Some Properties of ifs-Edge Regular Intuitionistic Fuzzy Soft Graph

T.K. Mathew Varkey  
Assistant Professor,  
Department of mathematics,  
TKM College of Engineering,  
Kollam, Kerala, India  

A.M. Shyla  
Research Scholar,  
Kerala University Library,  
Palayam, Thiruvananthapuram,  
Kerala, India  

Abstract  
We present a novel framework for handling intuitionistic fuzzy soft information by combining the theory of intuitionistic fuzzy soft sets with graphs. Here we present the concept of Intuitionistic fuzzy soft graph through a real life situation. We present the concepts ifs-order, ifs-size of an intuitionistic fuzzy soft graph, ifs-regular intuitionistic fuzzy soft graph, ifs-edge regular intuitionistic fuzzy soft graph. We illustrate these concepts by describing several examples. We investigate some of their properties.

Keywords: Intuitionistic Fuzzy Soft Graph, ifs-regular intuitionistic fuzzy soft graph, ifs-edge regular intuitionistic fuzzy soft graph, ifs-totally edge regular intuitionistic fuzzy soft graph.

1 Introduction  
Molodtsov [7] introduced the concept of soft set that can be seen as a new mathematical theory for dealing with uncertainties. The soft set theory has been applied to many different fields with greatness. P. K. Maji [6] worked on theoretical study of soft sets in detail. The most appreciate theory to deal with uncertainties is the theory of fuzzy sets, developed by Zadeh [12] in 1965. But it has an inherent difficulty to set the membership function in each particular cases. The generalization of Zadeh’s fuzzy set called intuitionistic fuzzy set was introduced by Atanassov [1] which is characterized by a membership function and a non-membership function.


In this paper we present a novel framework for handling intuitionistic fuzzy soft information by combining the theory of Intuitionistic fuzzy soft sets with graphs. Here we present the concept of Intuitionistic Fuzzy Soft graph through a real life situation. We present the concepts ifs-order, ifs-size of an Intuitionistic fuzzy soft graph, ifs-regular intuitionistic fuzzy soft graph, ifs-edge regular intuitionistic fuzzy soft graph and ifs-totally edge regular intuitionistic fuzzy soft graph. We illustrate these concepts by describing several examples. We investigate some of their properties.

2 Preliminaries

Definition 2.1 A Fuzzy Set of a base set \( V = \{v_1, v_2, \ldots, v_n\} \) (non-empty set) is specified by its membership function \( \sigma : V \rightarrow [0,1] \) for each \( v_i \in V \), the degree or grade to which \( v_i \) belongs to \( \sigma \).

Definition 2.2 A Fuzzy graph \( G = (\sigma, \mu) \) is a pair of function \( \sigma : V \rightarrow [0,1] \) and \( \mu : V \times V \rightarrow [0,1] \), where for all \( v_i, v_j \in V \) we have \( \mu(v_i, v_j) \leq \sigma(v_i) \wedge \sigma(v_j) \) for each \( (v_i, v_j) \in V \times V \). Here \( \sigma \) and \( \mu \) are respectively called fuzzy vertex and fuzzy edge of the fuzzy graph \( G = (\sigma, \mu) \).

Definition 2.3 An Intuitionistic Fuzzy Graph is defined as \( G = (V, E, \mu, \gamma) \) where \( V = \{v_1, v_2, \ldots, v_n\} \) (non-empty set) such that \( \mu_i : V \rightarrow [0,1] \) and \( \gamma_i : V \rightarrow [0,1] \) denote the degree of membership and non-membership of the element \( v_i \in V \), respectively and

1. \( 0 \leq \mu_i(v_i) + \gamma_i(v_i) \leq 1 \) for every \( v_i \in V, i = 1,2,\ldots n \).
2. \( E \subseteq V \times V \), where \( \mu_2, \gamma_2 : V \times V \rightarrow [0,1] \) are such that (i). \( \mu_2(v_i, v_j) \leq \min\{\mu_i(v_i), \mu_i(v_j)\} \).
(ii). \( \gamma_2(v_i, v_j) \leq \max\{\gamma_i(v_i), \gamma_i(v_j)\} \) and
(iii). \( 0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1 \).

0 \leq \mu_2(v_i, v_j), \gamma_2(v_i, v_j) \leq 1 \), where
for every \((v_i, v_j) \in E, i, j = 1, 2, \ldots, n\) .

