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Visco-Elastic Effects on Free Convective Flow Confined between a Long Vertical Wavy Wall and a Parallel Flat Wall of Equal Transpiration

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Abstract

An analysis of the steady free convective flow and heat transfer of a visco-elastic fluid confined between a long vertical wavy wall and a parallel flat wall of equal transpiration has been presented. The x axis is taken along the length of the walls while the walls are given by $y=\varepsilon\cos(Kx)$ and $y=d$ and the transpirations at the walls are taken as $v=-\varepsilon v_0\sin(Kx)$.The equations governing the fluid flow and heat transfer have been solved by perturbation technique. Expressions for the zeroth-order and first order velocity, temperature, skin friction, heat transfer coefficient at the walls and pressure drop are obtained. The first order velocity, pressure drop, and skin friction coefficient have been presented graphically to observe the visco-elastic effects in combination of other flow parameters involved in the solution.

Keywords and phrases: Free convective, visco-elastic, perturbation technique, Prandtl number, skin friction.

Introduction

Analysis of fluid over a wavy wall attracts the interest of many researchers because of its application in different areas such as transpiration, cooling of re-entry vehicles and rocket boosters, cross hatching on ablative surfaces and film vaporization in combustion chambers. Benjamin [1] was probably the first to consider the problem of the flow over a wavy wall. The steady streaming generated by an oscillatory viscous flow over a wavy wall under the assumption that the amplitude of the wave is smaller than the Stokes boundary layer thickness has been studied by Lyne [2]. A linear analysis of compressible boundary layer flows over a wavy wall was presented by Lekoudis *et al*. [3]. The Rayleigh Problem for a wavy wall was studied in detail by Shankar and Sinha [4]. As the liquid is dragged along the wall, it was found that the importance of the waviness of the wall quickly ceases at low Reynolds number, while at large Reynolds number, the effects of viscosity are found in a thin layer close to the wall. The effect of small amplitude wall waviness on the stability of the laminar boundary layer was studied by Lessen and Gangwani [5]. In the above problems, the wavy wall is taken to be horizontal. Vajravelu and Sastri [6] have studied the free convection of heat transfer in a viscous incompressible fluid between a long vertical wavy wall and a parallel flat wall. Yao [7] has studied the natural convection along a vertical wavy surface and presented a correct method for calculation of heat transfer co-efficient at the wavy wall taking the derivative of the wall temperature along the normal to the wavy surface. Das and Ahmed [8] have extended this problem by including the effects of transverse magnetic field. Das and Deka [9] have discussed a numerical approach of this problem.

On the other hand, the laminar flow control (LFC) has become very important in the fields of aeronautical engineering for its application to reduce drag and therefore to increase the power of the vehicle by a considerable amount. The boundary layer suction is one of the best effective methods of reducing the drag co-efficient which entails large energy losses. Keeping in view the importance of the suction velocity, various scholars studied the effects of different arrangements and configuration extensively, both theoretically and practically, and Lachmann [10] has compiled the development since World War II on the subject. Messiha [11] has studied the free convective flow past an infinite porous plate in case of constant free suction. Also, the laminar boundary layer in oscillatory flow along an infinite flat plate with variable suction has been studied by Messiha [12].Singh *et al*. [13] have studied the effect of variable suction on the free convective flow and heat transfer along a vertical porous plate. Aziz *et al*. [14] have studied the effects of suction and injection on the free convective steady flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall of equal transpiration. Ahmed *et al.* [15] have studied the free convective heat transfer through a porous medium confined between a long vertical wavy wall and a parallel flat wall. The effects of applied magnetic field on free convective flow in a vertical channel with heat transfer have been studied by Ahmed [16].

In modern technology and industrial applications, visco-elastic fluids play an important role. Increasing emergence of visco-elastic fluids in great varieties of industries like petroleum and chemical processes has stimulated a considerable amount of interest in the study of the behavior of such fluids when in motion. The present paper is concerned with the free convective flow of a visco-elastic fluid characterized by Walters liquid (Model B′) confined between a long vertical wavy wall and a parallel flat wall of equal transpiration. The first-order velocity, pressure drop and skin friction coefficient have been presented graphically for various values of the visco-elastic parameter. It has been observed that the visco-elastic parameter influences the flow characteristics significantly.

The constitutive equation for Walters liquid (Model B′) is

$$\sigma^{ik} = -pg_{ik} + 2\eta_0 e^{ik} - 2k_0 e'^{ik} \quad (1.1)$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, v^i is the velocity vector, the contravariant form of e'^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e^{ik}{}_{,m} - v^i{}_{,m} e^{im} - v^i{}_{,m} e^{mk} \quad (1.2)$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v^i{}_{,k} + v^k{}_{,i} \quad (1.3)$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau)d\tau \text{ and } k_0 = \int_0^\infty \tau N(\tau)d\tau \quad (1.4)$$

$N(\tau)$ being the relaxation spectrum as introduced by Walters [17, 18]. This idealized model is a valid approximation of Walters liquid (Model B′) taking very short memories into account so that terms involving

$$\int_0^\infty \tau^n N(\tau)d\tau, n \geq 2 \quad (1.5)$$

have been neglected.

Mathematical formulation

We consider the free convective flow of a Walters liquid (Model B′) confined between a long vertical wavy wall and a parallel flat wall of equal transpiration. The x-axis is taken vertically upwards along the length of the wavy wall and y-axis perpendicular to it. The wavy and flat walls are represented by $y = \varepsilon \cos(K\bar{x})$ and $y=d$ respectively. $\bar{T}_w$ and $\bar{T}_1$ being the constant temperatures of the wavy and flat walls respectively. The transverse velocity at the walls are taken as $\bar{v} = -\varepsilon^* v_0 \sin(K\bar{x})$, $v_0>0$. It may be noted that the transverse velocity $\bar{v}$ at the walls becomes suction at some lengths of the walls and injection at other areas.

