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Reliability Analysis of Paper Plant using Boolean Function with Fuzzy Logic Technique

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Abstract

Assessment of reliability analysis of paper plant with fuzzy logic technique has carried out in this paper. Fuzzy Weibull distribution and lifetime of a component is also determined. The paper plant is a complex system under consideration which consists of various subsystems such as chipping, feeding, pulp preparation, washing, screening, bleaching and paper production etc. The mathematical model has developed based on Boolean function technique. Reliability of paper plant is obtained in three different cases. Reliability of each component is discussed as when all failures follow fuzzy Weibull distribution and when all repair follow exponential distribution and mean time to failure is also determined. Finally, a numerical analysis is presented to illustrate the characteristics of fuzzy reliability and their α - cuts.

Keywords: Fuzzy reliability, fuzzy Weibull distribution, fuzzy hazard function, mean time to failure and fuzzy logic technique.

Introduction

Reliability analysis is benefited to the industry in terms of higher productivity and lower maintenance cost. This can also help the management to understand the effect of increasing or decreasing the repair rates of a particular component or subsystem in overall system. Several researchers for the last many years have discussed the various facts of reliability technology of the subsystems or systems in process industries at various level and a number of research papers have been published in this direction including Singh et al. [1] has given reliability of a fertilizer production supply problem. Gupta et al. [2] derived a numerical analysis of reliability and availability of the serial processes in butter-oil processing plant.

The lifetimes of a component are assumed to be random variables. The probability distributions of the random variables have crisp parameters. In many situations, the parameters are difficult to determine due to uncertainties and imprecision of data. So it is reasonable to assume the parameters to be fuzzy quantity. In this paper, the lifetimes and repair times of components are assumed to have Weibull distribution with fuzzy parameters.

Paper making process is a very complex system involving many unit operations and processes; mainly it consists of seven subsystems. The first subsystem is chipping unit and it takes input in the form of raw material. Where cutters start cutting in fine small pieces and then the pieces send to the second subsystem is feeding. After that it goes to the third subsystem namely pulp preparation unit, which has an identical unit digester in standby redundancy. The pieces are then crushed and convert into the pulp by mixing with fresh water. The pulp goes to the fourth subsystem, washing unit, which has an identical unit Decker in standby redundancy. After filtration process this pulp goes to the screening unit which has an identical unit cleaner in parallel redundancy. When the impurities are removed, purification is completed by bleaching process which is connected in series. Finally, the white pulp goes to the last subsystem which is known as paper production or finished product. All the subsystems are connected in series and series as shown below in figure-1. During the last 5 decades reliability concepts have been applied in various technological fields. This technique has also been applied to a number of industrial and transportation problems; a detailed discussion is contained in Dhillon et al. [3].

Literature Review

Among the various distributions, the Weibull distribution has been proven to be flexible and versatile at describing monotonic failure rate data. However, for many modern complex systems which exhibit unimdal or bathtub shaped failure rates, the Weibull distribution is inadequate by Cai et al. [4] has given a different insight by introducing the possibility assumption and fuzzy state assumption to replace the probability and binary state assumptions. Cai et al. [5] presented an introduction to system failure engineering and its use of fuzzy methodology. Pervaiz et al. [6] discussed the mathematical modelling and digestive system of a paper making plant based on queuing theory. Karpisek et al. [7] described two fuzzy reliability models that are based on the Weibull fuzzy distribution. Pervaiz et al. [8] developed a mathematical model for scheduling of jobs problem in a small scale measure of paper making industry. Baloui et al. [9] evaluated reliability function using fuzzy exponential lifetime distribution. Pervaiz et al. [10] discussed a mathematical modeling and performance analysis of stock preparation unit in paper plant industry using genetic algorithm. The parameter of the system is represented by a trapezoidal fuzzy number. Agarwal et al. [11] has discussed the assessment of reliability analysis of sugar manufacturing plant using Boolean function technique is analyzed. In this paper, we propose a general procedure to construct the reliability characteristics and its α-cuts set, when the parameters are fuzzy.

Assumptions

a) Initially, the complete system is in operable state.
b) Failure rates of components are s-independent.
c) There is no repair facility for a failed component.
d) Reliability of each component is known in advance.
e) This system is in fuzzy arrangement.

List of Notations

θ_1 : State of chipping unit.
θ_2 : State of feeding unit.
θ_3 : State of pulp preparation unit.
θ_4 : Digester.
θ_5 : State of washing unit.
θ_6 : Decker.
θ_7 : State of screening unit.
θ_8, θ_9 : Cleaners.
θ_{10} : State of bleaching unit.
θ_{11} : Finished product / Paper mills.
$\theta_i (i = 1,2,---,11)$: 1 in good state.
0 in bad state.
$\tilde{\theta}$: Fuzzy number.
$\tilde{S}(t)$: Fuzzy reliability functions of a component with time t.
$\xi_{\tilde{\theta}}$: Membership functions of a fuzzy number.
$\wedge / \vee$: Conjunction/Disjunction.