Let \(U\) be an initial universal set. \(P\) be a set of parameters, \(\mathcal{P}(U)\) be the power set of \(U\) and \(A \subseteq P\).

**Definition 2.4** A pair \((F, A)\) is called a soft set over \(U\) if and only if \(F\) is a mapping of \(A\) into the set of all subsets of the set \(U\).

**Definition 2.5** A pair \((\bar{F}, A)\) is called an intuitionistic fuzzy soft set over \(U\), where \(\bar{F}\) is a mapping given by \(\bar{F} : A \rightarrow IF^U\); \(IF^U\) denotes the collection of all intuitionistic fuzzy subsets of \(U\); \(A \subseteq P\).

### 3 Intuitionistic Fuzzy Soft Graph

**Definition 3.1** [10] Let \(V = \{v_1, v_2, \ldots, v_n\}\) be a non-empty set, \(E \subseteq V \times V\). \(P\) (parameter set) and \(A \subseteq P\). Also let (1) \(\mu_i : A \rightarrow IF^U(V)\) (\(IF^U(V)\) denotes collection of all intuitionistic fuzzy subsets in \(V\)), 
\[ a \mapsto \mu_i(a) = \mu_{i_a} \text{ (say)}, a \in A \text{ and} \]
\[ \mu_{i_a} : V \rightarrow [0,1], v_i \mapsto \gamma_{i_a}(v_i). \]
where \((A, \mu_i)\) Intuitionistic fuzzy soft vertex of membership function and
\[ \gamma_i : A \rightarrow IF^U(V)\] (\(IF^U(V)\) denotes collection of all intuitionistic fuzzy subsets in \(V\)), 
\[ a \mapsto \gamma_i(a) = \gamma_{i_a} \text{ (say)}, a \in A \text{ and} \]
\[ \gamma_{i_a} : V \rightarrow [0,1], v_i \mapsto \gamma_{i_a}(v_i). \]
where \((A, \gamma_i)\) Intuitionistic fuzzy soft vertex of non-membership function such that 
\[ 0 \leq \mu_{i_a}(v_i) + \gamma_{i_a}(v_i) \leq 1\]
and for every \(v_i \in V, i = 1, 2, \ldots, n\) and for every \(a \in A\).

(2) \(\mu_2\) is a membership function defined on \(E\) by
\[ \mu_2 : A \rightarrow IF^U(V \times V)\] (\(IF^U(V \times V)\) denotes collection of all intuitionistic fuzzy subsets in \(E\)), 
\[ a \mapsto \mu_2(a) = \mu_{2_a} \text{ (say)}, \]
\[ a \in A \text{ and } \mu_{2_a} : V \times V \rightarrow [0,1], (v_i, v_j) \mapsto \mu_{2_a}(v_i, v_j). \]
\[ \gamma_2 \] is the non-membership function defined on \(E\) by
\[ \gamma_2 : A \rightarrow IF^U(V \times V)\] (\(IF^U(V \times V)\) denotes collection of all intuitionistic fuzzy subsets in \(V \times V\)), 
\[ a \mapsto \gamma_2(a) = \gamma_{2_a} \text{ (say)}, a \in A \text{ and} \]
\[ \gamma_{2_a} : V \times V \rightarrow [0,1], (v_i, v_j) \mapsto \gamma_{2_a}(v_i, v_j). \]
where \((A, \mu_2), (A, \gamma_2)\) are intuitionistic fuzzy soft edge of membership function and non-membership function satisfying
(a) \(\mu_{2_a}(v_i, v_j) \leq \min\{\mu_{i_a}(v_i), \mu_{i_a}(v_j)\}\]
(b) \(\gamma_{2_a}(v_i, v_j) \leq \max\{\gamma_{i_a}(v_i), \gamma_{i_a}(v_j)\}\)
(c) \(0 \leq \mu_{2_a}(v_i, v_j) + \gamma_{2_a}(v_i, v_j) \leq 1\).

0 \leq \mu_{2_a}(v_i, v_j) + \gamma_{2_a}(v_i, v_j) \leq 1, for every \((v_i, v_j) \in E, i, j = 1, 2, \ldots, n\) and for every \(a \in A\). Then 
\[ G^* = (V, E, (A, \mu), (A, \gamma), (A, \mu), (A, \gamma)) \]
was said to be the Intuitionistic Fuzzy Soft Graph (IFSG) and this IFSG is denoted by \(G^*_A\).