Assuming the wave length of the wavy wall to be large (i.e. K<<1), neglecting the viscous dissipative effects in the energy equation and the volumetric heat source/sink term in the energy equation to be constant, the governing equations of the flow and heat transfer are as follows:

The momentum equations are

$$\rho\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = -\frac{\partial P^*}{\partial\bar{x}}+\eta_0\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}\right) - K_0\left(\bar{u}\frac{\partial^3\bar{u}}{\partial\bar{x}^3}+\bar{u}\frac{\partial^3\bar{u}}{\partial\bar{x}\,\partial\bar{y}^2}+\bar{v}\frac{\partial^3\bar{u}}{\partial\bar{y}^3}+\bar{v}\frac{\partial^3\bar{u}}{\partial\bar{x}^2\,\partial\bar{y}}-3\frac{\partial\bar{u}}{\partial\bar{x}}\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial\bar{u}}{\partial\bar{x}}\frac{\partial^2\bar{u}}{\partial\bar{y}^2} - \frac{\partial\bar{u}}{\partial\bar{y}}\frac{\partial^2\bar{v}}{\partial\bar{x}^2}-\frac{\partial\bar{u}}{\partial\bar{y}}\frac{\partial^2\bar{u}}{\partial\bar{x}\,\partial\bar{y}}-2\frac{\partial\bar{v}}{\partial\bar{x}}\frac{\partial^2\bar{u}}{\partial\bar{x}\,\partial\bar{y}}\right)-\rho g \quad (2.1)$$

$$\rho\left(\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right) = -\frac{\partial P^*}{\partial\bar{y}}+\eta_0\left(\frac{\partial^2\bar{v}}{\partial\bar{x}^2}+\frac{\partial^2\bar{v}}{\partial\bar{y}^2}\right) - K_0\left(\bar{u}\frac{\partial^3\bar{v}}{\partial\bar{x}^3}+\bar{u}\frac{\partial^3\bar{v}}{\partial\bar{x}\,\partial\bar{y}^2}+\bar{v}\frac{\partial^3\bar{v}}{\partial\bar{x}^2\,\partial\bar{y}}+\bar{v}\frac{\partial^3\bar{v}}{\partial\bar{y}^3}-2\frac{\partial\bar{u}}{\partial\bar{y}}\frac{\partial^2\bar{v}}{\partial\bar{x}\,\partial\bar{y}} - \frac{\partial\bar{v}}{\partial\bar{x}}\frac{\partial^2\bar{v}}{\partial\bar{x}\,\partial\bar{y}}-\frac{\partial\bar{v}}{\partial\bar{x}}\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+\frac{\partial\bar{v}}{\partial\bar{y}}\frac{\partial^2\bar{v}}{\partial\bar{x}^2}-3\frac{\partial\bar{v}}{\partial\bar{y}}\frac{\partial^2\bar{v}}{\partial\bar{y}^2}\right) \quad (2.2)$$

The equation of continuity is

$$\frac{\partial\bar{u}}{\partial\bar{x}}+\frac{\partial\bar{v}}{\partial\bar{y}}=0 \quad (2.3)$$

And the energy equation is

$$\rho C_p\left(\bar{u}\frac{\partial\bar{T}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{T}}{\partial\bar{y}}\right)=k\left(\frac{\partial^2\bar{T}}{\partial\bar{x}^2}+\frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right)+Q \quad (2.4)$$

where $\bar{u}, \bar{v}$ are the velocity components, P^* the pressure, ρg the buoyancy force, Q the constant heat addition/absorption and the other symbols have their usual meanings.

The boundary conditions relevant to the problem are taken as

$$\bar{u}=0,\ \bar{v}=-\varepsilon^* v_0 \sin(K\bar{x}),\ \bar{T}=\bar{T}_w \text{ on } \bar{y}=\varepsilon^*\cos(K\bar{x})$$

$$\bar{u}=0,\ \bar{v}=-\varepsilon^* v_0 \sin(K\bar{x}),\ \bar{T}=\bar{T}_1 \text{ on } \bar{y}=d \quad (2.5)$$

It may be mentioned that we have chosen a sinusoidal form of suction/injection velocity at the walls, because this form of derivation of the solution of the boundary value problem in a way similar to that in the case without suction/injection. Of course, it is difficult to maintain such a suction/injection velocity in a physical situation. In fact, we were interested in the theoretical aspect of the problem and could offer a consistent solution of the problem with the assumed sinusoidal form of suction/injection at the walls. Some numerical results for skin friction and heat transfer at the walls have also been obtained for λx=π/4. However, for some scientists or engineers who are interested in studying a similar physical situation, our solution and results may serve to throw some insight into the problem.

