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Isomorphism among Kinematic Chains and Distinct Mechanisms

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Abstract

This paper presents a new method for the identification of the distinct mechanisms (DM) from a given mechanism kinematic chain (KC). The method is based on Weighted Physical Connectivity Matrix [WPCM] of the given KC. The two structural invariants namely sum of absolute characteristic polynomial coefficients [WPCMP∑] and Maximum absolute value of characteristic polynomial coefficient [WPCMPmax] have been derived and used as the identification numbers of the KCs.

Keywords: Kinematic chain; Distinct mechanism; Kinematic pairs.

NOMENCLATURE,

C: Cylinder lower pair,F: Planer lower pair,G: Spheric lower pair,HP: Higher pairs (point contact),HL: Higher pairs (line contact) ,P: Prismatic lower pairs,R: Revolute lower pairs,SL: Screw lower pairs,

1. INTRODUCTION

Over the past several years much work has been reported in the literature on the structural synthesis of kinematic chains and mechanisms. Undetected isomorphism results in duplicate solutions and unnecessary effort. Therefore, the need for a reliable and efficient algebraic method for this purpose is necessary. Identifying isomorphism among kinematic chains using characteristic polynomials of adjacency matrices of corresponding kinematic chains are simple methods Raicu [1], Mruthyunjaya and Raghavan [2] , Yan and Hall [3]. But the reliability of these methods was in questions as several counter examples were found by Mruthyunjaya and Balasubramanium [4]. The test proposed by [4] is based on characteristic coefficients of the 'Degree matrix' of the graph of the kinematic chains. The elements of the degree matrix were sum of the degree of vertices (degree or type of links) or unity in a link-link adjacency matrix. Later on this test was also found unreliable. Mruthyunjaya [5] proposed the representation polynomial for detecting isomorphism between two kinematic chains. The representation polynomial is the determinant of the generalized adjacency matrix, called representation matrix of the kinematic chain. But the representation matrix requires the use of a large number of symbols, the calculation and comparison of the representation polynomials is not as easy as that of the characteristic coefficients of the adjacency matrix.. One important aspect of structural synthesis is to develop the all possible arrangements of kinematic chain and their derived mechanisms for a given number of links, joints and degree of freedom, so that the designer has the liberty to select the best or optimum mechanisms according to his requirements. In the course of development of kinematic chains and mechanisms, duplication may be possible. For determining the distinct mechanisms of a kinematic chain, Several other methods like Mruthyunjaya [6] , Rao [7-13], Agrawal [14-15], Ambeker [16-17], Hasan [18-21,23], Hwang [24,27], Uicker [25], Yadav [26], Zhang [28], Yang [29], and Zou [30], etc. are available in the literature. But in most of the methods, either there is a lack of uniqueness or consume more time. Hence, it is needed to develop a computationally efficient method to determine the distinct mechanisms of a kinematic chain. In the present case, a new , easy and reliable method is used to determine the distinct mechanisms of a kinematic chains of 8 Links, 10 Joints, 1degree of freedom . Critical study of kinematic chain and mechanism structure has revealed that the performance of the joints is affected by the degree of the links (types of links). From the [JJ] matrix, the two structural invariants [SCPC] and [MCPC] are derived based on the characteristic polynomial coefficients of the [JJ] matrix using the software MAT LAB. These structural invariants are same for identical equivalent mechanisms and different for distinct mechanisms. Hence, in this way, it is possible to identify all distinct mechanisms derived from a given kinematic chain.

2. REPRESENTATION OF KINEMATIC PAIRS

The two links are connected either by lower pair or by higher pair. To distinguish them in physical connectivity matrix [PCM], the lower pairs and higher pairs are represented by numeric ‘1’ and ‘2’ respectively. Lower pairs and higher pairs are further distinguished based on the respective pairs with the number followed by decimal. All the kinematic pairs (KP) are distinguished by assigning different numeric values. Let R=1.1, P=1.2, C=1.3, SL=1.4, F=1.5, G=1.6, HP=2.1 and HL=2.2. These values are assumed to distinguish the different KP.

2.1 Physical connectivity matrix [PCM]

Once the links of the mechanism have been numbered from 1 to n, [PCM] is defined as a square symmetric matrix of order n. The elements of [PCM] are entered with either zero or the type of kinematic pair and given by equation (1).

$$[PCM] = \{P_{ij}\}_{nxn,} \text{ --(1)}$$

Where, P_{ij} = Type of kinematic pair between i^{th} link and j^{th} link that are directly connected.