**Remark 3.2** For every IFSG \(G^*_A\), the degree of indeterminacy of the intuitionistic fuzzy soft vertex \(v_i \in V\) for the parameter \(a \in A\) is 
\[ \varepsilon_{i_a}(v_i) = 1 - \mu_{i_a}(v_i) - \gamma_{i_a}(v_i) \]
and the degree of indeterminacy of the intuitionistic fuzzy soft edge \((v_i, v_j) \in E\) for the parameter \(a \in A\) is 
\[ \varepsilon_{2_a}(v_i, v_j) = 1 - \mu_{2_a}(v_i, v_j) - \gamma_{2_a}(v_i, v_j). \]

**Example 3.3** [10] Consider a simple graph \(G = (V, E)\) where 
\[ V = \{v_1, v_2, v_3, v_4\} \text{ and} \]
\[ E = \{(v_1, v_2), (v_2, v_3), (v_1, v_3), (v_1, v_4)\}. \]
Let \(A = \{a_1, a_2, a_3\}\) be the parameter set. Then the intuitionistic fuzzy soft graph (IFSG), 
\[ G^* = (V, E, (A, \mu), (A, \gamma), (A, \mu), (A, \gamma)) \]
is described in Table 1 and Figure 1.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_{i_a} v_1 v_2 v_1 v_4)</td>
</tr>
<tr>
<td>(a_1)</td>
</tr>
<tr>
<td>(a_2)</td>
</tr>
<tr>
<td>(a_3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 1(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_{2_a} (v_1, v_2) (v_1, v_3) (v_2, v_3) (v_1, v_4))</td>
</tr>
<tr>
<td>(a_1)</td>
</tr>
<tr>
<td>(a_2)</td>
</tr>
<tr>
<td>(a_3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 1(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma_{2_a} (v_1, v_2) (v_1, v_3) (v_2, v_3) (v_1, v_4))</td>
</tr>
<tr>
<td>(a_1)</td>
</tr>
<tr>
<td>(a_2)</td>
</tr>
<tr>
<td>(a_3)</td>
</tr>
</tbody>
</table>
In today’s world the need of providing proper career guidance to students is essential. Many students fail to achieve success in life because of improper choice of career. For example, we can see bulk of failed Engineering students around us because they choose engineering for higher education without interest or academic performance. So it is convenient that students should be given sufficient information on career determination on or before their twelth level. Academic Performance, interest, Personality etc., are some factors to be taken into consideration in a student’s career determination procedure where as the academic performance is much important.

Here we represent a problem relating career determination through an Intuitionistic Fuzzy Soft Graph. A teacher wants to determine suitable career option for his four students by analyzing their performance in twelth level exam. Let us represent the situation by an Intuitionistic fuzzy soft graph. Let $S_1,S_2,S_3$ and $S_4$ be four students. Let us take these four students as four parameters. Based on their interest, teacher consider Medicine, Engineering, Pharmacy and Biochemist as the professions. Consider the average mark obtained for the students in last four exams in hundred in four subjects – Mathematics (Ma), Physics (Ph), Chemistry (Ch) and Biology (Bi). Let us take these four subjects as vertices and represent it as $\{Ma,Ph,Ch,Bi\}$. Let us use these four vertices as subjects and represent it as $\{Ma,Ph,Ch,Bi\}$. Table 2(a)&2(b) represent the membership degree and non-membership degree of each student for different subject combinations and it is the intuitionistic fuzzy soft edges. Figure 2 is the intuitionistic fuzzy soft graph corresponding to the situation. We can solve this problem by analyzing the above data and using the properties of IFSG.