We introduce the following non-dimensional parameters:

$$x = \frac{\bar{x}}{d}, y = \frac{\bar{y}}{d}, u = \frac{\bar{u}d}{\nu}, v = \frac{\bar{v}d}{\nu}, \theta = \frac{\bar{T} - \bar{T}_s}{\bar{T}_w - \bar{T}_s}, \lambda = Kd, \bar{P} = \frac{P^* d^2}{\rho \nu^2},$$

$$\alpha = \frac{Qd^2}{k(\bar{T}_w - \bar{T}_s)}, m = \frac{\bar{T}_1 - \bar{T}_s}{\bar{T}_w - \bar{T}_s}, \bar{P}_s = \frac{{P_s}^* d^2}{\rho \nu^2}, \text{where } \nu = \frac{\eta_0}{\rho},$$

$\varepsilon = \frac{\varepsilon^*}{d}$, a small amplitude parameter,

$P_r = \frac{\mu C_p}{k}$, the Prandtl number, where C_p is the specific heat at constant pressure,

$G = \frac{d^3 g\beta(\bar{T}_w - \bar{T}_s)}{\nu^2}$, the Grashoff number, where β is the volume expansion of heat transfer and the subscript s denotes quantities in the static condition,

$n = \frac{v_w}{\lambda} = \frac{v_0 d^2}{\lambda \nu}$, the suction/injection parameter. (2.6)

Introducing the non-dimensional parameters (2.6) in the governing equations for velocity and temperature, we obtain

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial \bar{P}}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - K_1\left(u\frac{\partial^3 u}{\partial x^3} + u\frac{\partial^3 u}{\partial x\,\partial y^2} + v\frac{\partial^3 u}{\partial y^3} + v\frac{\partial^3 u}{\partial x^2\,\partial y} - 3\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\,\partial y} - 2\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x\,\partial y}\right) - \frac{gd^3}{\nu^2} \tag{2.7}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial \bar{P}}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - K_1\left(u\frac{\partial^3 v}{\partial x^3} + u\frac{\partial^3 v}{\partial x\,\partial y^2} + v\frac{\partial^3 v}{\partial x^2\,\partial y} + v\frac{\partial^3 v}{\partial y^3} - 2\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x\,\partial y} - \frac{\partial v}{\partial x}\frac{\partial^2 v}{\partial x\,\partial y} - \frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial x^2} - 3\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial y^2}\right) \tag{2.8}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.9}$$

$$P_r\left(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right) = \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \alpha \tag{2.10}$$

subject to the boundary conditions

$u = 0$, $v = -\varepsilon\, v_w \sin(\lambda x)$, $\theta = 1$ on $y = \varepsilon \cos(\lambda x)$

$u = 0$, $v = -\varepsilon\, v_w \sin(\lambda x)$, $\theta = m$ on $y = 1$ (2.11) and $K_1 = \frac{K_0}{\rho}$ is the visco-elastic parameter.

Then on using the well known Boussinesq approximation $\rho = \rho_s[1 - \beta(\overline{T} - \overline{T}_s)]$ in equation (2.7) in the static fluid condition and adopting the perturbation scheme

$$u(x, y) = u_0(y) + u_1(x, y)$$
$$v(x, y) = v_1(x, y)$$
$$\overline{P}(x, y) = P_0(x) + P_1(x, y)$$
$$\theta(x, y) = \theta_0(y) + \theta_1(x, y) \qquad (2.12)$$

where the perturbation quantities u_1, v_1, P_1 and θ_1 are all small compared with the mean or zeroth order quantities.

Then equations (2.6) to (2.9) give the following non-dimensional forms

$$\frac{d^2u_0}{dy^2} + G\theta_0 = 0$$
$$\frac{d^2\theta_0}{dy^2} + \alpha = 0 \qquad (2.13)$$

to zeroth order. (The constant pressure gradient $\frac{\partial}{\partial x}(P_0 - \overline{P}_s)$ has been neglected following Ostrach [19]), and

$$u_0\frac{\partial u_1}{\partial x} + v_1\frac{du_0}{dy} = -\frac{\partial p_1}{\partial x} + \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} - K_1\left(u_0\frac{\partial^3 u_1}{\partial x^3} + u_0\frac{\partial^3 u_1}{\partial x\,\partial y^2} + v_1\frac{d^3u_0}{dy^3} + \frac{\partial u_1}{\partial x}\frac{d^2u_0}{dy^2} - \frac{du_0}{dy}\frac{\partial^2 v_1}{\partial x^2} - \frac{du_0}{dy}\frac{\partial^2 u_1}{\partial x\,\partial y}\right) + G\theta_1 \qquad (2.14)$$

$$u_0\frac{\partial v_1}{\partial x} = -\frac{\partial p_1}{\partial y} + \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} - K_1\left(u_0\frac{\partial^3 v_1}{\partial x^3} + u_0\frac{\partial^3 v_1}{\partial x\,\partial y^2} - 2\frac{du_0}{dy}\frac{\partial^2 v_1}{\partial x\,\partial y} - \frac{\partial v_1}{\partial x}\frac{d^2u_0}{dy^2}\right) \qquad (2.15)$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \qquad (2.16)$$

$$P_r\left(u_0\frac{\partial\theta_1}{\partial x} + v_1\frac{d\theta_0}{dy}\right) = \frac{\partial^2\theta_1}{\partial x^2} + \frac{\partial^2\theta_1}{\partial y^2} \qquad (2.17)$$

to the first order.

In view of (2.12), the boundary conditions (2.11) can be split up into the following two parts:

$u_0 = 0,\ \theta_0 = 1$ on $y = 0$

$u_0 = 0,\ \theta_0 = m$ on $y = 1$ (2.18)

$u_1 = -\varepsilon\, u'_0 \cos(\lambda x),\ v_1 = -\varepsilon\, v_w \sin(\lambda x),\ \theta_1 = -\varepsilon\, \theta'_0 \cos(\lambda x)$ on $y = 0$

$u_1 = 0,\ v_1 = -\varepsilon\, v_w \sin(\lambda x),\ \theta_1 = 0$ on $y = 1$ (2.19)

where dash denotes differentiation w.r.t. y.

