International Journal of Advanced Materials Science
ISSN 2231-1211 Volume 16, Number 1 (2025), pp. 01-15

http://www.ripublication.com

Nonlocal thermoelasticity theory for Monoclinic thermoviscoelastic material under Lord-Shulman model

Leena Rani and Prerak Atreja*

Professor, Dept. of mathematics, School of Basic and Applied Sciences, Galgotias university, Greater Noida, U.P, INDIA
**Hansraj Model school, Punjabi bagh, New Delhi*

Abstract

The present research deals with the time-harmonic deformation in homogeneous, thermally conducting, Monoclinic, thermoviscoelastic material due to uniformly distributed thermal source for nonlocal thermoelasticity theory. The matrix differential equation is formed by using Fourier transforms into the considered equations of displacement, stresses and temperature distribution, which are solved by eigenvalue approach. The nonlocality effect has been studied numerically and presented graphically for zinc crystal-like material in context of Lord-Shulman (L-S) model.

Key words: Generalized thermoelasticity, monoclinic crystal, relaxation time, Fourier transform

1. Introduction

Anisotropic materials are widely used in Materials Science and Engineering, Structural Engineering, Geomechanics, Biomechanics, Aerospace Engineering, Automotive Engineering, Civil Engineering and Nanotechnology. The thermoelastic stresses and strains that form inside anisotropic structures as a result of thermal treatments or temperature changes in the surrounding environment cause the structural integrity to deteriorate. For this reason, thermoelastic analysis of anisotropic materials has persisted as a crucial subject in engineering practice.

The Fourier law-based classical model of thermoelasticity offers accurate approximations for the description in a variety of engineering applications. It does, however, give rise to the dilemma of the infinite heat pulse propagation speed and, in certain real-world scenarios, may result in an insufficient explanation of heat conduction. Since the middle of the 20th century, numerous hyperbolic thermoelastic models have been created in an effort to address these flaws in conventional thermoelasticity. The Maxwell-Cattaneo law, which generalised the Fourier law and took into account a single relaxation period, replaced the Lord and Shulman (1967) model's Fourier law of heat conduction. Noor and Camin (1976) studied symmetry considerations for anisotropic shells. Zhang and Evans (1988) discussed numerical prediction of the mechanical properties of anisotropic composite materials. Li (1992) investigated generalized theory of thermoelasticity for an anisotropic medium.

Bhaskar, Varadan and Ali (1996) explained thermoelastic solutions for orthotropic and anisotropic composite laminates. Chao and Gao (2001) discussed mixed boundary-value problems of two-dimensional anisotropic thermoelasticity with elliptic boundaries. Kuo and Chen (2005) investigated steady and transient Green's functions for anisotropic conduction in an exponentially graded solid. Kumar and Rani (2007) studied disturbances due to thermomechanical sources in orthorhombic thermoelastic material. Hou, Leung and He (2008) discussed three-dimensional Green's functions for transversely isotropic thermoelastic biomaterials. Shiah and Lee (2011) discussed boundary element modeling of 3D anisotropic heat conduction involving arbitrary volume heat source. Ramp type heating in thermally conducting cubic crystal has been studied by Abbas et al. (2015). Rani and Singh (2018) studied thermal disturbances in twinned orthotropic thermoelastic material. Rani and Shekhar (2020) investigated the response of ramp-type heating in a monoclinic generalized thermoelastic material. Hobiny and Abbas (2021) discussed the generalized thermoelastic interaction in a two-dimensional orthotropic material caused by a pulse heat flux. Hobiny and Abbas (2023) considered the generalized thermoelastic interaction in orthotropic media under variable thermal conductivity using the finite element method

In the nonlocal elasticity model, Eringen [2002] assumed that the stress field at a particular point in an elastic continuum not only depends on the strain field at that point but also on strains at all other points of the body. Hence, the nonlocal continuum theory contains information about long-range forces of atoms or molecules and, thus, an internal length scale parameter should be introduced in the formulation. Over the past forty years, numerous mathematicians have expanded the notion of nonlocal elasticity to include nonlocal thermoelsticity. Eringen and Edelen (1972) discussed non-local elasticity. Eringen (1974) proposed theory of nonlocal thermoelasticity. Cracium (1996) studied the nonlocal thermoelsticity. Non-local effects in radial heat transport in silicon thin layers and grapheme sheets has been discussed by Sellitto, Jou and Bafaluy (2011). Yu, Tian and Xiong (2016) investigated the nonlocal thermoelasticity based on nonlocal heat conduction and nonlocal elasticity. Luo, Li and Tian (2021) discussed nonlocal thermoelasticity and its application in thermoelastic problem with temperature-dependent thermal conductivity. Mallick and Biswas (2024) discussed thermoelastic diffusion in nonlocal orthotropic medium with porosity.

The present article is concerned with a two-dimensional nonlocal thermoelasticity theory for Monoclinic thermoviscoelastic material for a homogeneous,isotropic, thermally conducting half-space whose surface is subjected to a thermal shock. The eigen value approach has been used to obtain the exact analytical solutions of the problem. Numerical results for the nonlocality effect have been shown for the temperature, displacements and thermal stresses distribution. To the best of the authors' knowledge the problems of Nonlocal thermoelasticity theory for monoclinic thermoviscoelastic material has not been treated systematically in the scientific literature.