### Table 2(a)

<table>
<thead>
<tr>
<th>Subjects →</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ma</td>
<td>(0.75,0.20)</td>
<td>(0.85,0.13)</td>
<td>(0.92,0.06)</td>
<td>(0.95,0.05)</td>
</tr>
<tr>
<td>Ph</td>
<td>(0.80,0.15)</td>
<td>(0.89,0.09)</td>
<td>(0.90,0.09)</td>
<td>(0.90,0.08)</td>
</tr>
<tr>
<td>Ch</td>
<td>(0.95,0.05)</td>
<td>(0.92,0.07)</td>
<td>(0.85,0.13)</td>
<td>(0.88,0.10)</td>
</tr>
<tr>
<td>Bi</td>
<td>(0.93,0.05)</td>
<td>(0.94,0.05)</td>
<td>(0.82,0.16)</td>
<td>(0.84,0.13)</td>
</tr>
</tbody>
</table>

### Table 2(b)

<table>
<thead>
<tr>
<th>Subjects →</th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject Combination →</td>
<td>$S_1$</td>
<td>$S_2$</td>
<td>$S_3$</td>
<td>$S_4$</td>
</tr>
<tr>
<td>(Ma,Ph)</td>
<td>(0.75,0.15)</td>
<td>(0.84,0.11)</td>
<td>(0.90,0.06)</td>
<td>(0.90,0.06)</td>
</tr>
<tr>
<td>(Ma,Ch)</td>
<td>(0.75,0.18)</td>
<td>(0.84,0.12)</td>
<td>(0.85,0.10)</td>
<td>(0.88,0.08)</td>
</tr>
<tr>
<td>(Ma,Bi)</td>
<td>(0.75,0.15)</td>
<td>(0.85,0.11)</td>
<td>(0.82,0.13)</td>
<td>(0.84,0.10)</td>
</tr>
<tr>
<td>(Ph,Ch)</td>
<td>(0.80,0.05)</td>
<td>(0.89,0.07)</td>
<td>(0.85,0.11)</td>
<td>(0.87,0.08)</td>
</tr>
<tr>
<td>(Ph,Bi)</td>
<td>(0.80,0.05)</td>
<td>(0.89,0.07)</td>
<td>(0.82,0.10)</td>
<td>(0.84,0.11)</td>
</tr>
<tr>
<td>(Ch,Bi)</td>
<td>(0.93,0.05)</td>
<td>(0.92,0.05)</td>
<td>(0.82,0.11)</td>
<td>(0.84,0.10)</td>
</tr>
</tbody>
</table>

![Figure 2](image2.png)
Definition 3.5
Ifs-order of an IFSG $G^*_A\forall E\times (V,E,(A,\mu_1),(A,\gamma_1), (A,\mu_2),(A,\gamma_2))$ is defined as
$$O(G^*_A\forall E\times (V,E,(A,\mu_1),(A,\gamma_1), (A,\mu_2),(A,\gamma_2))) = \left( \sum_{a \in A} \left( \sum_{v \in V} \mu_{aV}(v) \right), \sum_{a \in A} \left( \sum_{v \in V} \gamma_{aV}(v) \right) \right)$$
for every $a \in A$ and $v \in V$.
In example 3.2., the order of IFSG is
$$O(G^*_A\forall E\times (V,E,(A,\mu_1),(A,\gamma_1), (A,\mu_2),(A,\gamma_2))) = \left( \sum_{a \in A} \left( \sum_{v \in V} \mu_{aV}(v) \right), \sum_{a \in A} \left( \sum_{v \in V} \gamma_{aV}(v) \right) \right) = ((0.8+0.6+0.0)+(0.9+0.65+0.5)+(0.2+0.75+0.8),(0.1+0.2+0.0)+(0.05+0.25+0.4)+(0.6+0.1+0.15)) = (5.5, 7.1)

Definition 3.6
Ifs-size of an IFSG $G^*_A\forall E\times (V,E,(A,\mu_1),(A,\gamma_1), (A,\mu_2),(A,\gamma_2))$ is defined as
$$S(G^*_A\forall E\times (V,E,(A,\mu_1),(A,\gamma_1), (A,\mu_2),(A,\gamma_2))) = \left( \sum_{a \in A} \sum_{v \in V} \mu_{2a}(v,v'), \sum_{a \in A} \sum_{v \in V} \gamma_{2a}(v,v') \right)$$
In example 3.2., the size of IFSG is
$$S(G^*_A\forall E\times (V,E,(A,\mu_1),(A,\gamma_1), (A,\mu_2),(A,\gamma_2))) = \left( \sum_{a \in A} \sum_{v \in V} \mu_{2a}(v,v'), \sum_{a \in A} \sum_{v \in V} \gamma_{2a}(v,v') \right) = ((0.1+0.0+0.0)+(0.15+0.25+0.1)+(0.01+0.1+0.2),(0.1+0.0+0.0)+(0.2+0.1+0.2)+(0.5+0.2+0.01)) = (0.91,1.31).