Solution of the problem

The solutions for the zeroth order equations (2.13) subject to the boundary condition (2.17) are

$\theta_0(y) = H_1 y^2 + H_2 y + 1$

$u_0(y) = G(H_3 y - y^2 - H_2 y^3/3 - H_1 y^4/6)/2$ (3.1)

where $H_1 = -\alpha/2,\ H_2 = m + \alpha/2 - 1,\ H_3 = 1 + H_2/3 + H_1/6$

To solve the first order equations, we introduce the stream function $\bar{\psi}_1$, defined by

$$u_1 = -\frac{\partial \bar{\psi}_1}{\partial y},\ v_1 = \frac{\partial \bar{\psi}_1}{\partial x} \qquad (3.2)$$

Eliminating P_1 from (2.14) and (2.15) we get,

$$u_0(\bar{\psi}_{1,xyy} + \bar{\psi}_{1,xxx}) - u''_0 \bar{\psi}_{1,x} = 2\bar{\psi}_{1,xxyy} + \bar{\psi}_{1,yyyy} + \bar{\psi}_{1,xxxx} + K_1\left[u^{iv}_0 \bar{\psi}_{1,x} - u_0(2\bar{\psi}_{1,xxxyy} + \bar{\psi}_{1,xyyyy} + \bar{\psi}_{1,xxxxx})\right] - G\theta_{1,y} \qquad (3.3)$$

And (2.17) becomes

$$P_r(u_0 \theta_{1,x} + \theta'_0 \bar{\psi}_{1,x}) = \theta_{1,xx} + \theta_{1,yy} \qquad (3.4)$$

We assume that

$$\bar{\psi}_1(x, y) = \varepsilon e^{i\lambda x} \psi(y),\ \theta_1(x, y) = \varepsilon e^{i\lambda x} t(y) \qquad (3.5)$$

from which we infer that

$u_1(x, y) = \varepsilon e^{i\lambda x} \bar{u}_1(y)$ where $\bar{u}_1(y) = -\psi'(y)$

$v_1(x, y) = \varepsilon e^{i\lambda x} \bar{v}_1(y)$ where $\bar{v}_1(y) = i\lambda\psi(y)$ (3.6)

$\theta_1(x, y) = \varepsilon e^{i\lambda x} \bar{\theta}_1(y)$ where $\bar{\theta}_1(y) = i\lambda t(y)$

using (3.6) in (3.3) and (3.4) we get

$$\Psi^{iv} - i\lambda[u_0(\psi'' - \lambda^2\psi) - u''_0\psi] - \lambda^2(2\psi'' - \lambda^2\psi) + K_1 i\lambda[u^{iv}_0 - u_0(-2\lambda^2 \psi'' + \psi^{iv} + \lambda^4\psi)] = Gt' \qquad (3.7)$$

$$t'' - \lambda^2 t = i\lambda P_r (u_0 t + \theta'_0 \psi) \qquad (3.8)$$

Now, we shall restrict our attention to the real parts of the solutions for perturbed quantities

$\bar{\psi}_1$, θ_1, u_1, v_1.

The boundary conditions (2.19) can now be written in terms of $\bar{\psi}_1$ as

$$\frac{\partial\bar{\psi}_1}{\partial y} = \varepsilon\, u'_0 \cos(\lambda x),\ \frac{\partial\bar{\psi}_1}{\partial x} = -\,\varepsilon\, v_w \sin(\lambda x) \text{ on } y = 0$$

$$\frac{\partial\bar{\psi}_1}{\partial y} = 0,\ \frac{\partial\bar{\psi}_1}{\partial x} = -\,\varepsilon\, v_w \sin(\lambda x) \text{ on } y = 1 \qquad (3.9)$$

Considering small values of λ (or K<<1) and substituting

$$\Psi(y)= \Psi(\lambda, y)= \psi_0(y)+\lambda\psi_1(y)+\ldots\ldots\ldots\ldots..$$

$$t(y) = t(\lambda, y) = t_0(y)+\lambda t_1(y)+\ldots\ldots\ldots\ldots\ldots\ldots \qquad (3.10)$$

into (3.7) to (3.9) we get the following set of differential equations and corresponding boundary conditions

$$\Psi_0^{iv}= Gt_0' \,;\ t_0'' = 0 \qquad (3.11)$$

$$\Psi_1^{iv}- iu_0\psi_0''+iu_0''\psi_0+iK_1(u_0^{iv}\psi_0-u_0\psi_0^{iv}) = Gt_1' \qquad (3.12)$$

$$t_1'' = iP_r\,(u_0t_0 +\theta_0'\psi_0) \qquad (3.13)$$

$$\psi_0' = u_0',\ \psi_0 = n,\ t_0 = -\theta_0' \text{ on } y = 0$$

$$\psi_0' = 0,\ \psi_0 = n,\ t_0 = 0 \text{ on } y = 1 \qquad (3.14)$$

$$\psi_1' = 0,\ \psi_1 = n,\ t_1 = 0 \text{ on } y = 0$$

$$\psi_1' = 0,\ \psi_1 = n,\ t_1 = 0 \text{ on } y = 1 \qquad (3.15)$$

Solving the equations (3.11) to (3.13), subject to the boundary conditions (3.14) and (3.15), we get

$$t_0(y) = H_2(y-1)$$

$$\psi_0(y) = GH_2y^4/24+C_3y^3/6+C_4y^2/2+C_5y+C_6 \qquad (3.16)$$

$$t_1(y) = i\ t_{10}(y)$$

$$\Psi_1(y) = i\ \Psi_{10}(y) \qquad (3.17)$$

Here

$$t_{10} = Pr\ (C_7y^6/30+C_8y^5/20+C_9y^4/12+C_{10}y^3/6+C_{11}y^2/2-C_{12}y)$$

$$\Psi_{10} = C_{13}y^9/3024+C_{14}y^8/1680+C_{15}y^7/840+C_{16}y^6/360 +C_{17}y^5/120+C_{18}y^4/24+C_{21}y^3/6-C_{22}y^2/2$$

where the constants C_3 to C_{22} are obtained but not presented here due to brevity.