2. Formulation Of The Problem

We consider a homogenous, monoclinic, nonlocal thermoelastic half-space in undeformed state at uniform temperature T_0. The rectangular Cartesian co-ordinate system (x,y,z) having origin on the plane surface z=0 with z-axis pointing vertically into medium is introduced. The surface of the half-space is subjected to thermal source acting at z=0. We consider a plane strain problem with displacement vector $\vec{u}$ = (u,0,w) and temperature change T(x, z, t), then the field equations and constitutive relations for such a medium in the absence of body forces and heat sources can be written, by following the equations given by Dhaliwal and Sherief (1980), Eringen (2002) and L- S (1967) as

$$c_{11}\frac{\partial^2 u}{\partial x^2} + c_{55}\frac{\partial^2 u}{\partial z^2} + (c_{13} + c_{55})\frac{\partial^2 w}{\partial x \partial z} - \beta_1 \frac{\partial T}{\partial x} = \rho(1 - \epsilon^2 \nabla^2)\frac{\partial^2 u}{\partial t^2}, \tag{1}$$

$$c_{55}\frac{\partial^2 w}{\partial x^2} + c_{33}\frac{\partial^2 w}{\partial z^2} + (c_{13} + c_{55})\frac{\partial^2 u}{\partial x \partial z} - \beta_3 \frac{\partial T}{\partial z} = \rho(1 - \epsilon^2 \nabla^2)\frac{\partial^2 w}{\partial t^2}, \tag{2}$$

$$K_{ij}T_{,ij} = \rho c_e (\dot{T} + \tau_o \ddot{T}) + T_o \beta_{ij}(\dot{u}_{i,j} + \tau_o \delta_{1k}\ddot{u}_{i,j}), \tag{3}$$

and

$$(1 - \epsilon^2 \nabla^2)t_{zz} = c_{13}\frac{\partial u}{\partial x} + c_{33}\frac{\partial w}{\partial z} - \beta_3 T, \tag{4}$$

$$(1 - \epsilon^2 \nabla^2)\, t_{zx} = c_{55}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right), \tag{5}$$

where c_{ij} are isothermal elastic parameters, ρ , c_e are the density, specific heat at constant strain.β_1, β_3 are thermal moduli along x and z axis. K_1 and K_3 are the coefficient of thermal conductivity. u and w are displacement components along x and z directions respectively, t is time, $\nabla = \hat{\imath}\frac{\partial}{\partial x} + \hat{k}\frac{\partial}{\partial z}$, T_o is the temperature of the medium in its natural state. T is temperature change. t_{zx} and t_{zz} are stresses. $\epsilon = e_0 a$ is elastic non local parameter (1984), a is the internal characteristic length (e.g. atomic lattice parameter in crystal, average granular distance in granular solids etc.) and e_0 is material constant. Note that in the absence of nonlocality (i.e., $\in = 0$), the equations of motion given in (1)–(2) reduce to those for a local monoclinic thermoelastic solid. Here τ_0 is the thermal relaxation time given by L-S theory.

Initially the displacements, temperature and their velocities are zero.

Regularity conditions are given by

$$\lim_{z\to\infty} u = 0, \quad \lim_{z\to\infty} w = 0, \lim_{z\to\infty} T = 0. \tag{6}$$

In order to account for the material damping behavior the material coefficients c_{ij} are assumed to be function of the time operator $D = \frac{\partial}{\partial t}$, i.e.

$$c_{ij} = c_{ij}^* \tag{7}$$

where $c_{ij}^* = c_{ij}(D)$

Assuming that the viscoelastic nature of the material is described by the Voigt model of linear viscoelasticity (1963), we write

$$c_{ij}(D) = c_{ij}\,(1+\tau_0 \frac{\partial}{\partial t}), \tag{8}$$

where τ_0 is the relaxation time assumed to be identical for each c_{ij}.

3. Solution of The Problem

Assuming time harmonic behavior

$$(u, w, T)(x, z, t) = (u, w, T)(x, z)e^{i\omega t}. \tag{9}$$

with ω is the circular frequency.

We introduce dimensionless quantities as

$$x' = \frac{\omega_1^* x}{v_1}, \quad z' = \frac{\omega_1^* z}{v_1}, \quad t' = \omega_1^* t, \quad u' = \frac{\rho v_1 \omega_1^*}{\beta_1 T_0} u, \quad w' = \frac{\rho v_1 \omega_1^*}{\beta_1 T_0} w,$$

$$\mathbf{c}_3^* = \frac{c_{33}^*}{c_{11}^*}, \quad \mathbf{c}_1^* = \frac{c_{55}^*}{c_{11}^*}, \quad \mathbf{c}_2^* = \frac{c_{13}^* + c_{55}^*}{c_{11}^*}, \quad \bar{K} = \frac{K_3}{K_1},$$

$$\tau_0' = \omega_1^* \tau_0, \omega' = \frac{\omega}{\omega_1^*}, \ \in_1 = \frac{\beta_1^2 T_0}{\rho^2 c_e v_1^2}, \quad T' = \frac{T}{T_0} \tag{10}$$

$$t'_{zz} = \frac{t_{zz}}{\beta_1 T_0}\ , \qquad t'_{zx} = \frac{t_{zx}}{\beta_1 T_0}\ , \tag{11}$$

and $v_1 = \left(\frac{c_{11}^*}{\rho}\right)^{\frac{1}{2}}$ and $\omega_1^* = \frac{c_e\, c_{11}^*}{K_1}$ are, respectively, the velocity of compressional waves in x-direction and characteristic frequency of the medium.

