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Review on Analysis of Functionally Graded Material Beam Type Structure

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Abstract

Functionally Graded Materials are those in which the volume fraction of the two or more constituent materials is varied continuously as a function of position along certain dimension(s) of the structure. This paper presents a review of the developments in analysis of FGM beam type structures with an emphasis on the recent work published. Diverse areas relevant to various aspects of analysis of FGM beam type structures are reflected in this paper. They include various deformation theories, effect of property distribution laws along coordinate, boundary conditions and system parameters effect. Some other aspects in analysis of FGM beam which remain as open problems are outlined in the conclusions and can be undertaken as Scope for future work.

Keywords: Functionally graded materials (FGMs); beams; deformation theory; system parameter.

1. Introduction

FGMs possess a number of advantages, including an improved residual stress distribution, high temperature withstanding ability, higher fracture toughness, improved strength, and reduced stress intensity factors, which make them attractive in many engineering sector applications such as aerospace, aircraft, automobile, defence, biomedical and electronic industries. Many primary and secondary structural elements, such as helicopter rotor blades, turbine blades, robot arms and space erectable booms, can be idealised as beam type structure. As the applications of FGMs are gaining increasing importance in the aforesaid sectors, wherein these components are subjected to vibration and instability, an analysis of FGM beam type structures may be worth of an important research review topic.

The present review concentrates on several aspects, to be reviewed for analysis, include the following:

- Various deformation theories for analysis of beams
- Effect of property distribution laws along coordinate
- Effect of other system parameters like effect of environment, effect of rotation and effect of pre-twist angle etc.

1.1 Various Deformation Theories

The oldest and the well-known beam theory is the Euler–Bernoulli beam theory (or classical beam theory—CBT) which assumed that straight lines perpendicular to the mid-plane before bending remain straight and perpendicular after bending. As a result of this assumption, transverse shear strain is neglected. Although this theory is useful for slender beams and plates, it does not give accurate solutions for thick beams and plates. The next theory is the Timoshenko beam theory (the first order shear deformation theory—FSDT) which assumed that straight lines perpendicular to the mid-plane before bending remain straight, but no longer remain perpendicular to the mid-plane after bending. In FSDT, the distribution of the transverse shear stress with respect to the thickness coordinate is assumed constant. The second order shear deformation plate theory further relaxes the kinematic hypothesis by removing the straightness assumption; i.e., the straight normal to the middle plane before deformation may become cubic curves after deformation. The third-order shear deformation theory (TSDT) which assumed parabolic distribution of the transverse shear stress and strain with respect to the thickness coordinate was proposed for beams with rectangular cross-sections (Wang *et al*, 2000). Also, zero transverse shear stress condition of the upper and lower fibres of the cross-section is satisfied without a shear correction factor in TSDT. The most significant difference between the classical and shear deformation theories is the effect of including transverse shear deformation on the predicted deflections, frequencies, and buckling loads Reddy (2007).

1.2 Property Distribution Laws

There are various models developed to determine the material properties of FGM. Such as Rules of mixtures employ bulk constituent properties assuming no interaction between phases, in Variational approach, variational principles of thermo-mechanics used to derive the bounds for effective thermo-physical properties and Micromechanical approaches include information about spatial distribution of the constituent materials.

The material properties variation for FGM is derived from either Power Law Gradation or Exponential Gradation Law. The most commonly used models for most of the literature that express the variation of material properties in FGMs is the power law distribution of the volume fraction given by,

$$P\left(z\right)=(P_m-p_c)\left(\frac{2z+h}{2z}\right)^n+P_c \tag{1}$$

Where Pc and Pm are the corresponding material properties of the ceramic and metal, and n is the volume fraction exponent, which have values greater than or equal to zero.

In exponential gradation the materials properties are assume to vary according to,

$$P\left(z\right)=P_0\,e^{k\left(z+\frac{t}{2}\right)} \tag{2}$$

Where P_0 refers the material properties of bottom surface of the FGM structure and '*k*' is the parameter describing the gradation across the thickness direction. This law reflects the simple rule of mixtures.

1.3 Effect of System Parameters

Various methods used by several researchers for the analysis of vibration and stability for FGM and Effect of system parameters like effect of thermal environment, effect of rotation and effect of pre-twist angle is presented in the section of literature review.