Using (3.16) and (3.17) in (3.6) we get,

$$u_1(x, y) = -\,\varepsilon\,[\psi_0'(y) \cos(\lambda x)- \lambda\psi_{10}'(y) \sin(\lambda x)] \qquad (3.18)$$

$v_1(x, y) = -\varepsilon \lambda [\psi_0(y) \sin(\lambda x) + \lambda\psi_{10}(y) \cos(\lambda x)]$ (3.19)

$\theta_1(x, y) = \varepsilon [t_0(y) \cos(\lambda x) - \lambda t_{10}(y) \sin(\lambda x)]$ (3.20)

Pressure drop

Using equations (2.12), (2.14) and (3.5), we obtain the fluid pressure at any point (x, y) as

$\widehat{P}(x, y) = -K'x + \mathrm{Re}\,[\varepsilon \{i\, \varepsilon e^{i\lambda x})\}/\lambda]\, Z(y) + L$ (3.21)

where L and K' are arbitrary constants and

$Z(y) = \psi''' - \lambda^2\psi' - i\lambda\,(u_0\psi' - u_0'\psi) - Gt + i\lambda K_1(-u_0\,\psi''' + \lambda^2 u_0\,\psi'$

$+ u_0'''\psi - u_0''\,\psi' + \lambda^2 u_0'\psi + u_0'\psi'')$ (3.22)

The pressure drop $\widehat{P}$ indicates the difference between the pressure at any point y in the flow field and that at the flat wall, with x- fixed and is given by

$\widehat{P} = \overline{P}(x, y) - \overline{P}(x, 1)$

$= \frac{\varepsilon}{\lambda}\mathrm{Re}\,[i\, e^{i\lambda x}\{Z(y) - Z(1)\}]$ (3.23)

Using (3.16) and (3.17), the equation (3.23) can now be written as

$\widehat{P} = \varepsilon\, [\cos(\lambda x)\{\varphi_2(y) - \varphi_2(1)\} - \{\sin(\lambda x)/\lambda\}\{\varphi_1(y) - \varphi_1(1)\}]$ (3.24)

where

$\varphi_1(y) = \psi_0'''(y) - \lambda^2\psi_0'(y) + \lambda^2\psi_{10}'(y)u_0(y) - \lambda^2 u_0'(y)\,\psi_{10}(y) - Gt_0(y)\; - \lambda K_1\{-\lambda u_0(y)\,\psi_{10}'''(y) + \lambda^3 u_0(y)\,\psi_{10}'(y)$

$+ \lambda u_0'''(y)\,\psi_{10}(y) - \lambda u_0''(y)\,\psi_{10}'(y) + \lambda^3 u_0'(y)\,\psi_{10}(y) + \lambda u_0'(y)\,\psi_{10}''(y)\}$

$\varphi_2(y) = -\psi_{10}'''(y) + \lambda^2\psi_{10}'(y) + u_0(y)\psi_0'(y) - u_0'(y)\,\psi_0(y) + Gt_{10}(y) + K_1\{u_0(y)\,\psi_0'''(y) - \lambda^2 u_0(y)\,\psi_0'(y)$

$- u_0'''(y)\,\psi_0(y) + u_0''(y)\,\psi_0'(y) - \lambda^2 u_0'(y)\,\psi_0(y) - u_0'(y)\,\psi_0''(y)\}$

Skin friction Heat transfer Coefficient (Nusselt number) at the walls

The shear stress in dimensionless form is given by

$$\sigma_{xy} = \frac{\overline{\sigma}_{xy} d^2}{\rho \nu^2} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - K_1\left(u\frac{\partial^2 u}{\partial x\,\partial y} + u\frac{\partial^2 v}{\partial x^2} + v\frac{\partial^2 u}{\partial y^2} + v\frac{\partial^2 v}{\partial x\,\partial y} - 2\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}\right) \quad (4.1)$$

At the wavy wall y = ε cosλx and at the flat wall y = 1, the shear stress σ_{xy} becomes

$\sigma_w = u_0'(0) + \varepsilon\ \cos(\lambda x)[u_0''(0) - \psi_0''(0) - \lambda^2\psi_0(0)\ \ - K_1\lambda\{\lambda u_0(0)\psi_{10}''(0) + \lambda^3 u_0(0)\psi_{10}(0) -$

$\lambda u_0''(0)\ \psi_{10}(0)\}]$

$-\varepsilon\sin(\lambda x)[-\lambda\psi_{10}''(0)-\lambda^3\psi_{10}(0)+K_1\lambda\{u_0(0)\psi_0''(0)+\lambda^2 u_0(0)\psi_0(0)-u_0''(0)\psi_0(0)\}]$ (4.2)

and

$\sigma_1= u_0'(1)+\varepsilon \cos(\lambda x)[-\psi_0''(1)-\lambda^2\psi_0(1)+K_1\lambda\{-\lambda u_0(1)\psi_{10}''(1)-\lambda^3 u_0(1)\psi_{10}(1)+\lambda u_0''(1)\psi_{10}(1)\}]$

$+\varepsilon\sin(\lambda x)[\lambda\psi_{10}''(1)+\lambda^3\psi_{10}(1)-K_1\lambda\{u_0(1)\psi_0''(1)+\lambda^2 u_0(1)\psi_0(1)-u_0''(1)\psi_0(1)\}]$ (4.3)

respectively.