Equations (1)-(3) with the help of equations (7)-(11), can be written in non-dimensional form as (dropping the dashes for convenience)

$$\frac{\partial^2 u}{\partial x^2} + c_1^* \frac{\partial^2 u}{\partial z^2} + c_2^* \frac{\partial^2 w}{\partial x \partial z} - \frac{\partial T}{\partial x} = \{1 - \delta_1(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2})\} \frac{\partial^2 u}{\partial t^2}\ , \tag{12}$$

$$c_1^* \frac{\partial^2 w}{\partial x^2} + c_3^* \frac{\partial^2 w}{\partial z^2} + c_2^* \frac{\partial^2 u}{\partial x \partial z} - \beta^* \frac{\partial T}{\partial z} = \{1 - \delta_1(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2})\}^* \frac{\partial^2 w}{\partial t^2}, \tag{13}$$

$$T_{,xx} + \bar{K} T_{,zz} = (\dot{T} + \tau_0 \ddot{T}) + \epsilon_1 \{(\dot{u} + \tau_0 \ddot{u})_{,x} + \beta^*(\dot{w} + \tau_0 \ddot{w})_{,z}\}, \tag{14}$$

where dot notation represents time differentiation,

$$\delta_1 = \frac{\epsilon^2 \omega_1^{*2}}{v_1^2},\ \beta^* = \frac{\beta_3}{\beta_1},\ \bar{K} = \frac{K_3}{K_1}$$

Applying the Fourier transforms

$$\tilde{f}(\xi,\ z,\ t) = \int_{-\infty}^{\infty} \hat{f}(x,\ z,\ t)\, e^{i\xi x} dx\ . \tag{15}$$

on the resulting expressions, we obtain

$$\frac{d^2\tilde{u}}{dz^2} = M_{11}\tilde{u} + M_{13}\tilde{T} + M_{15}\frac{d\tilde{w}}{dz}, \tag{16}$$

$$\frac{d^2\tilde{w}}{dz^2} = M_{22}\tilde{w} + M_{24}\frac{d\tilde{u}}{dz} + M_{26}\frac{d\tilde{T}}{dz}, \tag{17}$$

$$\frac{d^2\tilde{T}}{dz^2} = M_{31}\tilde{u} + M_{33}\tilde{T} + M_{35}\frac{d\tilde{w}}{dz}. \tag{18}$$

where

$$M_{11} = -\frac{\{\omega^2(1 + \delta_1\xi^2) - \xi^2\}}{(c_1^* - \delta_1\omega^2)}, \quad M_{13} = \frac{-i\xi c_2^*}{(c_1^* - \omega^2\delta_1)}, \quad M_{15} = \frac{i\xi}{(c_1^* - \delta_1\omega^2)},$$

$$M_{22} = \frac{\xi^2(c_1^* - \delta_1\omega^2) - \omega^2}{(c_3^* - \delta_1\omega^2)}, M_{26} = -\frac{c_2^* i\xi}{(c_3^* - \delta_1\omega^2)}, M_{24} = -\frac{\beta^*}{(c_3^* - \delta_1\omega^2)},$$

$$M_{31} = -\frac{i\xi\varepsilon_1 N_1}{\bar{K}}, \qquad M_{33} = \frac{\{\xi^2 + N_1\}}{\bar{K}}, \qquad M_{35} = \frac{\beta^*\varepsilon_1 N_1}{\bar{K}},$$
$$N_1 = i\omega - \tau_0\omega^2.$$

The equations (16)-(18) can be written as

$$\frac{d}{dz}W(\xi,z,p) = A(\xi,p)W(\xi,z,p), \tag{19}$$

where

$$W = \begin{bmatrix} U \\ U' \end{bmatrix}, \qquad A = \begin{bmatrix} O & \mathrm{I} \\ A_1 & A_2 \end{bmatrix}, \qquad U = \begin{bmatrix} \tilde{u} \\ \tilde{w} \\ \tilde{T} \end{bmatrix}, \qquad U' = \begin{bmatrix} \tilde{u}' \\ \tilde{w}' \\ \tilde{T}' \end{bmatrix},$$
$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0 & \mathrm{M}_{15} & 0 \\ M_{24} & 0 & \mathrm{M}_{26} \\ 0 & \mathrm{R}_{35} & 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} M_{11} & 0 & \mathrm{M}_{13} \\ 0 & \mathrm{R}_{22} & 0 \\ M_{31} & 0 & \mathrm{M}_{33} \end{bmatrix},$$

To solve the equation (16),we take

$$W(\xi,z,\omega) = X(\xi,\omega)\, e^{qz} \tag{20}$$

so that

$$A(\xi,\omega)W(\xi,z,\omega) = qW(\xi,z,\omega)$$

which leads to an eigenvalue problem. The characteristic equation corresponding to matrix A is given by

$$\det[A\text{-}qI]=0 \tag{21}$$

which on expansion leads to

$$q^6 - \lambda_1 q^4 + \lambda_2 q^2 - \lambda_3 = 0 \tag{22}$$

where

$\lambda_1 = M_{15} M_{24} + M_{33} + M_{22} + M_{11} + M_{26} M_{35}$

$$\lambda_2 = M_{15}M_{24}M_{33} - M_{13}M_{24}M_{35} + M_{22}M_{33} + M_{11}M_{26}M_{35} - M_{31}M_{15}M_{26} + M_{11}M_{33} - M_{31}M_{13} + M_{11}M_{22}$$

$$\lambda_3 = M_{22}(M_{11}M_{33} - M_{31}M_{13}),$$

The roots of equation (19) are $\pm q_\ell (\ell = 1,2,3)$.