= 0, when i^{th} and j^{th} link are not connected directly.

Off course; $P_{ii} = 0$

2.2 Degree vector (V)

The degree of link actually represents the type of link like binary, ternary, quaternary etc. Let

d (v_i) =2, for binary link, d (v_i) =3, for ternary link, d (v_i) = 4, for quarter nary link, d (v_i) =k, for k-nary link. The degree vector (V) represents the degree of individual link and is given by equation (2).

$$V = [v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ - \ - \ - \ - \ v_n] \text{ ----------------(2)}$$

2.3 Relative weight of degree of vector

Critical study of kinematic structure has revealed that there is a strong correlation between the network formed by the different types of links, KP, and the machine performance like wear, reliability, and susceptibility to manufacturing error because of link tolerances and joint/bearing clearance etc. So, this information should also be added in the [PCM] matrix in the form of relative importance of v_i to v_j and vice versa to construct a [WPCM] matrix shown in equation (3).

$$\mathbf{W_{ij}} = \frac{[\mathbf{v_i / v_j + v_j / v_i}]}{2} \text{ -------------(3)}$$

3. WEIGHTED PHYSICAL CONNECTIVITY MATRIX [WPCM]

For increasing the discrimination power for detection of isomorphism and identification of DM, the mutual effect of W_{ij} are introduced into each element of [PCM] matrix and so [WPCM] is represented by equation (4).

$$[WPCM] = \{g_{ij}\}_{nxn} \text{ ------------(4)}$$

Where . $g_{ij} = (P_{ij}) \times (W_{ij})$

4. METHODOLOGY

The characteristic polynomial is generally derived from (0, 1) adjacency matrix. Many researchers have reported co-spectral graphs (the non-isomorphic KC having same Eigen spectrum derived from (0, 1) adjacency matrix). But the Proposed [WPCM] matrix has additional information about the types of KP and types of links existing in a mechanism KC. It has been verified that the characteristic polynomial and characteristic polynomial coefficients of [WPCM] matrix are unique to clearly identify the mechanisms and even KC with co-spectral graphs. The proposed [WPCM] matrix provides distinct set of characteristic polynomial coefficients of the kinematic chain with co-spectral graph also. Here the new composite invariants [WPCMP∑], [WPCMPmax] of [WPCM] matrix are proposed and are obtained by using software MATLAB.

The mechanisms are obtained by fixing the links of the KC turn in turn. If link 1 is fixed, the diagonal element P_{11} of [PCM] is changed from 0 to 1. Then it will be the representation of the first mechanism with link 1 fixed. The structural invariants of [WPCM] matrix of the first mechanism are then calculated. These invariants are the characteristic numbers of the first mechanism. This process is repeated for the second link and so on. In this way, sets of invariants equal to the number of the links are obtained. Some of them are same and others are different. The same structural invariants represent the corresponding structurally equivalent links that constitute one DM.

5. Examples-1

Considering 2 KCs with 10- bars of class IV (h) [22] as shown in Fig. 1 and Fig. 2 in which the number of binary (n_2) and 5-nary (n_5) links are same. [WA] and [WB] represent the [WPCM] matrices of chain 1 and 2 shown in Fig. 1 and Fig. 2 respectively. The [WPCM] matrices of the inversions of mechanism KC are obtained by fixing the links in turn and are represented by suffix 1, 2, 3, ----------10.

[WPCMP∑] for WA, WA1, -----WA10 = 1.1243e+003, 1.2580e+003, 1.2580e+003, 1.5393e+003, 1.5393e+003, 1.5393e+003, 1.5393e+003, 1.5393e+003, 1.5393e+003, 1.5393e+003 and 1.5393e+003 respectively.

[WPCMP∑] for WB, WB1, ---------WB10 = 1.2763e+003, 1.3792e+003, 1.3792e+003, 1.5034e+003, 1.5034e+003, 1.5034e+003, 1.5034e+003, 1.5034e+003, 1.5034e+003, 1.8071e+003 and 1.8071e+003 respectively.

[WPCMPmax] for WA, WA1, -----WA10 = 393.6130, 393.6130, 393.6130, 393.6130, 393.6130, 393.6130, 393.6130, 393.6130, 393.6130, 393.6130, 393.6130 and 393.6130 respectively

[WPCMPmax] for WB, WB1, -----WB10 = 397.7265, 397.7265, 397.7265, 366.3287, 366.3287, 366.3287, 366.3287, 366.3287, 366.3287, 488.1475 and 488.1475.