The dimensionless form of the heat transfer coefficient (Nusselt Number) is given by

$$Nu = \theta_0'(y)+Re\,[\varepsilon\, e^{i\lambda x}t'(y)] \qquad (4.4)$$

Using (3.10) we get the Nusselt number at the wavy wall,

$$Nu_w=\theta_0'(0)+\varepsilon[\cos(\lambda x)\{\theta_0''(0)+t_0'(0)\}-\lambda\sin(\lambda x)t_{10}'(0)] \qquad (4.5)$$

and at the flat wall,

$$Nu_1=\theta_0'(1)+\varepsilon[\cos(\lambda x)t_0'(1)-\lambda\sin(\lambda x)t_{10}'(1)] \qquad (4.6)$$

respectively.

Discussion

The purpose of this study is to bring out the effects of visco-elastic parameter on the free convective flow confined between a long vertical wavy wall and a flat wall of equal transpiration as the effects of other flow parameters have been discussed by Aziz *et al*. The visco-elastic effect is exhibited through the non-dimensional parameter K_1. The corresponding results for Newtonian fluid are obtained by setting $K_1=0$ and it is worth mentioning that these results show conformity with that of Aziz *et al*.

The expressions for u_1, v_1, and θ_1 are the first-order solutions or the disturbed parts due to waviness of the wall. The expression for the total velocity field (u, v) and the total temperature field θ may be obtained from (2.12) by using (3.1) and (3.18) to (3.20). The profiles of u_1 and v_1 are depicted against y in the figures 1, 2 and 3, 4 respectively to observe the visco-elastic effects. It is to be noted that the zeroth order quantities u_0, v_0 and θ_0 are not affected by the visco-elastic parameter K_1. The numerical calculations are to be carried out for $\lambda x=\pi/4$. This means that at the neighbourhood of $\lambda x=\pi/4$ the transverse velocity v or v_1 is suction at the wavy wall and injection at the flat wall, the magnitudes of suction and injection being equal. We have considered $\lambda=.05$ and $\varepsilon=.01$ throughout the discussion. It is noticed that the velocity profile u_1 increases with the increase of the visco-elastic parameter K_1 up to y=.5 and then diminishes (figure 1), the velocity profile v_1 decreases with the increase of visco-elastic parameter K_1 up to y=.55 and then increases (figure 3) and the profile of pressure drop against y increases with the increase of the visco-elastic parameter K_1 (figure 5) in comparison with the Newtonian fluid (K_1=0) for different values of other flow parameters. The flow patterns remain the same if we increase the suction / injection parameter n with that of the visco-elastic parameter K_1 (figures 1, 2; figures 3, 4; figures 5, 6).The skin friction coefficient against the heat source/sink parameter α increases at the wavy wall (figure 7) but decreases at the flat wall (figure 9) in both Newtonian and non-Newtonian cases. Again a growth of skin friction is observed at both the walls with the increasing values of visco - elastic parameter as compared to Newtonian fluid (K_1= 0). By rising the values of n with that of K_1, the results remain unaltered (figures 7, 8; figures 9, 10).

The temperature field is not significantly affected by the visco-elastic parameter.

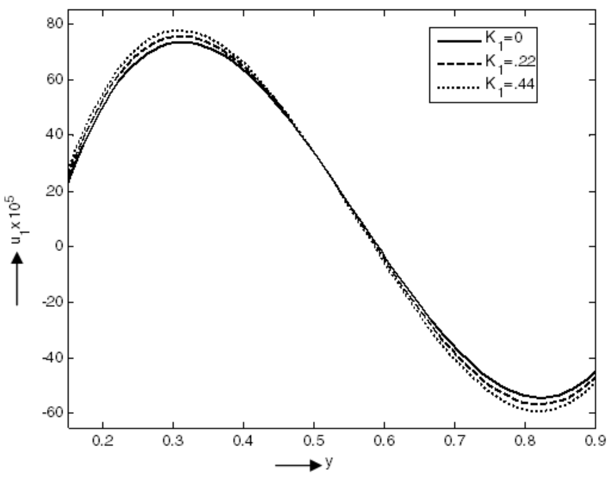

Figure 1: Variation of u_1 against y for α= -3, n=2.5, m=-1, G=5, P_r=5.

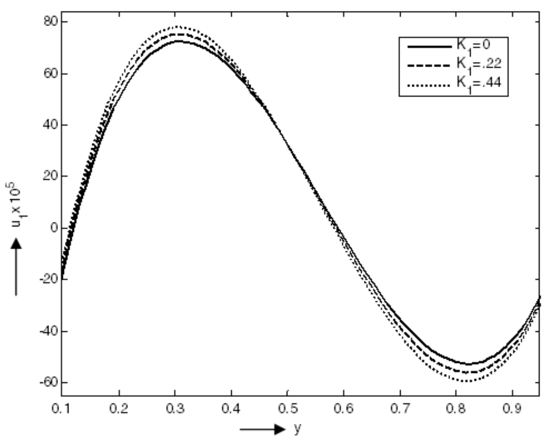

Figure 2: Variation of u_1 against y for α= -3, n=3.5, m=-1, G=5, P_r=5.

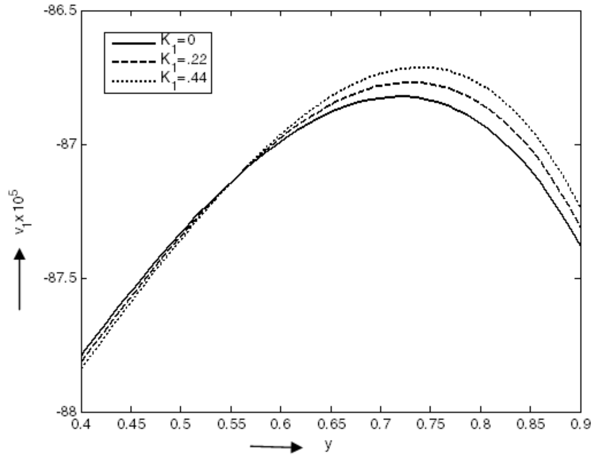

Figure 3: Variation of v_1 against y for α= -3, n=2.5, m=-1, G=5, P_r=5.