2. Literature Review

2.1 Bases on Deformation Theories

There are many studies related with the problem of free vibration of beams based on CBT and FSDT (Timoshenko et al 1955; Hurty et al 1967; Farghaly 1994; Banerjee 1998; Nallim et al 1999; Kim & Kim 2001; Lee et al 2003; Auciello et al 2004; Zhou 2001; Lee et al 2004; S et al 2005a, b; Kocat et al 2005a, b). The relationship between the bending solution of TSDT and those of CBT and FSDT was presented (Wang et al 2000). The exact stiffness matrix was derived from the solutions of differential equations according to TSDT for isotropic beams (Eisenberger 2003). Frequency equations and characteristic functions of homogeneous orthotropic beams having different boundary conditions were obtained, and the first six natural frequency parameters was tabulated for different values of stiffness ratios and values of thickness-to-length ratios (Soldatos et al 2001). Static deflections of the laminated composite beams subjected to uniformly distributed load were studied using the classical, the first-order, the second-order and the third-order beam theories (Khdeir et al 1997).

2.2 Bases on Distribution Laws and System Parameters

Santare et al (2000) have developed integral finite elements to estimate the dynamic characteristics of elastic-viscoelastic composite (EVC) structures such as sandwich beam, plate and shell structures with viscoelastic material as core layer. Paulino et al (2001) have made an attempt to show that the correspondence principle can be applied to the study of viscoelastic FGM under the assumption that the relaxation moduli for shear dilation are separable functions in space and time. Chakraborty et al (2003) have developed a beam finite element to study the thermo elastic behaviour of FGM beam structures with exponential and power law variation of material properties along thickness. Zhu et al (2004) have developed a method to solve 2-D elasticity equations for an FGM beam. Chaofeng et al (2006) have investigated the stress distribution in thick FGM beam subjected to mechanical and thermal loads with arbitrary end conditions. Nirmala et al (2006) have derived an analytical expression to determine the thermoelastic stresses in a three layered composite beam system having an FGM as the middle layer. Bhangale et al (2006) have studied the static and dynamic behaviour of FGM sandwich beam in thermal environment having constrained viscoelastic layer using finite element method. Li (2008) has developed a unified approach to analyze static and dynamic behaviour of FGM beam of Timoshenko, Euler-Bernoulli and Rayleigh type. Salai et al (2009) have presented a theoretical analysis of FGM beams using sigmoid function. Aydogdu et al (2007) studied free vibration analysis of functionally graded beams with simply supported edges. Kapuria et al (2008) have used zigzag theory to investigate both the static and dynamic behaviour of beams made of FGM for different end conditions. The effects of material properties and inertia of the moving load on the dynamic behaviour of an FGM beam were studied by Khalili et al (2010) using a mixed Ritz-DQ method. Simsek (2010a, b, c) presented the dynamic analysis of FGM beams using different higher order theories. Akhtar et al (2008) presented the static behaviour of various FGM beams. Aminbaghai et al. (2012) have carried out the modal analysis of second order shear deformable FGM-beams considering property variations in both transverse and longitudinal directions.

Sladek et al (2006) have developed an efficient numerical method to calculate the fracture parameters such as stress intensity factor and T-stresses of a FG orthotropic beam subjected to thermal and impact mechanical load. Huang et al. (2009) investigated the bending problem of a FG anisotropic beam subjected to thermal and uniformly distributed load using a polynomial stress function. Jurij et al (2007) have investigated the effect of thermal load on the natural frequencies of simply supported beam and clamped beam. A study of thermal buckling and vibration of sandwich beam with composite facings and viscoelastic core is carried out by Pradeep et al (2007). Evandro et al (2008) have used finite element method to evaluate nonlinear response of structures subjected to thermo-mechanical loading. A beam made up of FGM simply supported at both the ends and subjected to lateral thermal shock loads is investigated by Babai et al (2008). Guo et al (2009) have investigated the coupled thermoelastic vibration characteristics of axially moving beams using differential quadrature (DQ) method. Mahi et al (2010) have studied the free vibration of FG beams with temperature dependent properties. The effects of material constants, transverse shear deformation, temperature-dependent material properties, in-plane loading and boundary conditions on the nonlinear behaviour of FGM beams are investigated by Ma et al (2011) using a shooting method. Chen et al (1999) studied the effect of temperature on frequency, loss factor and control of a flexible beam with a constrained viscoelastic layer and shape memory alloy layer. The effects of material composition, temperature dependent properties, slenderness ratio on thermal buckling and vibration of FG beams are investigated by Wattanasakulpong et al (2011).