Definition 3.7
Ifs-degree of a vertex $v_i$ for the parameter $a \in A$ in an intuitionistic fuzzy soft graph $G^*_A\forall E\times (V,E,(A,\mu_1),(A,\gamma_1), (A,\mu_2),(A,\gamma_2))$ is defined as
$$d_{va}(v_i) = \left( \sum_{v \in V} \mu_{va}(v,v_i), \sum_{v \in V} \gamma_{va}(v,v_i) \right)$$
In example 3.4., ifs-degree of each vertex for the student $S_1$ is
$$d_{va}(Ma) = \left( 0.75+0.75+0.75+0.15+0.18+0.15 \right) = (3.35,0.48),$$
$$d_{va}(Ph) = \left( 0.75+0.80+0.80+0.15+0.05+0.05 \right) = (2.35,0.25),$$
$$d_{va}(Ch) = \left( 0.75+0.80+0.93+0.18+0.05+0.05 \right) = (2.48,0.28),$$
and $d_{va}(Bi) = \left( 0.75+0.80+0.93+0.15+0.05+0.05 \right) = (2.48,0.25).$
Let }E\in G_{A,V,E}^*(Ma,Bi)\text{ be the parameter set. Then the if- total edge degree of an edge for every parameter }a\in A\text{ is constant for every edge }E_{vv}ji \in E\text{ and for each parameter }a\in A\text{, then }G_{A,V,E}^*\text{ is said to be a } (l,l')\text{-if-regular intuitionistic fuzzy soft graph.}

Example 3.11
Consider a simple graph }G=(V,E)\text{ where }V=\{(v_1,v_2),(v_2,v_3),(v_3,v_4),(v_1,v_4),(v_2,v_4)\}\text{. Let }A=\{a_1,a_2\}\text{ be the parameter set. Then the intuitionistic fuzzy soft graph }G_{A,V,E}^*= (V,E,(A,\mu_{a_1}), (A,\mu_{a_2}),(A,\nu_{a_1}),(A,\nu_{a_2}))\text{ is described in Table 3 and Figure 3.}

In example 3.10,  
\begin{align*}
d_{G_{a_1}}(v_1,v_2) &= d_{G_{a_2}}(v_2,v_3) = d_{G_{a_2}}(v_2,v_4) =  
d_{G_{a_1}}(v_1,v_2) &= d_{G_{a_1}}(v_1,v_3) = d_{G_{a_1}}(v_1,v_4) = (1.0,0.8) \text{ and }  
d_{G_{a_2}}(v_1,v_2) &= d_{G_{a_2}}(v_2,v_3) = d_{G_{a_2}}(v_3,v_4) =  
d_{G_{a_2}}(v_1,v_2) &= (1.0,0.8). \text{ Therefore example 3.10 represents an (1.0,0.8) -if-regular intuitionistic fuzzy soft graph.}
\end{align*}

Figure 3

Table 3

<table>
<thead>
<tr>
<th>$\nu_{a_1}$</th>
<th>$v_1$</th>
<th>$v_2$</th>
<th>$v_3$</th>
<th>$v_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.7</td>
<td>0.6</td>
<td>0.6</td>
<td>0.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\mu_{a_2}$</th>
<th>$(v_1,v_2)$</th>
<th>$(v_2,v_3)$</th>
<th>$(v_3,v_4)$</th>
<th>$(v_1,v_4)$</th>
<th>$(v_2,v_4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Definition 3.12
Let }G_{A,V,E}^*\text{ be an IFSG and let }E_{vv}ji \in E\text{ be an edge for some }a\in A\text{ in }G_{A,V,E}^*\text{. Then the if-total edge degree of an edge }E_{vv}ji \in E\text{ for the parameter }a\in A\text{ is defined as }
td_{a_1}(v_i,v_j) = (td_{\mu_{a_1}}(v_i,v_j), td_{\nu_{a_2}}(v_i,v_j))\text{, where }
td_{\mu_{a_1}}(v_i,v_j) = \sum_{v_k \in E \setminus E_{vv}ji} \mu_{a_1}(v_k,v_j) + \mu_{a_1}(v_i,v_j),
\begin{align*}
td_{\nu_{a_2}}(v_i,v_j) &= \sum_{v_k \in E \setminus E_{vv}ji} \nu_{a_2}(v_k,v_j) + \nu_{a_2}(v_i,v_j),
\end{align*}