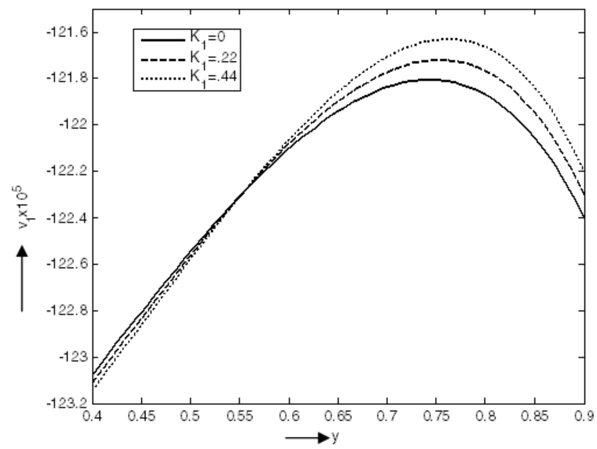

Figure 4: Variation of v_1 against y for α= -3, n=3.5, m=-1, G=5, P_r=5.

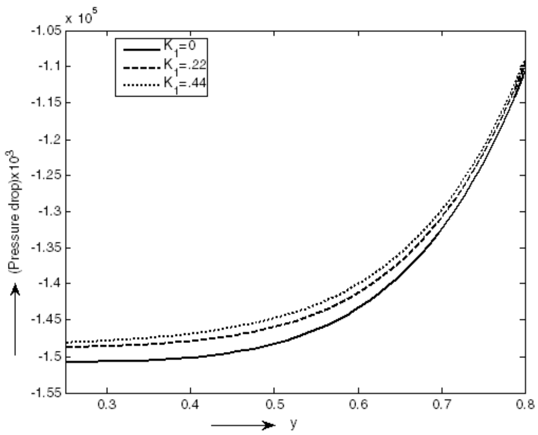

Figure 5: Variation of pressure drop $\widehat{P}$ against y for α= -3, n=2.5, m=-1, G=5, P_r=5

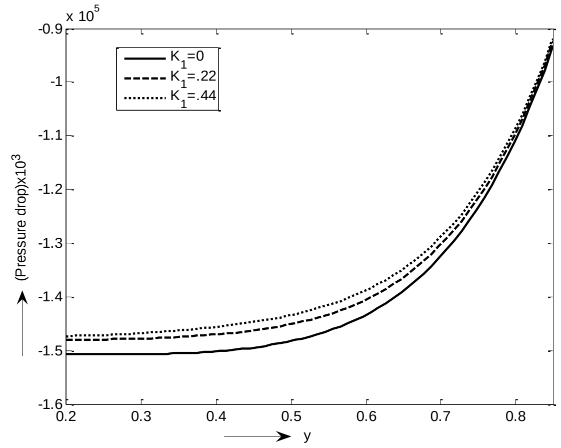

Figure 6: Variation of pressure drop $\widehat{P}$ against y for α= -3, n=3.5, m=-1, G=5, Pr=5

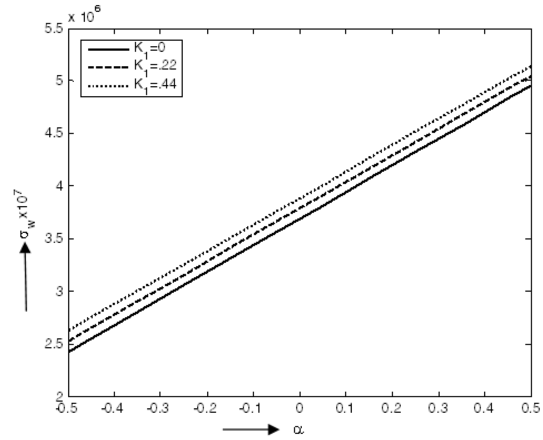

Figure 7: Variation σ_w against α for n=2.5, m=-1, G=5, P_r=5, y=0.

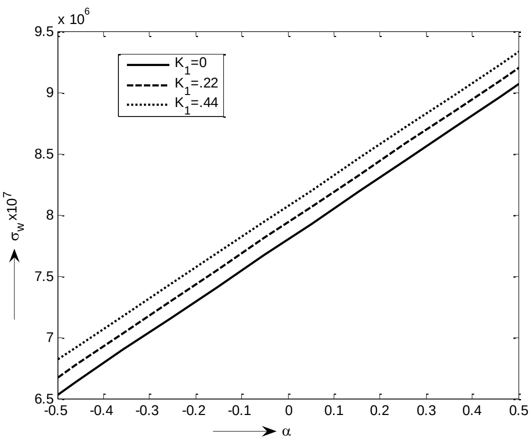

Figure 8: Variation σ_w against α for n=3.5, m=-1, G=5, P_r=5, y=0.

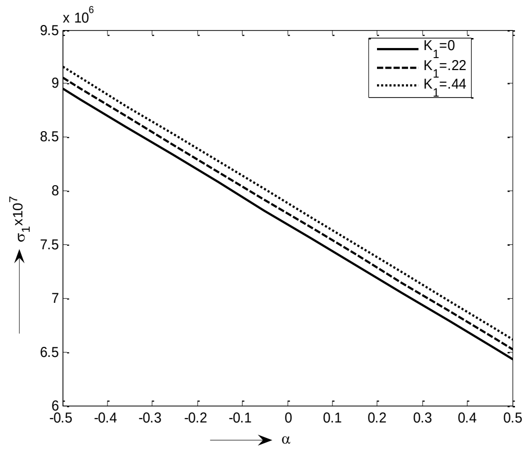

Figure 9: Variation σ_1 against α for n=2.5, m=-1, G=5, P_r=5, y=1.