The eigenvalues of the matrix A are roots of equation (22).The eigenvector $X(\xi,\omega)$ corresponding to the eigenvalues q_ℓ can be determined by solving the homogeneous equation

$$[A\text{-}qI]\, X(\xi,\omega)=0 \tag{23}$$

The set of eigenvectors $X_\ell(\xi,\omega)$, (ℓ =1,2,3,4,5,6) may be obtained as

$X_\ell\ (\xi,\omega)= \begin{bmatrix} X_{\ell 1}(\xi,\omega) \\ X_{\ell 2}(\xi,\omega) \end{bmatrix}$

Where

$X_{\ell 1}\ (\xi,\omega)= \begin{bmatrix} -\xi \\ a_\ell q_\ell \\ b_\ell \end{bmatrix}, \qquad X_{\ell 2}(\xi,\omega) = \begin{bmatrix} -\xi q_\ell \\ a_\ell q_\ell^2 \\ b_\ell q_\ell \end{bmatrix}, \qquad \mathrm{q} = q_\ell, \ell = 1,2,3.$

$X_{\ell_a 1}\ (\xi,\ \omega)= \begin{bmatrix} -\xi \\ -a_\ell q_\ell \\ b_\ell \end{bmatrix}, \quad \mathrm{X}_{\ell_a 2}(\xi\ ,\omega) = \begin{bmatrix} \xi q_\ell \\ a_\ell q_\ell^2 \\ -b_\ell q_\ell \end{bmatrix}, \quad \ell_a = \ell + 3, \quad \mathrm{q} = -q_\ell, \ell =$ 1,2,3.

and

$$a_\ell = \frac{\{(\beta^* - c_2^*)\xi^2 - \omega^2\beta^* - c_1^*\beta^* q_\ell^2\}}{\Delta_\ell},$$

$$b_\ell = \frac{\{c_1^* q_\ell^2 \xi - (\xi^2 - \omega^2)\xi\}\{(c_1^*\xi^2 - \omega^2) - q_\ell^2(c_3^* - c_1^*\beta^*)\} - q_\ell^2 c_2^* \xi \quad \{(\xi^2 - \omega^2) - c_1^* q_\ell^2\}\beta^* - c_2^*\xi^2\}}{\xi \Delta_\ell},$$

$$\Delta_\ell = i\{(c_1^*\xi^2 - \omega^2) - (c_3^* - c_2^*\beta^*)q_\ell^2\}, \qquad \ell = 1,2,3.$$

The solution of equation (23) is given by

$$W(\xi,z,p) = \sum_{\ell=1}^{3}[B_\ell X_\ell(\xi,p)\ exp(\,q_\ell z) + B_{\ell+3}X_{\ell+3}(\xi,p)\ exp(-q_\ell z)], \quad \textbf{(24)}$$

where$B_\ell(\ell = 1,2,3,4,5,6)$ are arbitrary constants.

Thus equation (24) represents the solution of the general problem in the plane strain case of generalized homogeneous thermoelasticity by employing the eigenvalue approach and therefore can be applied to a broad class of problems in the Fourier transforms. Displacements and temperature distribution that satisfy the regularity conditions (6) are given by

$$\tilde{u}(\xi,z,p) = \ -\xi(B_4\, \mathrm{e}^{-q_1 z} + B_5\, \mathrm{e}^{-q_2 z} + B_6\, \mathrm{e}^{-q_3 z}), \qquad \textbf{(25)}$$

$$\tilde{w}(\xi,z,p) = \ -(a_1 q_1 B_4\, \mathrm{e}^{-q_1 z} + a_2 q_2 B_5\, \mathrm{e}^{-q_2 z} + a_3 q_3 B_6\, \mathrm{e}^{-q_3 z}), \qquad \textbf{(26)}$$

$$\tilde{T}(\xi,z,p) = \ (b_1 B_4\, \mathrm{e}^{-q_1 z} + b_2 B_5\, \mathrm{e}^{-q_2 z} + b_3 B_6\, \mathrm{e}^{-q_3 z}), \qquad \textbf{(27)}$$

4. Application

Dynamic thermoelastic case:

Thermoelastic Interactions due to Thermal Source

The boundary conditions at the plane surface are

$$t_{zz} = 0, \quad t_{zx} = 0, \qquad \text{at } z = 0$$

$\frac{\partial T}{\partial z}(x, z=0) = r(x,t),$ for the temperature gradient boundary,

or

$T(x,z=0) = r(x,t),$ for the temperature input boundary,. **(28)**

where r(x,t)= $\eta(x)e^{i\omega t}$

Making use of equations (9)-(11), applying the transforms defined by (15) and with the help of equations (25)–(27) in the boundary conditions (28), we obtain the expressions for displacement components, stresses, temperature distribution as

$$\tilde{u} = \frac{-\xi\ \tilde{\eta}(\xi)}{\Delta}(\Delta_1''\bar{e}^{q_1 z} - \Delta_2''\bar{e}^{q_2 z} + \Delta_3''\bar{e}^{q_3 z})e^{i\omega t},$$

$$\widetilde{w} = \frac{-\tilde{\eta}(\xi)}{\Delta}(a_1 q_1 \Delta_1''\bar{e}^{q_1 z} - a_2 q_2 \Delta_2''\bar{e}^{q_2 z} + a_3 q_3 \Delta_3''\bar{e}^{q_3 z})e^{i\omega t},$$

$$\tilde{T} = \frac{\tilde{\eta}(\xi)}{\Delta}(b_1 \Delta_1''\bar{e}^{q_1 z} - b_2 \Delta_2''\bar{e}^{q_2 z} + b_3 \Delta_3''\bar{e}^{q_3 z})e^{i\omega t},$$

$$\tilde{t}_{zz} = \frac{\tilde{\eta}(\xi)}{\Delta}(E_{34}\Delta_1''\bar{e}^{q_1 z} + E_{35}\Delta_2''\bar{e}^{q_2 z} + E_{36}\Delta_3''\bar{e}^{q_3 z})e^{i\omega t},$$

$$\tilde{t}_{zx} = \frac{\tilde{\eta}(\xi)}{\Delta}(E_{41}\Delta_1''\bar{e}^{q_1 z} + E_{42}\Delta_2''\bar{e}^{q_2 z} + E_{43}\Delta_3''\bar{e}^{q_3 z})e^{i\omega t}, \qquad \textbf{(29)}$$

where

$\Delta_1^* = E_{34}(E_{42}b_3q_3 - b_2q_2E_{43})$-$E_{35}(E_{41}b_3q_3 - b_1q_1E_{43}) + E_{36}(E_{41}b_2q_2 - b_1q_1E_{42})$

$\Delta_2^* = E_{34}(E_{42}b_3 - b_2E_{43})$-$E_{35}(E_{41}b_3 - b_1E_{43}) + E_{36}(E_{41}b_2 - b_1E_{42})$

$\Delta_1'' = (E_{35}E_{43} - E_{42}E_{36}),$
$\Delta_2'' = -(E_{34}E_{43} - E_{41}E_{36}),$
$\Delta_3'' = (E_{34}E_{42} - E_{41}E_{35}), \quad \delta_2 = 1 + \delta_1\xi^2$

$E_{34} = (-\delta_1 q_1^2 + \delta_2)E_{31},$

$E_{35} = (-\delta_1 q_2^2 + \delta_2)E_{32},$

$E_{36} = (-\delta_1 q_3^2 + \delta_2)E_{33},$

$E_{31} = (-i\xi^2 c_{13}^* T^* + c_{33}^* T^* a_1 q_1^2 - \beta^* b_1),$

$E_{32} = (-i\xi^2 c_{13}^* T^* + c_{33}^* T^* a_2 q_2^2 - \beta^* b_2),$

$E_{33} = (-i\xi^2 c_{13}^* T^* + c_{33}^*\ T^* a_3 q_3^2 - \beta^* b_3),$

$T^* = \frac{1}{\rho v_1^2}$,

$E_{41} = -c_{55}^* T^* (i\xi a_1 - 1)(-\delta_1 q_1^2 + \delta_2) q_1$,

$E_{42} = -c_{55}^* T^* (i\xi a_2 - 1)(-\delta_1 q_2^2 + \delta_2) q_2$,

$E_{43} = -c_{55}^* T^* (i\xi a_3 - 1)(-\delta_1 q_3^2 + \delta_2) q_3$.

On replacing $\varDelta$ *by* $(\frac{\omega_1^* T_0}{v_1})\varDelta_1^*$ and $T_0\, \varDelta_2^*$, respectively, we obtain the expressions for temperature gradient boundary and temperature input boundary. $\tilde{\eta}(\xi)$ is the transformed function of $\eta(x)$.

The solution due to uniformly distributed source applied on the half-space surface is obtained by setting

$$\eta(x) = \begin{cases} 1 & \text{if } |x| \le a\,, \\ 0 & \text{if } |x| > a\,, \end{cases}$$

in equation (28), where 2a is the width of strip load. Using equations (9)-(11) (after suppressing the primes) and applying the transforms defined by equation (15), we get

$$\tilde{\eta}(\xi) = \left[2\, sin\left(\frac{\xi v_1 a}{\omega_1^*}\right)/\xi\right],\ \xi \neq 0.$$

6. Particular cases:

Transversely isotropic materials

This type of medium has only one axis of thermal and elastic symmetry. We take the z-axis along the axis of symmetry. Then the non-vanishing elastic and thermal parameters are

$$\boldsymbol{c_{11}} = \boldsymbol{c_{33}},\quad \mathrm{K}_1 = K_3,\quad \alpha_1 = \alpha_3,.$$

Cubic crystal

For cubic crystals, the nonvanishing elastic and thermal parameters are

$$c_{11} = c_{33},\quad \mathrm{K}_1 = K_3 = K,\quad \beta_1 = \beta_3 = \beta_,\quad \alpha_1 = \alpha_3 = \alpha_t$$