for }v_i,v_j,v_k \in V\text{ and for }a\in A\text{.}

Definition 3.13
An IFSG }G_{A,V,E}^*\text{ is said to be an if-s totally edge regular intuitionistic fuzzy soft graph if its-total edge degree is constant for every edge and for each parameter. If every edge in }G_{A,V,E}^*\text{ has the same if-s-total edge degree } (l,l')\text{ for every parameter }a\in A\text{, then }G_{A,V,E}^*\text{ is said to be a } (l,l')\text{-if-s totally edge regular intuitionistic fuzzy soft graph.}
Theorem 3.14

Let \( G^*_{A,V,E} \) be an IFSG. Then \( (\mu_{2a}, \gamma_{2a}) \) is a constant function for every parameter \( a \in A \) in \( G^*_{A,V,E} \) if and only if the following conditions are equivalent.

i. \( G^*_{A,V,E} \) is an ifs-edge regular IFSG.

ii. \( G^*_{A,V,E} \) is an ifs-totally edge regular IFSG.

Proof

Assume that \( (\mu_{2a}, \gamma_{2a}) \) is a constant function for every parameter \( a \in A \). Then \( \mu_{2a}(v_i, v_j) = k \) and \( \gamma_{2a}(v_i, v_j) = k' \) for every \( (v_i, v_j) \in E \) and for every \( a \in A \), where \( k \) and \( k' \) are constants. Assume that \( G^*_{A,V,E} \) is \((l,l')\)-ifs edge regular IFSG. Then
\[
d_{G^*_{A,V,E}}(v_i, v_j) = (l,l') \quad \text{for every} \quad (v_i, v_j) \in E \quad \text{and for every} \quad a \in A.
\]

Consider \( t d_{G^*_{A,V,E}}(v_i, v_j) = (t d_{\mu_{2a}}(v_i, v_j), t d_{\gamma_{2a}}(v_i, v_j)) = (t, l') \) for every \( v_i, v_j \in V \) and for every \( a \in A \). Therefore
\[
t d_{\mu_{2a}}(v_i, v_j) = (d_{\mu_{2a}}(v_i, v_j), d_{\gamma_{2a}}(v_i, v_j)) = (l, l')
\]
and \( \gamma_{2a}(v_i, v_j) = (l, l') \) for at least one pair of edges \((v_i, v_j) \in E\) and for some \( a \in A \). Let \( G^*_{A,V,E} \) be an \((l,l')\)-ifs edge regular IFSG. Then
\[
d_{G^*_{A,V,E}}(v_i, v_j) = (l,l')
\]
and \( \gamma_{2a}(v_i, v_j) = (l, l') \) for at least one pair of edges \((v_i, v_j) \in E\) and for every \( a \in A \). Therefore \( G^*_{A,V,E} \) is an ifs-totally edge regular IFSG.

Conversely assume that (i) and (ii) are equivalent.

That is \( G^*_{A,V,E} \) is an ifs-edge regular IFSG if and only if \( G^*_{A,V,E} \) is an ifs-totally edge regular IFSG. Suppose that \( (\mu_{2a}, \gamma_{2a}) \) is not constant for some parameter \( a \in A \). Then \( \mu_{2a}(v_i, v_j) \neq \mu_{2a}(v_i, v_j) \) and \( \gamma_{2a}(v_i, v_j) \neq \gamma_{2a}(v_i, v_j) \) for at least one pair of edges \((v_i, v_j) \in E\) and for some \( a \in A \). Let \( G^*_{A,V,E} \) be an \((l,l')\)-ifs edge regular IFSG. Then
\[
d_{G^*_{A,V,E}}(v_i, v_j) = (l,l')
\]
and \( \gamma_{2a}(v_i, v_j) = (l, l') \) for at least one pair of edges \((v_i, v_j) \in E\) and for every \( a \in A \). Therefore \( G^*_{A,V,E} \) is not an ifs-totally edge regular IFSG.

