{y.xp}^2 - \rho_{yp}^2\right)C_y^2\right] \geq 0 \quad (6.7)$$

(viii). Bahl and Tuteja (1991) exponential product estimator using auxiliary variable

$$\overline{Y_8} = \bar{y}\exp\left(\frac{\bar{x}' + \bar{x}}{\bar{x}' - \bar{x}}\right) \text{ vs } T_R \text{ (or } T_R^* \text{)}$$

$$MSE\left(\overline{Y_8}\right) - M = f_{nn'}\bar{Y}^2\left[\left(\frac{C_x}{2} + \rho_{yx}C_y\right)^2 + \left(R_{y.xp}^2 - \rho_{yx}^2\right)C_y^2\right] \geq 0 \quad (6.8)$$

(ix). Following Bahl and Tuteja (1991) and Sawan, N (2010), exponential ratio type estimator using auxiliary attribute

$$\overline{Y_9} = \bar{y}\exp\left(\frac{p' + p}{p' - p}\right) \text{ vs } T_R \text{ (or } T_R^* \text{)}$$

$$MSE\left(\overline{Y_9}\right) - M = f_{nn'}\bar{Y}^2\left[\left(\frac{C_p}{2} + \rho_{yp}C_y\right)^2 + \left(R_{y.xp}^2 - \rho_{yp}^2\right)C_y^2\right] \geq 0 \quad (6.9)$$

Thus, from above section, it can be easily seen that the proposed class of log-type estimators is far better than these above mentioned estimators, available in sampling literature.

7. EMPIRICAL STUDY

Population 1: Source: Advance Data from Vital and Health Statistics, Number 347, October 7,2004 (CDC) dealing the height of the people of different age group of the United States

Y : Height of the people, X : Weight of the people, ϕ : Sex of the people

$\overline{Y} = 140.18;\ \overline{X} = 39.63;\ P = 0.50;\ C_x = 0.482337;\ C_y = 0.191657$

$C_p = 1.014;\ \rho_{yx} = 0.973;\ \rho_{yp} = 0.07;\ \rho_{xp} = 0.073$

$R^2_{y.xp} = 0.94673;\ n = 18;\ n' = 25;\ N = 36$

Population 2: Source: Cochran, W. G. (1977): Sampling Techniques, 3rd ed. New York, John Wiley and Sons. Pg no .-34

Y : food cost, X : income of family, ϕ : size of the family greater than 3.

$\overline{Y} = 27.49;\ \overline{X} = 72.55;\ P = 0.52;\ C_x = 0.146;\ C_y = 0.369$

$C_p = 0.985;\ \rho_{yx} = 0.2521;\ \rho_{yp} = 0.388;\ \rho_{xp} = -0.153$

$R^2_{y.xp} = 0.249879;\ n = 15;\ n' = 25;\ N = 33$

Table2:

Percentage Relative Efficiency (PRE) of various estimators with respect to sample mean

Estimators	**Population 1**		**Population 2**	
	MSE	**PRE**	**MSE**	**PRE**
$\overline{Y}_1$	20.05021	100	2.858247	100
$\overline{Y}_2$	36.17583	55.42434	2.78951	102.4641
$\overline{Y}_3$	326.0257	6.149887	10.94799	26.10751
$\overline{Y}_4$	146.1538	13.71857	3.428137	83.3761
$\overline{Y}_5$	342.659	5.85136	17.57915	16.2593
$\overline{Y}_6$	10.33437	194.0149	2.761234	103.5134
$\overline{Y}_7$	94.46493	21.22503	4.051787	70.54286
$\overline{Y}_8$	65.32336	30.69379	3.080548	92.78372
$\overline{Y}_9$	102.7816	19.50759	7.367366	38.79605
$T_R\left(\text{or } T_R^*\right)$	**9.420215**	**212.8424**	**2.458286**	**116.2699**

CONCLUSION

In the light of the theoretical and empirical study given in preceeding sections, we easily conclude that the proposed class of estimators using the auxiliary information, both in the form of variable and attribute can be considered better than many of the estimators, available in the literature, under the double sampling technique.

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