Isotropic media

For isotropic material, every direction is a direction of elastic as well as thermal symmetry and the nonvanishing elastic and thermal parameters are

$$c_{11} = c_{33} = \lambda + 2\mu,\quad \mathrm{c}_{13} = \lambda,\quad \mathrm{c}_{55} = \mu,\quad \mathrm{K}_1 = K_3 = K,$$
$$\alpha_1 = \alpha_3 = \alpha_t,\qquad \beta_1 = \beta_3 = \beta = (3\lambda + 2\mu)\alpha_t$$

5. Inversion of the Transforms

To obtain the solution of the problem in the physical domain, we must invert the transforms in (30), for the L-S theory. These expressions are functions of z and the parameter of Fourier transform ξ and hence are of the form $\tilde{f}$ (ξ, z, t). To get the function f (x,z,t) in the physical domain, we invert the Fourier transform using

$$f(x,z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x}\,\tilde{f}\,(\xi,z,t)\,d\xi = \frac{1}{\pi}\int_{0}^{\infty}(\cos(\xi x)\,f_e - i\sin(\xi x) f_0)\,d\xi\,,$$

where f_e and f_0 are, respectively, the even and odd parts of the function $\tilde{f}$ (ξ, z, t). The method for evaluating this integral is described by Press et al. (1986), and it involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

6. Numerical Result and Discussion

Following Dhaliwal and Singh (1980) we take the case of zinc crystal-like material for numerical calculations. The physical constants used are:

ε = 0.0221, c_{11} = 1. 628 x 10^{11} Nm^{-2}, c_{13} = 0.508 x 10^{11} Nm^{-2},

ρ = 7.14 x 10^3 kgm^{-3} , c_{33}= 0.627 x 10^{11} Nm^{-2}, c_{55} = 0.770 x 10^{11} Nm^{-2},

c_e = 3.9 x 10^2 $Jkg^{-1}degree^{-1}$,$\omega_1^* = 5.01x10\,1^{11}$ s^{-1},K_1= K_3 = 1.24 x 10^2 $Wm^{-1}degree^{-1}$,

β_1 = 5.75 x 10^6 $Nm^{-2}degree^{-1}$, T_0 = 296 ^{0} K, β_3 = 5.17 x 10^6 $Nm^{-2}degree^{-1}$,.

The variations of normal boundary displacement w and boundary temperature field T

with distance x at non-dimensional time t = 0.1 are shown graphically in figures 1-4 for non-dimensional relaxation times $\tau_0 = 0.02$. The computations were carried out for z=1.0 in the range $0 \le x \le 10$ for L-S theory. The results for distributed thermal source are presented for dimensionless width a=1.

Thermal source on the surface of half-space (Temperature gradient boundary)

Uniformly Distributed Thermal Source

Figure 1 depicts the variation of normal displacement w with distance x. The values of normal displacement for nonlocality parameter $\in= 0.01$ and $\in= 0.02$ shows same oscillatory behavior about zero in the whole range. The values of normal displacement for nonlocality parameter $\in= 0.05$ shows opposite oscillatory behavior in comparison to nonlocality parameter $\in= 0.01$ and $\in= 0.02$ in the range $0 \le x \le 10$. **Figure 2**. shows variation of temperature distribution T with distance x. The values of temperature T decreases as the value of nonlocality parameter increases in the range $0 \le x \le 4.5$ and shows opposite oscillatory pattern in rest of the range. **Figure 3**. depicts the variation of normal displacement w with distance x. The values of normal displacement for t=0.2 lies between the values for t=0.1 and t=0.5 in the whole range. The values of normal displacement for t=0.1 starts with sharp increase and for t=0.5 starts with sharp decrease then become oscillatory in the whole range. **Figure 4**. shows variation of temperature distribution T with distance x. Near the point of application of source the values of temperature for t=0.1 starts with sharp decrease and for t=0.2 and t=0.5 starts with sharp increase and then become oscillatory in the whole range. The values of temperature T decrease as values of time decrease from t=0.5 to t=0.1 in the range $0.5 \leq x \leq 4$ and shows reverse oscillatory pattern in rest of the range.

Conclusion

1. The strong nonlocality effect has been observed on the values of normal displacement and temperature distribution.
2. The values of normal displacement and temperature distribution for nonlocality parameter $\in = 0.02$ lies between $\in = 0.01$ and $\in = 0.05$ in the whole range.
3. The values of normal displacement and temperature distribution for time t=0.2 and t=0.5 show same oscillatory pattern and for t=0.1 show opposite oscillatory pattern in the whole range.

4. References

[1] H.W.Lord and Y. Shulman A generalized dynamical theory of thermoelasticity J. Mech. Phys.Solids, 15**(**1967), pp.299-309

[2] Noor,A.K, and Camin,R.A (1976): Symmetry considerations for anisotropic shells, Computer Methods in Applied Mechanics and Engineering Volume 9, Issue 3, November–December 1976, Pages 317-335

[3] W.C. Zhang and K.E. Evans(1988) Numerical prediction of the mechanical properties of anisotropic composite materials, Computers & Structures, Volume 29, Issue 3, Pages 413-422

[4] X.U.Li, A generalized theory of thermoelasticity for an anisotropic medium. International Journal of Engineering Science 30(1992), pp. 571-577

[5] Bhaskar, K, Varadan, T.K. and Ali, J.S.M (1996) Thermoelastic solutions for orthotropic and anisotropic composite laminates, Composites Part B: Engineering, Volume 27, Issue 5, 1996, Pages 415-420.