Theorem 3.15

If an intuitionistic fuzzy soft graph \( G^*_{A,V,E} \) is both ifs-edge regular and ifs-totally edge regular IFSG, then \( (\mu_{2a}, \gamma_{2a}) \) is a constant function for every parameter \( a \in A \).

Proof

Suppose \( G^*_{A,V,E} \) is both ifs-edge regular and ifs-totally edge regular IFSG. Then \( d_{G^*_{A,V,E}}(v_i, v_j) = (l,l') \) and \( \gamma_{2a}(v_i, v_j) = (l, l') \) for every \( v_i, v_j \in V \) and for every \( a \in A \). Therefore \( G^*_{A,V,E} \) is not an ifs-totally edge regular IFSG. Contradiction to our assumption. Therefore \( (\mu_{2a}, \gamma_{2a}) \) is constant for every \( (v_i, v_j) \in E \) and for every parameter \( a \in A \). Similarly, let \( G^*_{A,V,E} \) be a \((t,t')\)-ifs-totally edge regular IFSG. Then
\[
d_{G^*_{A,V,E}}(v_i, v_j) = (t,t') \quad \text{for every} \quad (v_i, v_j) \in E \quad \text{and for every} \quad a \in A.
\]
and \( \gamma_{2a}(v_i, v_j) = (t,t') \) for at least one pair of edges \((v_i, v_j) \in E\) and for some \( a \in A \). Let \( G^*_{A,V,E} \) be an \((t,t')\)-ifs edge regular IFSG. Then
\[
d_{G^*_{A,V,E}}(v_i, v_j) = (t,t')
\]
and \( \gamma_{2a}(v_i, v_j) = (t,t') \) for at least one pair of edges \((v_i, v_j) \in E\) and for every \( a \in A \). Therefore \( G^*_{A,V,E} \) is not an ifs-totally edge regular IFSG. Contradiction to our assumption. 

Therefore \( (\mu_{2a}, \gamma_{2a}) \) is constant for every \( (v_i, v_j) \in E \) and for every parameter \( a \in A \).
\( v_i, v_j \in V \) and for every \( a \in A \).

\[ \Rightarrow (\mu_{2a}, \gamma_{2a}) \text{ is a constant function for every parameter } a \in A. \]

**Remark 3.16**

Converse of the above theorem is not true.

If \((\mu_{2a}, \gamma_{2a})\) is a constant function for every parameter \( a \in A \) in an intuitionistic fuzzy soft graph \( G_{A,V,E}^* \), does not imply that \( G_{A,V,E}^* \) is an ifs-edge regular IFSG or ifstotally edge regular IFSG.

**Theorem 3.17**

Let \( G_{A,V,E}^* \) be an Intuitionistic fuzzy soft graph on a crisp graph \( G=(V,E) \). If \((\mu_{2a}, \gamma_{2a})\) is a constant function for every parameter \( a \in A \), then \( G_{A,V,E}^* \) is ifs-edge regular intuitionistic fuzzy soft graph if and only if \( G \) is edge regular.

**Proof**

Given that \((\mu_{2a}, \gamma_{2a})\) is a constant function for every parameter \( a \in A \). Let \( \mu_{2a}(v_i, v_j) = p \) and \( \gamma_{2a}(v_i, v_j) = q \) for every \((v_i, v_j) \in E \) and for every \( a \in A \), where \( p \) and \( q \) are constants. Assume that \( G_{A,V,E}^* \) is an ifs-edge regular IFSG. To prove \( G \) is edge regular. Suppose \( G \) is not edge regular. Let \( K_a \) be the crisp graph corresponding to the parameter \( a \in A \) in \( G_{A,V,E}^* \). \( d_{K_a}(v_i, v_j) \) is the degree of the edge \((v_i, v_j)\) and \( (v_i, v_j) \in E \) for \( a \in A \), where \( d_{K_a}(v_i, v_j) \) and \( d_{K_a}(v_j, v_i) \) is the degree of the edge \((v_j, v_i)\) and \( (v_j, v_i) \in E \) in \( K_a \). The degree of the edge \((v_i, v_j)\) and \( (v_j, v_i) \in E \) in \( K_a \) is \( d_{K_a}(v_i, v_j) = (\mu_{2a}(v_i, v_j), \gamma_{2a}(v_i, v_j)) \) where

