

On Fuzzy Game Theoretical Model for the prediction of stock market based on Triangular Fuzzy Number: A Study

^aUMA.K*, ^bVIDHYA.G

^a ASSISTANT PROFESSOR, POOMPUHAR COLLEGE, NAGAPATTINAM, 609 107, INDIA.

^b LECTURER, KUNTHAVAI NACCHIYAR GOVT. ARTS COLLEGE(W), THANJAVUR, 613007, INDIA.

Abstract:

The aim of this paper is to develop an effective methodology for solving constraint matrix games with payoff triangular fuzzy numbers. The concept of new ranking method for constrained matrix with payoff triangular fuzzy numbers and values are introduced. Stock market prediction is one of the most interesting research fields for ages. The stock market prices are in a dynamic, non-linear complex and chaotic in nature. We used new algorithm depending on a ranking function to solve the fuzzy game problem utilizing triangular fuzzy numbers and also trying to get a desirable gain. Fuzzy decision system is constructed based on new proposed ranking algorithm to analyze future opportunist of stocks.

Keywords: Triangular Fuzzy Number, ranking function, Stock Market values.

Introduction:

The theory of matrix games is a mathematical theory that deals with general features of competitive situations[1]. Matrix games have been extensively studied and successfully applied to many fields such as economics, business, management, and e-commerce as well as advertising. It is usually used when two or more individuals or organizations with conflicting objectives try to make decisions[8]. The set of objective functions in the game may have uncertain values where the way to deal with uncertainty is to use the concept of fuzzy games [7]. A ranking function is used which helps us not just to find the solution but also to find the best gain for the fully fuzzy game problem. Medinechiene. M, Zavadskas. E. K, Turskis. Z at (2011) described a model of dwelling selection using fuzzy games theory on buildings [10]. Jawad. M. A at (2012) represented fuzzy sets and fuzzy processes with game theory to address the uncertainty in data for mobile phone companies in Iraq [6].

Stock market prediction has been in focus for many years, but it can yield significant profits. Even though, predicting the stock prices has not been a simple task due to various factors intractable in such prediction. However, investors have developed a number of prediction methods that ought to help them to predict the direction of stock price movement[5]. In this paper we have focused on short term price prediction on general stock using financial time series data. Many authors have been utilized in this data to find poor results and low accuracy.

After some analysis, we have noticed that some methods increase their accuracy to evaluate in these models. Geo [3] has constructed a fuzzy trading rule but the rule was evolved with a genetic algorithm. Compose provided crisp solutions with inter prediction of fuzzy semantics. Garrison et. al. coevolved fuzzy trading rules from market trend features they have formulated as a **zero sum** competitive game to match trading strategies to be evaluated by broken age firm.

Deng-feng et [3] all have developed an efficient methodology for solving constrained matrix game with payoff of triangular fuzzy numbers. The objective of this paper is to propose a new algorithm depending on a ranking function to solve the fuzzy game problem using triangular fuzzy numbers and trying to get a desirable gain. This paper contains four sections: in section two a review of some fuzzy theory concepts, section three defines the fuzzy games problem, Ranking function and a new proposed algorithm, finally in section four Experimental results is represented depending on the new proposed algorithm.

Basic concepts:

Definition: Let a, b and c be real numbers with $a < b < c$. Then the **Triangular Fuzzy Number** (TFN) $A = (a, b, c)$ is the FN with membership function:

$$y = m(x) = \begin{cases} \frac{x-a}{b-a}, & x \in [a, b] \\ \frac{c-x}{c-b}, & x \in [b, c] \\ 0, & x < a \text{ and } x > c \end{cases}$$

Obviously we have that $m(b)=1$, while b need not be in the “middle” of a and c. It is well known that for a TFN $A = (a, b, c)$, the x-cut

$$A^x = [A_l^x, A_r^x] = [a+x(b-a), c-x(c-b)]$$

Definition: α – Cut Set (α – Level Set)

The crisp set of elements that belong to the fuzzy set \tilde{A} at least to the degree α is called the **α - level set**

$$A_\alpha = \{ x \in X / \mu_{\tilde{A}} \geq \alpha \}$$

$A'_\alpha = \{ x \in X / \mu_{\tilde{A}} > \alpha \}$ is called strong α -level set or strong α -cut. Convex Fuzzy Set [13]

Definition:A fuzzy set \tilde{A} is **convex** if

$$\mu_{\tilde{A}} [\alpha x_1 + (1 - \alpha)x_2] \geq \min \{ \mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2) \} \quad x_1, x_2 \in X, \alpha \in [0, 1].$$

Definition:

A fuzzy set is called **Normal** if $h(A) = 1$. A nonempty fuzzy set can always be normalized by dividing (x) by $hgt(A)$.

Definition:

A **Fuzzy number** \tilde{A} is a fuzzy set on the real line R, must satisfy the following conditions:

1. There exist at least one $x_0 \in R$ with $\mu_{\tilde{A}} = 1$.
2. $\mu_{\tilde{A}}(x)$ is piecewise continuous.
3. \tilde{A} must be normal and convex.