[6] Chao, C-K and Gao, B (2001) Mixed boundary-value problems of two-dimensional anisotropic thermoelasticity with elliptic boundaries, International Journal of Solids and Structures, Volume 38, Issues 34–35, August 2001, Pages 5975-5994.

[7] Kuo, H-Y and Chen, T(2005) Steady and transient Green’s functions for anisotropic conduction in an exponentially graded solid, Volume 42, Issues 3–4, February 2005, Pages 1111-1128.

[8] R.Kumar and L.Rani, Disturbance due to mechanical and thermal sources in orthorhombic thermoelastic material IJAME, 12(2007), 677-692.

[9] Hou, P.F, Leung, A.Y.T, He, Y.J. (2008): Three-dimensional Green's functions for transversely isotropic thermoelastic biomaterials, International Journal of Solids and Structures, Volume 45, Issue 24, 1 December 2008, Pages 6100-6113.

[10] Shiah, Y.C. and Lee, R.F. Boundary element modeling of 3D anisotropic heat conduction involving arbitrary volume heat source, Mathematical and Computer Modelling, Volume 54, Issues 9–10, November 2011, Pg 2392-2402.

[11] I.A.Abbas, R. Kumar and L.Rani Thermoelastic interaction in a Thermally conducting cubic crystal subjected to ramp-type heating. Applied Mathematics and computation, 254(2015), pp. 360-369

[12] L. Rani, and V. Singh Thermal disturbances in twinned orthotropic thermoelastic material Int. J. of Applied Mechanics and Engineering, 23(2018), pp.897-910,.

[13] L. Rani and S. Shekhar (2020) Response of ramp-type heating in a monoclinic ssgeneralized thermoelastic material Multidiscipline Modeling in Materials and Structures, © Emerald Publishing Limited 1573-6105, DOI 10.1108/MMMS-12-2019-0214.

[14] Eringen, A.C. (1974) Theory of nonlocal thermoelasticity, International Journal of Engineering Science, Volume 12, Issue 12, Pages 1063-1077

[15] Cracium, B "On nonlocal thermoelsticity," Ann. St. Univ. Ovidus Constanta, vol. 5, pp. 29–36, 1996.

A. Sellitto, D. Jou and J. Bafaluy, Non-local effects in radial heat transport in silicon thin layers and graphene sheets, 2011, https://doi.org/10.1098/rspa.2011.0584

[16] Yu, Y.J, Tian,X-G, Xiong, Q-L (2016): Nonlocal thermoelasticity based on nonlocal heat conduction and nonlocal elasticity, European Journal of Mechanics - A/Solids, Volume 60, Pages 238-253

[17] Luo, Pengfei, Li, Xiaoya and Tian Xiaogeng (2021): Nonlocal thermoelasticity

and its application in thermoelastic problem with temperature-dependent

[18] thermal conductivity, European Journal of Mechanics - A/Solids, Volume 87, 104204.

[19] Mallick, A and Biswas, S(2024): Thermoelastic diffusion in nonlocal orthotropic medium with porosity, Composite Structures, Volume 337, 1 June 2024, 118043

[20] W.H.Press, S.A.Teukolshy, W.T.Vellerling and B.P. Flannery, Numerical Recipes

p.134, Cambridge University Press, Cambridge, 1986.

[21] R.S.Dhaliwal and H.H.Sherief Generalized thermoelasticity for anisotropic media

Quart. of Applied Mathematics 38(1980), pp.1-8

[22] Eringen AC. Nonlocal continuum field theories. New York (NY): Springer; 2002.

[23] Eringen, A.C. (1984): Plane waves in nonlocal micropolar elasticity,

[24] Int. J. Engng Sci., Vol. 22, No.8-10, pp. 1113-1121.

[25] Kaliski, S. (1963) *Proc.Vib.Prob.* **4:** 319- 329.

[26] Kumar R, Chawla V (2011) A study of plane wave propagation in anisotropic three-phase-lag and two-phase-lag model. Int. Comm. Heat Mass Tran. 38:1262-1268.

