Unsteady MHD Flow and Heat Transfer Over a Porous Stretching Plate

1A. K. Jhankal*, 2R. N. Jat and 3Deepak Kumar

1Department of Mathematics, Army Cadet College, Indian Military Academy, Dehradun – 248 007, India.
2,3Department of Mathematics, University of Rajasthan, Jaipur-302 004, India.*Corresponding author E-mail: anujjankal@yahoo.com

Abstract

This paper considers the problem of unsteady MHD flow over a porous stretching plate. The governing boundary layer equations are transformed into ordinary differential equations using similarity transformation which are then solved numerically using shooting technique. The effects of unsteadiness parameter, magnetic parameter, porosity, Eckert number and Prandtl number on the flow and heat transfer characteristics are presented and discussed.

Keywords: MHD, boundary layer, stretching plate, porosity, similarity solution.

NOMENCLATURE

\(a, b, c\) Constants, [-]
\(A\) Unsteady parameter \((c/a)\), [-]
\(B_0\) Constant applied magnetic field, \([Wb m^{-2}]\)
\(C_p\) Specific heat at constant pressure, \([J Kg^{-1}K^{-1}]\)
\(f\) Dimensionless stream function, [-]
\(Ec\) Eckert number \((= u^2/C_p(T_w - T_\infty))\), [-]
\( k_0 \) Permeability of porous medium, [Darcy]

\( M \) Magnetic parameter \((= \sigma_e B_0^2 / \rho a)\), [-]

\( Pr \) Prandtl number \((= \mu C_p / \kappa)\), [-]

\( S_p \) Porosity parameter \((= \nu / \rho a)\), [-]

\( t \) Dimensionless time, [s]

\( T \) Temperature of the fluid, [K]

\( u, v \) Velocity component of the fluid along the x and y directions, respectively, [m s\(^{-1}\)]

\( x, y \) Cartesian coordinates along the surface and normal to it, respectively, [m]

**Greek symbols**

\( \rho \) Density of the fluid, [Kg m\(^{-3}\)]

\( \mu \) Viscosity of the fluid, [Kg m s\(^{-1}\)]

\( \sigma_e \) Electrical conductivity, [m\(^2\) s\(^{-1}\)]

\( \eta \) Dimensionless similarity variable, \n\( \left[ = (U_\infty \nu x) \right] \)

\( \nu \) Kinematic viscosity, [m\(^2\) s\(^{-1}\)]

\( \kappa \) Thermal conductivity, [W m\(^{-2}\) K\(^{-1}\)]

\( \psi \) Stream function, \n\( \left[ = (\nu x U_\infty)^{1/2} f(\eta) \right] \)

\( \theta \) Dimensionless temperature, \n\( \left[ = (T - T_\infty) / (T_w - T_\infty) \right] \)

**Superscript**

Derivative with respect to \( \eta \)

**Subscripts**

\( \omega \) Properties at the plate

\( \infty \) Free stream condition

**INTRODUCTION**

The Study of MHD flow plays an important role in various industrial applications. Some important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aero dynamics. The problems of heat and mass transfer in the
boundary layers on stretching surfaces have attracted considerable attention during the last few decades. It is importance in connection with many engineering problems, such as wire drawing, glass-fiber and paper production, drawing of plastic films, metal and polymer extrusion and metal spinning. Both the kinematics of stretching and simultaneous heating or cooling during such processes have a decisive influence on the quality of the final products.

In his pioneering work, Sakiadis [1] developed the flow field due to a flat surface, which is moving with a constant velocity in a quiescent fluid. Crane [2] extended the work of Sakiadis [1] for the two-dimensional problem where the surface velocity is proportional to the distance from the flat surface. As many natural phenomena and engineering problems are worth being subjected to MHD analysis, the effect of transverse magnetic field on the laminar flow over a stretching surface was studied by Pavlov [3]. Andersson [4] then demonstrated that the similarity solution derived by Pavlov [3] is not only a solution to the boundary layer equations, but also represents an exact solution to the complete Navier-Stokes equations. Liu [5] extended Andersson’s results by finding the temperature distribution for non-isothermal stretching sheet, both in the prescribed surface temperature and prescribed surface heat flux conditions, in which the surface thermal conditions are linearly proportional to the distance from the origin.