Definition:

A **Triangular fuzzy matrix** of order $m \times n$ is defined as $A = (a_{ij})_{m \times n}$, where $a_{ij} = \langle m_{ij}, \alpha_{ij}, \beta_{ij} \rangle$ is the ij th element of A , m_{ij} is the mean value of a_{ij} and α_{ij}, β_{ij} are the left and right spreads of a_{ij} respectively.

As for classical matrices defined the following operations on TFMs. Let $A = (a_{ij})$ and $B = (b_{ij})$ be two TFMs of same order. Then we have the following.

- (i) $A + B = (a_{ij} + b_{ij})$
- (ii) $A - B = (a_{ij} - b_{ij})$,
- (iii) For $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{n \times p}$, $A \cdot B = (c_{ij})_{m \times p}$, where $c_{ij} = \sum_{k=1}^n a_{ik} \cdot b_{kj}$, $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, p$
- (iv) $A^T = A'$ (the transpose of A)
- (v) $k \cdot A = (ka_{ij})$, where k is a scalar.

Definition:

A TFM is said to be a **pure null TFM** if all its entries are zero, i.e., all elements are $\langle 0, 0, 0 \rangle$. This matrix is denoted by 0 .

Definition:

A TFM is said to be a **Fuzzy null TFM** if all elements are of the form $a_{ij} = \langle 0, e_1, e_2 \rangle$, where $e_1 \cdot e_2 \neq 0$.

Definition:

A square TFM is said to be a **pure unit TFM** if $a_{ii} = \langle 1, 0, 0 \rangle$ and $a_{ij} = \langle 0, 0, 0 \rangle$, $i \neq j$, for all i, j . It is denoted by I .

Definition: Fuzzy Unit TFM

A square TFM is said to be a **Fuzzy unit TFM** if $a_{ii} = \langle 1, e_1, e_2 \rangle$ and $a_{ij} = \langle 0, e_3, e_4 \rangle$ for $i \neq j$ for all i, j , where $e_1 \cdot e_2 \neq 0$, $e_3 \cdot e_4 \neq 0$.

Some basic properties of TFM:

I For any three TFMs A, B and C of order $m \times n$ we have:

- (i) $A + B = B + A$,
- (ii) $A + (B + C) = (A + B) + C$,
- (iii) $A + A = 2A$,
- (iv) $A - A$ is a fuzzy null TFM ,
- (v) $A + 0 = A - 0 = A$.

II Let A and B be two TFMs of the same order and k, l be two scalars. Then:

- (i) $k(lA) = (kl)A$,
- (ii) $k(A + B) = kA + kB$,
- (iii) $(k+l)A = kA + lA$, if $k, l \geq 0$,

(iv) $k(A-B)=kA-kB$.

III If A and B be two TFMs such that $A + B$ and $A.B$ are defined then:

- (i) $(A0)0 = A$,
- (ii) $(A + B)0 = A0 + B0$,
- (iii) $((A.B)0 = B0.A0$.

IV Let A be a square TFM then

- (i) $A.A0$ and $A0.A$ are both symmetric,
- (ii) $A + A0$ is symmetric,
- (iii) $A-A0$ is fuzzy skew-symmetric.

Fuzzy Game Problem:

The fuzzy game problem is where all the payoffs of the game matrix are fuzzy quantities. Now the formula of the fully fuzzy game problem is as follows:

$$\tilde{A} = (\tilde{a}_{ij})_{m \times n} = \begin{matrix} & \beta_1 & \beta_2 & \dots & \beta_n \\ \delta_1 & \left(\begin{matrix} \tilde{a}_{11} & \tilde{a}_{12} & \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \dots & \tilde{a}_{2n} \\ \vdots & \vdots & \dots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \dots & \tilde{a}_{mn} \end{matrix} \right) \\ \delta_2 & & & & \\ \vdots & & & & \\ \delta_m & & & & \end{matrix},$$

Where $\tilde{a}_{ij} = (a_{ij}^l, a_{ij}^m, a_{ij}^r)$ ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$) are a matrix game with payoffs of triangular fuzzy numbers is expressed with \tilde{A} .

Ranking Function:

If $F(R)$ is the set of all fuzzy numbers defined on R the set of real numbers then a ranking function $R: F(R) \rightarrow R$ maps each fuzzy number into a real ordinary number where there is a natural order, the order rules are as follows [5]:

$\tilde{A} > \tilde{B}$ if and only if $R(\tilde{A}) > R(\tilde{B})$.

$\tilde{A} = \tilde{B}$ if and only if $R(\tilde{A}) = R(\tilde{B})$.

$\tilde{A} < \tilde{B}$ if and only if $R(\tilde{A}) < R(\tilde{B})$.

Where A and B are two fuzzy numbers belong to $F(R)$.