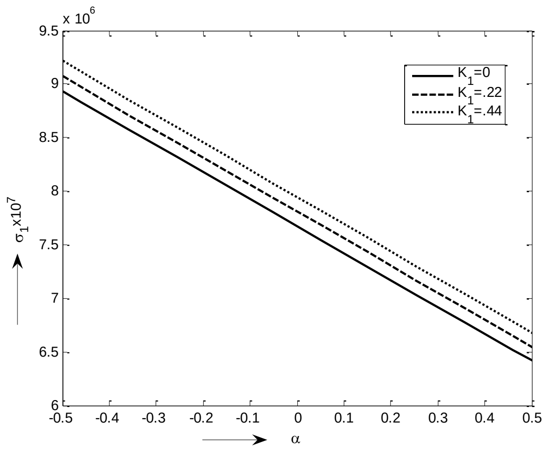

Figure 10: Variation σ_1 against α for n=3.5, m=-1, G=5, P_r=5, y=1.

Conclusion

An analysis of the visco-elastic effects on free convective flow confined between a long vertical wavy wall and a parallel flat wall of equal transpiration has been presented for different values of visco-elastic parameter K_1 in combination of other flow parameters.

From this study, we make the following conclusions:

- The velocity field is considerably affected by the variation of visco-elastic parameter as well as suction/injection parameter.
- The profile of pressure drop is enhanced by the rising of visco-elastic parameter as well as suction/injection parameter in comparison with Newtonian fluid.
- The skin friction coefficient against the heat source/sink parameter increases at the wavy wall but decreases at the plane wall in both Newtonian and non-Newtonian cases. Also, the growth of skin friction coefficient is observed by the increasing trend of visco-elastic parameter as well as suction/injection

parameter.

- The effect of visco-elastic parameter is not prominent in the temperature field.

References

[1] Benjamin, T. B., 1959, "Shearing flow over a wavy boundary, " J. Fluid Mech., 6, pp. 161- 205.
[2] Lyne, W. H., 1971, "Unsteady viscous flow over a wavy wall, " J. Fluid Mech., 50, pp. 33- 48.
[3] Lekoudis, S. G., Nayfeh, A. H., and Saric, W. S., 1976, "Compressible boundary layers over wavy wall, " Phys. Fluids, 19, pp. 514-519.
[4] Shankar, P. N., and Sinha, U. N., 1976, "The Rayleigh problem for a wavy wall, " J. Fluid Mech., " 77, pp. 243-256.
[5] Lessen, M., and Gangwani, S. T., 1976, "Effects of small amplitude wall waviness upon the stability of the boundary layer, " Phys. Fluids, 19, pp. 510-513.
[6] Vajravelu, K., and Sastri, K. S., 1978, "Free convective heat transfer problem between a long vertical wavy wall and a parallel flat wall, " J. Fluid. Mech., 86, pp. 365-383.
[7] Yao, L. S., 1983, "Natural convection along a vertical wavy surface, " ASME J. Heat Transfer, 105, pp. 465-468.
[8] Das, U. N., and Ahmed, N., 1992, "Free convective MHD flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall, " Indian J. Pure Appl. Math., 23(4), pp. 295-304.
[9] Das, U. N., and Deka, R. K., 1992, "Free convection in a viscous incompressible fluid confined between a long vertical wavy wall and parallel flat wall: A numerical approach, " J. Assam Sc. Soc., 34(4), pp. 121-126.
[10] Lachmann, G. V., 1961, "Boundary layer and Flow Control. Its Principles and Applications, " Vol I and II, Pergamon Press, London.
[11] Messiha, S.A.S., 1965, "Free convection boundary layer from a vertical flat plate with constant suction, " Proc. Phys. Soc. Egypt, 29, pp. 93-101.
[12] Messiha, S.A.S., 1966, "Laminar boundary layers in oscillatory flow along an infinite flat plate with variable suction, " Proc. Camb. Phil. Soc. 62, pp. 329-337.
[13] Singha, P., Sharma, V. P., and Misra, U. N., 1978, "Three dimensional free convection flow and heat transfer along a porous vertical plate, " Appl. Sci. Res. 34, pp. 105-115.
[14] Aziz, A., Das, U. N., and Ahmed, S., 1999, "Free convective steady flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall of equal transpiration, " Far East J. Appl. Math., 3(3), pp. 263-286.

[15] Ahmed, N., Sarma, D., and Das, U.N., 2001, “Free convective heat transfer through a porous medium confined between a long vertical wavy wall and a parallel flat wall, ” Ultra Science, 13(2), pp. 231-238.
[16] Ahmed, S., 2008, “Effects of applied magnetic field on free convective flow in a vertical channel with heat transfer, ” Bull. Cal. Math. Soc., 100(4), pp. 349-358.
[17] Walters, K., 1960, “The motion of an elastic-viscous liquid contained between co-axial cylinders (II), ” Quart. J. Mech. Appl. Math., 13, pp. 444-461.
[18] Walters, K., 1962, “The solution of flow problems in the case of materials with memories, ” J. Mecanique, 1, pp. 473-478.
[19] Ostrach, S., 1951, “Laminar natural convection flow and heat transfer of fluids with and without heat sources in channels with constant wall temperature, ” N. A. C. A. Tech. Note, No. 2863.