The heat transfer aspect for the problem posed by Crane [2] was studied by Grubka and Bobba [6], who reported the solution for the energy equation in terms of Kummer’s functions. Several closed form analytical solutions for specific conditions also reported. Chen and Char [7] investigated the effects of suction and injection on the heat transfer characteristics of a continuous, linearly stretching sheet for both the power law surface temperature and the power law surface heat flux variations. Char [8] then studied the case when the sheet immersed in a quiescent electrically conducting fluid in the presence of a transverse magnetic field. The effect of thermal radiation on the heat transfer over a nonlinear stretching sheet immersed in an otherwise quiescent fluid has been studied by Bataller [9]. The effect of transverse magnetic field on the laminar flow over a stretching surface was studied by number of researchers Chakrabarthy and Gupta [10], Chiam [11], Ghaly [12], Raptis [13], Ishak et al. [14], Muhaimin et al. [15], Noor et al. [16], Jat and Chaudhary [17], Jhankal and Kumar [18] etc.

The unsteady boundary layer flow over a stretching sheet has been studied by Devi et al. [19], Elbashbeshy and Bazid [20] and quit recently by Tsai et al. [21]. In the present study, we consider the problem of unsteady flow over a porous stretching plate in presence of transverse magnetic field. The governing partial differential equations are transformed into ordinary differential equations using similarity transformations which are then solved numerically using shooting technique.
MATHEMATICAL FORMULATION OF THE PROBLEM

Let us consider an unsteady, laminar two-dimensional boundary layer flow over a continuously porous stretching plate. The fluid is an electrically conducting incompressible viscous fluid. It is assumed that external fluid owing polarization of charges and Hall-effect are neglected. At time \(t=0\), the plate is impulsively stretched with the velocity \(U_\infty(x, t)\) along x-axis, keeping the origin fixed in the fluid of ambient temperature \(T_\infty\). The stationary Cartesian coordinate system has its origin located at the leading edge of the plate with the positive x-axis extending along the plate, while y-axis is measured normal to the surface of the plate. A transverse magnetic field of strength \(B_0\) is assumed to be applied in the positive y-axis, normal to the surface. Under the usual boundary layer approximations, the governing equation of continuity, momentum and energy (Pai [22], Schlichting [23], Bansal [24]) under the influence of externally imposed transverse magnetic field (Jeffery [25], Bansal [26]) are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots(1)
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e B_0^2 u}{\rho} - \frac{\nu u}{k_0} \quad \ldots(2)
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\sigma_e B_0^2 u^2}{\rho C_p} \quad \ldots(3)
\]

Accompanied by the boundary conditions:

\(y = 0: u = U_w, v = 0, T = T_w\)

\(y \to \infty: u \to 0, T \to T_\infty\) \quad \ldots(4)

We assume that the stretching velocity \(U_w(x, t)\) and surface temperature \(T_w(x, t)\) are of the form:

\[U_w(x, t) = ax \frac{1}{1-ct}, T_w(x, t) = T_\infty + bx \frac{1}{1-ct} \quad \ldots(5)\]

The governing partial differential equations (1) \(\ldots(3)\) can be reduced to ordinary differential equations by introducing the following transformation

\[\eta = \left(\frac{U_w}{v_x}\right)^{1/2}, y, \Psi = (\nu x U_\infty)^{1/2} f(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} \quad \ldots(6)\]

The continuity equation (1) is satisfied by introducing a stream function \(\Psi\) such that

\[u = \frac{\partial \Psi}{\partial y} \text{ and } v = -\frac{\partial \Psi}{\partial x}.\]

The transformed nonlinear ordinary differential equations are:

\[f'''' + ff'' - f'^2 - Mf' - Spf' - A \left(f' + \frac{1}{2} f'' \eta\right) = 0 \quad \ldots(7)\]
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\[
\frac{\theta''}{Pr} + f0' - f'\theta + EcMf^{-2} - A\left(\theta + \frac{1}{2}\eta\theta'\right) = 0 \quad \text{...}(8)
\]

The transformed boundary conditions are:

\[
f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1 \quad \text{and} \quad f'(\infty) \to 0, \quad \theta(\infty) \to 0. \quad \text{...}(9)
\]

Where prime denotes differentiation with respect to \(\eta\), \(M = \frac{\sigma_e B_0^2}{\rho a}\) is the magnetic parameter, \(S_p = \frac{\nu}{\rho a}\) is the porosity parameter, \(Pr = \frac{\mu c_p}{\kappa}\) is the Prandtl number and \(Ec = \frac{u_w^2}{c_p(T_w - T_\infty)}\) is the Eckert number.