Our method reports that chain 1 and 2 shown in Fig. 1 and Fig. 2 are non-isomorphic as the set of structural invariants [WPCMP∑] and [WPCMPmax] are different for both the KC. It is evident that 2 distinct mechanisms can be derived from chain 1(shown in Fig.1) because there are only two different values of the structural invariant [WPCMP∑] i.e. 1.2580e+003, 1.5393e+003 for the mechanisms obtained by fixing the links 1 to 10 in turns. Similarly, **3** distinct mechanisms can be derived from chain 2 (shown in Fig.2), as there are 3 different values of [WPCMP∑] i.e. 1.3792e + 003, 1.5034e + 003 and 1.8071e + 003 for the mechanisms obtained by fixing the link 1 to 10 in turn. Note that the result obtained by using other methods given in the literature is the same as we obtained.

6.1 Example –2

The second example also concerns the mechanism KC of 10-bars as shown in Fig. 3. Our methods reports that 10 DM can be derived from this chain as there are 10 different values of [WPCMP∑] of the inversion mechanism of KC- 3.

$$[WA]=\begin{pmatrix} 0 & 1.1 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 \\ 1.1 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 0 \\ 0 & 1.595 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.595 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.595 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 & 0 \\ 1.595 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.595 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 \\ 1.595 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 \\ 0 & 1.595 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.1 \\ 1.595 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 \end{pmatrix}$$

$$[WB]=\begin{pmatrix}
0 & 0 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 1.595 & 1.595 \\
0 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 0 & 1.595 & 1.595 \\
0 & 1.595 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.595 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.595 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
1.595 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.595 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 \\
1.595 & 0 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 \\
1.595 & 1.595 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.595 & 1.595 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$$[WC1]=\begin{pmatrix}
1 & 1.1458 & 0 & 0 & 0 & 0 & 1.5 & 1.5 & 0 & 1.375 \\
1.1458 & 0 & 1.1917 & 0 & 0 & 0 & 0 & 0 & 1.1917 & 0 \\
0 & 1.1917 & 0 & 1.1917 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.1917 & 0 & 1.1917 & 0 & 0 & 1.1917 & 0 & 0 \\
0 & 0 & 0 & 1.1917 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.1 & 0 & 1.1 & 0 & 0 & 0 \\
1.5 & 0 & 0 & 0 & 0 & 1.1 & 0 & 0 & 0 & 0 \\
1.5 & 0 & 0 & 1.1917 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.1917 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.1917 \\
1.375 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.1917 & 0
\end{pmatrix}$$

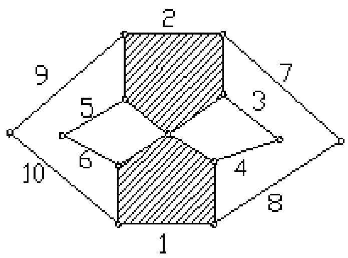

Fig. 1

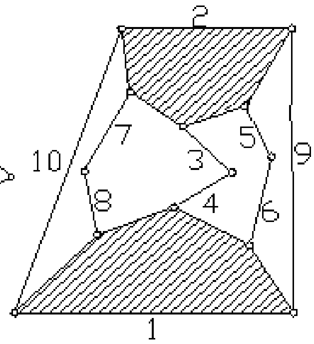

Fig.2

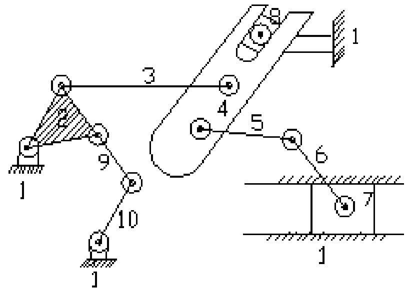

Fig.3

CONCLUSION

In the proposed method, the [PCM] matrix is able to distinguish the type of KP between two links. The [WPCM] matrix is derived from [PCM] matrix, which has the additional information about the type of links that are directly connected in the form of mutual interactive effect of the relative weights. The two structural invariants [WPCMP∑] and [WPCMPmax] are derived from the [WPCM]. These invariants are able to detect isomorphism among the mechanism KC and even the KC with co-spectral graph. According to the author exhaustive study, this method is a simple , easy to compute and reliable for the identification of the distinct mechanisms of kinematic chains up to 10 links and more work is needed for the kinematic chains of more than 10 links. Such a new identification system would be extremely selective and would minimize, if not completely, eliminate the possibility of duplicate identification for structurally different mechanisms.

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