A New proposed Algorithm :

We use the previous triangular membership function and by the α - cut, $\alpha \in [0, 1]$ then

$$\frac{x-a}{b-a} = \alpha \Rightarrow x = (b - a) + a = \inf \tilde{A}(\alpha)$$

$$\frac{c-x}{c-b} = \alpha \Rightarrow x = c - (c - b) = \sup \tilde{A}(\alpha)$$

Applying the following ranking function $R(\tilde{A}) = \int_0^1 [k \inf \tilde{A}(\alpha) + (1 - k) \sup \tilde{A}(\alpha)] d\alpha, k \in [0, 1]$ then

$$R(\tilde{A}) = \int_0^1 k [\alpha(b-a) + a] d\alpha + \int_0^1 (1-k)[c - \alpha(c-b)] d\alpha$$

$$R(\tilde{A}) = k \left[\frac{\alpha^2}{2} (b-a) + a \right] + (1-k) \left[c\alpha - \frac{\alpha^2}{2} (c-b) \right]$$

$$R(\tilde{A}) = k \left[\frac{1}{2} b - \frac{1}{2} a + a \right] + (1-k) \left[c - \frac{1}{2} c + \frac{1}{2} b \right].$$

$$R(\tilde{A}) = k \left[\frac{1}{2} a + \frac{1}{2} b \right] + (1-k) \left[\frac{1}{2} c + \frac{1}{2} b \right].$$

$$R(\tilde{A}) = \frac{k}{2} a + \frac{k}{2} b + \frac{c}{2} + \frac{b}{2} - \frac{k}{2} b - \frac{k}{2} c.$$

$$R(\tilde{A}) = \frac{1}{2} (c+b) + \frac{k}{2} (a-b).$$

Experimental results:

In this section, we present our work about the stock forecasting method based on fuzzy time series with triangular fuzzy number. In order to make the model easier to understand, we use Nifty fifty stock index from July 2017 to December 2017. The proposed method is presented as follows:

Date	Open	percentage change Value	Fuzzy
24.7.2017	9936.8	0.74%	A3
25.7.2017	10010.55	-0.268 %	A2
26.7.2017	9983.65	0.79 %	A3
-	-	-	-
-	-	-	-
-	-	-	-
-	-	-	-
12.7.2017	10324.9	-0.855%	A2
13.7.2017	10236.6	-0.071%	A2

This process repeated until some termination condition is the predicated value of the data can be calculated according to triangular fuzzy number and the payoff matrix. Then the root mean square error between predicated value and the actual value stock prices employed as new proposed algorithms.

Then, we find the value of the game for the following payoff matrix.

$$\begin{matrix} & \text{Player B} \\ \text{player A} & \begin{pmatrix} 0 & 0.6 & 0.4 & 0 \\ 0.071 & 0.5 & 0.357 & 0.007 \\ 0.047 & 0.381 & 0.524 & 0.047 \\ 0 & 0.5 & 0.333 & 0.16 \end{pmatrix} \end{matrix}$$

We transform the crisp game problem for the payoff matrix to fuzzy game problem and then solve it by applying the new proposed algorithm. Consider the payoff matrix as triangular fuzzy numbers.

Let $\Delta = 0.5$ for R_1 and R_4 , $\Delta = 0.4$ for R_2 , $\Delta = 0.25$ for R_3 where $R_i = (a_{ij} - 1, a_{ij} + 1, \Delta)$ for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$.

$$\begin{bmatrix} (-1, 1, 0.5) & (-0.4, 1.6, 0.5) & (-0.6, 1.6, 0.5) & (-1, 1, 0.5) \\ (-0.929, 1.071, 0.4) & (-0.5, 1.5, 0.4) & (-0.649, 1.351, 0.4) & (-0.927, 1.071, 0.4) \\ -0.953, 1.047, 0.25) & (-0.619, 1.381, 0.25) & (-0.476, 1.524, 0.25) & (-0.953, 1.04, 0.25) \\ (-1.1, 0.5) & (-0.5, 1.5, 0.5) & (-0.667, 1.333, 0.5) & (-0.833, 1.167, 0.5) \end{bmatrix}$$

Now, we applying the new proposed ranking algorithm

$R(\tilde{A}^k) = \frac{1}{2}(c + b) + \frac{1}{2}(a - c)$. $k \in [0, 1]$ for the fuzzy payoff matrix. Stating all the values of k neglecting (0) using principle of dominance and method of sub game where each single sub game is solved with probabilities represented in table as follows:

(k)	Value of the game
0.1	0.675
0.2	0.6
0.3	0.537
0.4	0.470
0.5	0.404
0.6	0.337
0.7	0.271
0.8	0.204
0.9	0.138
1	0.071

The best shrinkage constant (k) that gives the best gains is when $k = 0.1$ where player A when $k \in [0.1, 1]$ with an optimal strategy for A (0, 0.9, 0.1, 0) and for player B (0.4, 0.6, 0, 0).

Conclusion:

This paper presents, an analysis of the efficiency of stock market prediction using triangular fuzzy numbers based on new proposed algorithm depending on a ranking functions. By adopting triangular fuzzy number, our model can obtain more suitable partition of the universe, which can improve the forecasting results significantly. Furthermore, we deal with games of fuzzy payoffs problems while there is uncertainty in stock market data. We use the triangular membership function which make the data fuzziness and utilize the new proposed ranking function algorithm by using triangular fuzzy numbers for the decision maker to get the best gains. .

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