\[
\sum_{(v_i, v_j) \in E} (\mu_{2a}(v_i, v_j) + \gamma_{2a}(v_i, v_j)), \text{ for } v_i, v_j \in V, a \in A.
\]

\[ = p(d_{K_a}(v_i) - 1) + p(d_{K_a}(v_j) - 1), \text{ for } v_i, v_j \in V \text{ and for } a \in A.
\]

\[ = p(d_{K_a}(v_i) + d_{K_a}(v_j) - 2 \]
Proof
Let $G^*_{A,v,E}$ be an ifs-regular Intuitionistic fuzzy soft graph and $(\mu_{2a},\gamma_{2a})$ is a constant function for every parameter $a \in A$. Then $d_{G^*_{a}}(v_i) = (d_{\mu_a}(v_i), d_{\gamma_a}(v_i)) = d$ for every $v_i \in V$ and $\mu_{2a}(v_i,v_j) = k$ and $\gamma_{2a}(v_i,v_j) = k'$ for every $(v_i,v_j) \in E$ and for every $a \in A$, $k,k'$ are constants. By definition of ifs-degree of an edge for the parameter $a \in A$ in an intuitionistic fuzzy soft graph,

$$d_{G^*_{a}}(v_i,v_j) = (d_{\mu_a}(v_i,v_j), d_{\gamma_a}(v_i,v_j)) ,$$

where

$$d_{\mu_a}(v_i,v_j) = d_{\mu_a}(v_i) + d_{\mu_a}(v_j) - 2\mu_{2a}(v_i,v_j) = d + d - 2(k) = 2(d - k)$$

and

$$d_{\gamma_a}(v_i,v_j) = d_{\gamma_a}(v_i) + d_{\gamma_a}(v_j) - 2\gamma_{2a}(v_i,v_j) = d + d - 2(k') = 2(d - k')$$

Therefore $d_{G^*_{a}}(v_i,v_j) = (2(d - k),2(d - k'))$, this is true for every $(v_i,v_j) \in E$ and for every $a \in A$, since $G^*_{A,v,E}$ is regular intuitionistic fuzzy soft graph.

Thus $G^*_{A,v,E}$ is an edge regular intuitionistic fuzzy soft graph.

Conversely suppose that $G^*_{A,v,E}$ is an edge regular intuitionistic fuzzy soft graph.

$$d_{G^*_{a}}(v_i,v_j) = (l,l')$$

for every $(v_i,v_j) \in E$ and $a \in A$, $l,l'$ are constants.

$$d_{G^*_{a}}(v_i,v_j) = (d_{\mu_a}(v_i,v_j), d_{\gamma_a}(v_i,v_j)) ,$$

where

$$d_{\mu_a}(v_i,v_j) = d_{\mu_a}(v_i) + d_{\mu_a}(v_j) - 2\mu_{2a}(v_i,v_j)$$

$$l = d + d - 2\mu_{2a}(v_i,v_j)$$

$$\Rightarrow \mu_{2a}(v_i,v_j)$$

is a constant.

Similarly $d_{\gamma_a}(v_i,v_j) = d_{\gamma_a}(v_i) + d_{\gamma_a}(v_j) - 2\gamma_{2a}(v_i,v_j)$$

$$l' = d + d - 2\gamma_{2a}(v_i,v_j)$$

$$\Rightarrow \gamma_{2a}(v_i,v_j)$$

is a constant.

This is true for every $(v_i,v_j) \in E$ and for every $a \in A$.

**Theorem 3.19**

Let $G^*_{A,v,E}$ be an Intuitionistic fuzzy soft graph on $G$ and $(\mu_{2a},\gamma_{2a})$ is constant for every parameter $a \in A$. If $G^*_{A,v,E}$ is ifs-regular intuitionistic fuzzy soft graph, then $G^*_{A,v,E}$ is ifs- totally edge regular intuitionistic fuzzy soft graph.

**Conclusion**

Fuzzy Graph Theory is finding an increasing number of applications in modeling real time systems. Intuitionistic fuzzy models give more precision, flexibility and compatibility to the system as compared to the fuzzy models and also soft set theory provide us a new way of coping with uncertainty from the view point of parameterization. We have applied the concepts of Intuitionistic fuzzy soft sets to graphs.

**REFERENCES**