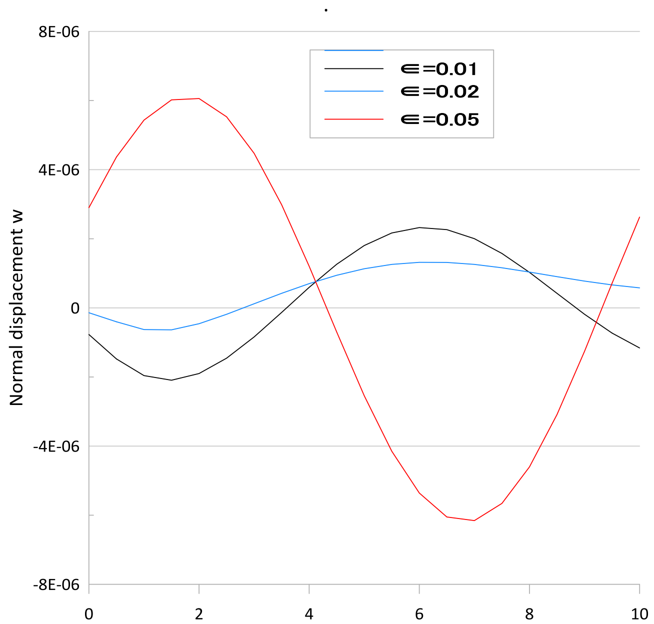

Figure 1: Variation of normal displacement w with distance x

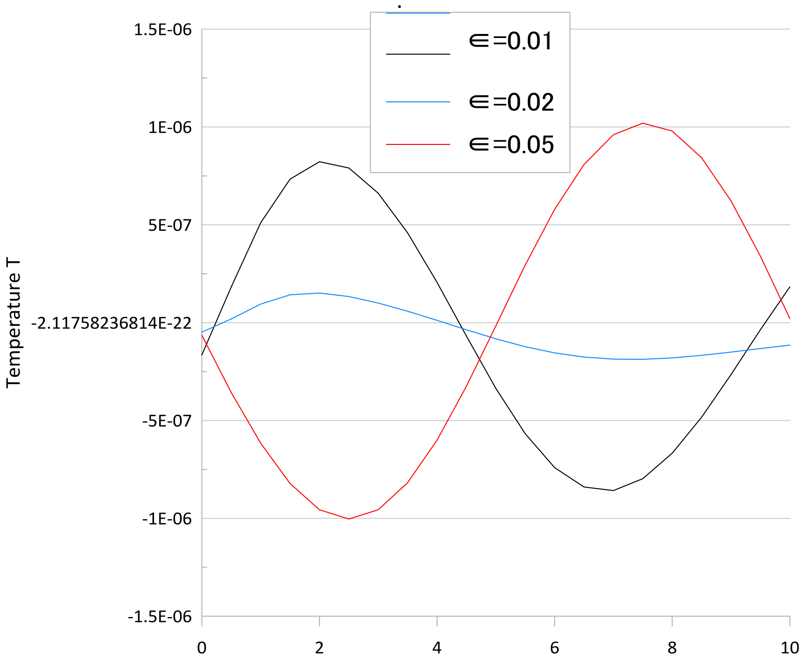

Figure 2: Variation of temperature T with distance x.

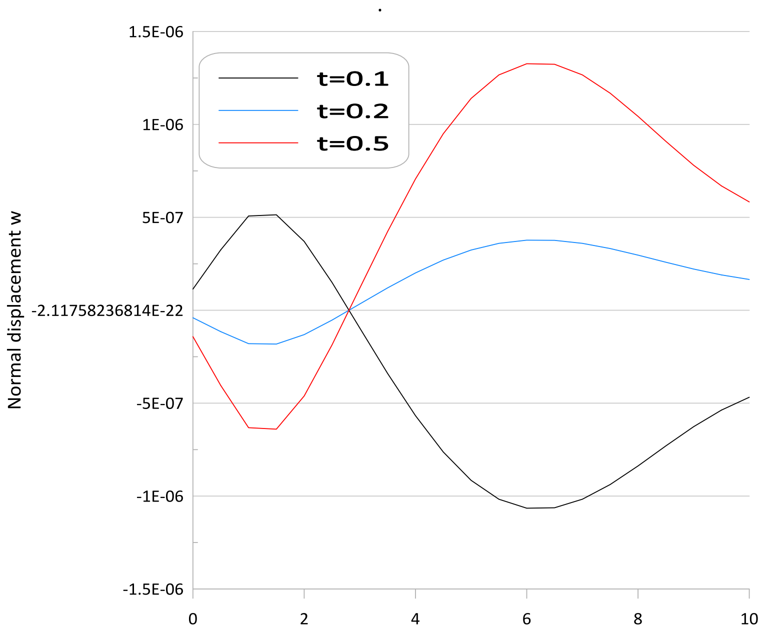

Figure 3 : Variation of normal displacement w with distance x.

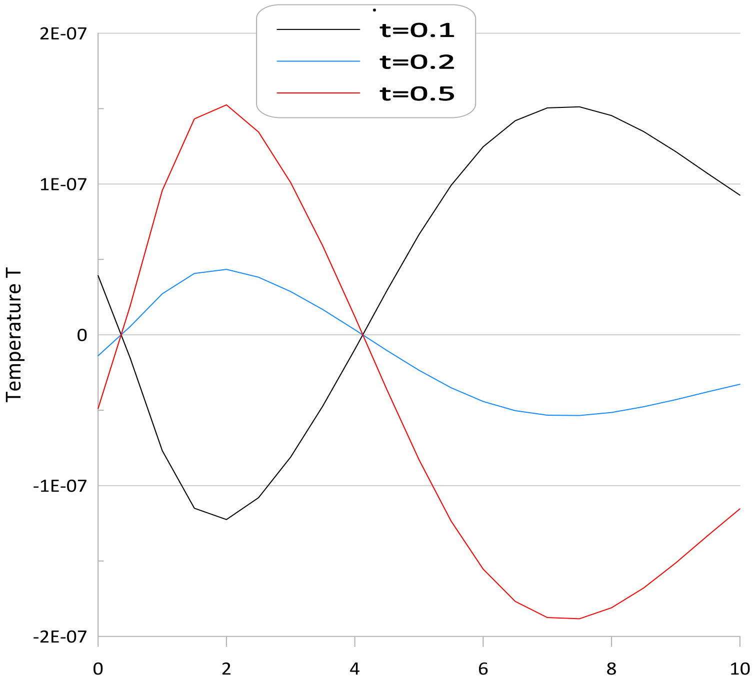

Figure 4 : Variation of temperature T with distance x.