Vinod et al (2007) have formulated an approximate spectral element for uniform as well as tapered rotating Euler-Bernouli beam in order to carry out both free vibration and wave propagation analysis. A super element having shape functions as a combination of polynomials and trigonometric functions is used by Gunda et al (2007) to study the dynamic analysis of rotating tapered beams. Bazoune (2007) has investigated the problem of free vibration of a rotating tapered beam by developing explicit expressions for the mass, elastic and centrifugal stiffness matrices in terms of the taper ratios. Lesaffre et al (2007) have done the stability analysis of rotating beams using the Routh-Hurwitz criterion. Lee et al (2007) have developed an exact power-series solution for free vibration of a rotating inclined Timoshenko beam. Das et al (2007) have studied the large displacement free vibration analysis of linearly tapered rotating beam. Ouyang et al (2007) have presented a dynamic model for the vibration of a rotating Timoshenko beam subjected to a three-directional load moving in the axial direction. Attarnejad et al (2008) have studied free vibration of non-prismatic rotating Euler-Bernoulli beams using differential transform method. Lin et al (2008) have modelled the blade of a horizontal-axis wind power turbine as a rotating Bernoulli-Euler beam with pre-cone angles and setting angles. Piovan et al (2009) have developed a rotating non-linear beam model accounting for arbitrary axial deformation to study the dynamics of rotating beams made of FGM. Yuksel et al (2009) have studied bending vibrations of a radially rotating beam with end mass subjected to different base excitations using the Lagrangian's approach. Ahmad et al (2009) have investigated the vibration characteristics of rotating FGM cylindrical shells using Budiansky and Sanders, thin shell theory. Hosseini et al (2009) have used multi-scale method to investigate free vibration analysis of simply supported rotating shaft with nonlinear curvature. Huang et al (2009) have provided a power series solution to free vibration of rotating inclined Euler beam. Divergence instability and vibration of a rotating Timoshenko beam with pre-cone and pitch angles are investigated by Lee et al (2009). Yardimoglu (2010) has used a finite element model based on the coupled displacement field for vibration analysis of rotating Timoshenko beam of equal strength.

Lin et al (2003) have studied the coupled bending–bending vibration of a rotating pre-twisted beam with an elastically restrained root and a tip mass, subjected to the external transverse forces and rotating at a constant angular velocity. Young et al (2003) have investigated the dynamic stability of a spinning pre-twisted beam subjected to random axial force by using stochastic averaging method along with mean square stability criterion. The effect of pre-twist angle of an aerofoil blade simplified as a rotating Euler as well as Timoshenko beam has been investigated by Subuncu et al (2006a, b) using finite element method. Jhung (2007) have studied the vibration characteristics of a rectangular twisted beam with pins surrounded with liquid and the safety assessment of the potential for fretting-wear damages caused by foreign particles. Mohanty (2007) has studied parametric instability of pre-twisted cantilever beam with localized damage. The effects of various parameters such as shroud dimensions, pre-twist angle, stagger angle, rotational speed and distance of shear centre from the centroid on the stability of the rotating pre-twisted blade packets of aerofoil cross-section are investigated by Sakar et al (2008) using finite element method. Hsu (2009) has investigated dynamic behaviour of pre-twisted beams using spline collocation method. Liu et al (2009) have carried out an investigation on the coupled axial-torsional vibration of pre-twisted beams. Leung et al (2010) have studied the influence of multiple kinds of initial stresses due to compression, shears, moments and torque on the natural vibration of pre-twisted straight beam based on the Timoshenko theory. Chen (2010a, b) has found the influence of thickness-to-width ratio, twist angle, spinning speed and axial load on the natural frequency, buckling load and instability zone of a pre-twisted Timoshenko beam by using finite element method.

3. Conclusions

Due to the broad and rapidly developing field of FGM, these conclusions cannot encompass all significant directions, trends, and needs. Nevertheless, they reflect some of the observations of the authors based on the published research and their own analysis of the subject.

Higher order shear deformation theory may be used to get precise results in case of thick beams. The most commonly used models for most of the literature that express the variation of material properties in FGMs is the power law distribution of the volume fraction. In practice, turbo-machinery blades are pre-twisted rotating blades. Hence the dynamic stability analysis of rotating pre-twisted blades may be undertaken as a future work of research. Moreover, the tapered beams can be considered for their parametric instability analysis. The study of dynamic stability of FGM beams considering material nonlinearity may be undertaken as a future work of research. Moreover, higher order stretching strain may be considered to include geometric nonlinearity. The problem that has to be addressed includes analysis of system under electro-thermo- mechanical environment as future work. The results obtained need to be verified with experimental results. Therefore experimental analysis of dynamic stability of functionally graded material beams may be taken as a future work in order to validate the used computational method and obtained theoretical results.

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