NUMERICAL SOLUTION AND DISCUSSION

The non-linear differential equations (7) and (8) subject to the boundary conditions (9) are solved by Runge-Kutta fourth order scheme with a systematic guessing of \(f'(0)\) and \(\theta'(0)\) by the shooting technique until the boundary conditions at infinity are satisfied. The step size \(\Delta \eta = 0.01\) is used while obtaining the numerical solution and accuracy upto the seventh decimal place i.e. \(1 \times 10^{-4}\), which is very sufficient for convergence. The computations were done by a programme which uses a symbolic and computer language Matlab.

Figure 1 and 2 show the velocity profile for different values of magnetic parameter (M) and unsteady parameter (A) respectively, when the other parameter is fixed. From both the figures, it is observed that the velocity gradient at the surface increases (in magnitude) with M and A. Thus, the magnitude of skin friction coefficient \(|f''(0)|\) increases as A increases (table 1). Physically, negative values of \(f''(0)\) means the solid surface exerts a drag force on the fluid. This is not surprising since the development of the velocity boundary layer is caused solely on the stretching plate. Further, the velocity is found to decreases as the distance increases from the surface increases and reaches the boundary condition at infinity asymptotically.

Figure 3, which illustrate the porosity parameter \(S_p\) on the velocity profile. We infer from this figure that the velocity profile decreases with increasing values of porosity parameter \(S_p\) but quit slowly. This phenomenon corresponds with the assumption of pure Darcy flow.

Figure 4 and 5 show that the temperature gradient at the surface increases as (in magnitude) as M and A increases respectively, which impales an increase of heat transfer rate at the surface \(|-\theta'(0)|\) as A increases (table 2).
Figure 6 which illustrate the effect of Prandtl number (Pr) on the temperature profiles. We infer from this figure that the temperature decreases with an increase in Prandtl number, which implies viscous boundary layer thickness than the thermal boundary layer. From these plots it is evident that large values of Prandtl number result in thinning of the thermal boundary layer. In this case temperature asymptotically approaches to zero in free stream region.

Figure 7, which is a graphical representation of the temperature profiles for different values of Eckert number (Ec) versus η. It is evident from these plots that temperature of the fluid decreases with increases in Eckert number. Physically it means that the heat energy is stored in the fluid due to the frictional heating.

Table 1. Numerical values of Skin friction coefficient, when $M = 0.5$ and $S_p = 0.4$

<table>
<thead>
<tr>
<th>$A$</th>
<th>$f'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.773</td>
</tr>
<tr>
<td>2</td>
<td>-1.951</td>
</tr>
<tr>
<td>3</td>
<td>-2.121</td>
</tr>
</tbody>
</table>

Table 2. Numerical values of Nusselt number when $Pr = 1.0$, $Ec = 0.05$, $M = 0.5$, $S_p = 0.4$

<table>
<thead>
<tr>
<th>$A$</th>
<th>$-\theta'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.8051</td>
</tr>
<tr>
<td>2</td>
<td>-2.0280</td>
</tr>
<tr>
<td>3</td>
<td>-2.2300</td>
</tr>
</tbody>
</table>

Figure 1: Velocity profile for various values of $M$ when $A=1$ and $S_p = 0.4$.

Figure 2: Velocity profile for various values of $A$ when $M=0.5$ and $S_p = 0.4$.

Figure 3: Velocity profile for various values of $S_p$ when $A=1.0$ and $M=0.5$. 
CONCLUSION

In the present study, we consider the problem of unsteady flow over a porous stretching plate in presence of transverse magnetic field. The governing partial differential equations are transformed into ordinary differential equations by means of similarity transformations. The resulting non-linear ordinary differential equations are solved using Runge-Kutta fourth order method along with shooting technique. The velocity and temperature distributions are discussed numerically and presented through graphs. The numerical values of Skin-friction coefficient and Nusselt number are derived, for various values of unsteady parameter A and presented through tables. From the study, following conclusions can be drawn:

- Velocity gradient at the surface increases (in magnitude) with M and A.
- Velocity profile decreases with increasing values of porosity parameter $S_p$ but quit slowly.
- Temperature gradient at the surface increases as (in magnitude) as M and A increases respectively.
- Temperature decreases with an increase in Prandtl number, which implies viscous boundary layer thickness than the thermal boundary layer.
- Temperature of the fluid decreases with increases in Eckert number.
- Shear stress and Nusselt number increase (in magnitude) due to increase parameter A.

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REFERENCES