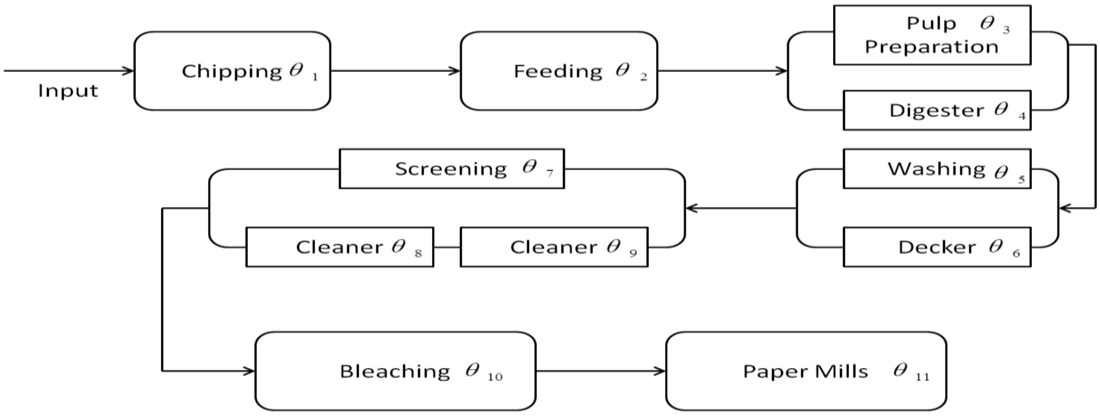

Figure-1: Flow chart of paper milling process

Mathematical Formation and Analysis

By using Boolean function technique, the conditions of capability for the successful operation of the system in terms of logical matrix are expressed as:

$$F(\theta_1,\theta_2,---,\theta_{11}) = \begin{bmatrix} \theta_1 & \theta_2 & \theta_3 & \theta_5 & \theta_7 & \theta_{10} & \theta_{11} & \\ \theta_1 & \theta_2 & \theta_3 & \theta_5 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} \\ \theta_1 & \theta_2 & \theta_3 & \theta_6 & \theta_7 & \theta_{10} & \theta_{11} & \\ \theta_1 & \theta_2 & \theta_3 & \theta_6 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} \\ \theta_1 & \theta_2 & \theta_4 & \theta_5 & \theta_7 & \theta_{10} & \theta_{11} & \\ \theta_1 & \theta_2 & \theta_4 & \theta_5 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} \\ \theta_1 & \theta_2 & \theta_4 & \theta_6 & \theta_7 & \theta_{10} & \theta_{11} & \\ \theta_1 & \theta_2 & \theta_4 & \theta_6 & \theta_8 & \theta_9 & \theta_{10} & \theta_{11} \end{bmatrix} \quad (1)$$

By the application of algebra of logic, equation (1) may be written as,

$$F(\theta_1,\theta_2,---,\theta_{11}) = [\theta_1\theta_2\theta_{10}\theta_{11} \wedge f(\theta_1,\theta_2,---,\theta_{11})] \quad (2)$$

Where,

$$F(\theta_1,\theta_2,---,\theta_{11}) = \begin{bmatrix} \theta_3 & \theta_5 & \theta_7 & \\ \theta_3 & \theta_5 & \theta_8 & \theta_9 \\ \theta_3 & \theta_6 & \theta_7 & \\ \theta_3 & \theta_6 & \theta_8 & \theta_9 \\ \theta_4 & \theta_5 & \theta_7 & \\ \theta_4 & \theta_5 & \theta_8 & \theta_9 \\ \theta_4 & \theta_6 & \theta_7 & \\ \theta_4 & \theta_6 & \theta_8 & \theta_9 \end{bmatrix} \quad (3)$$

Then

$$A_1 = [\theta_3 \quad \theta_5 \quad \theta_7] \quad (4)$$
$$A_2 = [\theta_3 \quad \theta_5 \quad \theta_8 \quad \theta_9] \quad (5)$$
$$A_3 = [\theta_3 \quad \theta_6 \quad \theta_7] \quad (6)$$
$$A_4 = [\theta_3 \quad \theta_6 \quad \theta_8 \quad \theta_9] \quad (7)$$
$$A_5 = [\theta_4 \quad \theta_5 \quad \theta_7] \quad (8)$$
$$A_6 = [\theta_4 \quad \theta_5 \quad \theta_8 \quad \theta_9] \quad (9)$$
$$A_7 = [\theta_4 \quad \theta_6 \quad \theta_7] \quad (10)$$
$$A_8 = [\theta_4 \quad \theta_6 \quad \theta_8 \quad \theta_9] \quad (11)$$

Fuzzy Weibull Distribution

The Weibull distribution is widely used in reliability, and its probability density function is as:

$$f(x) = \frac{\beta}{\theta}\left(\frac{x-\delta}{\theta}\right)^{\beta-1} e^{-\left(\frac{x-\delta}{\theta}\right)^{\beta}} \quad (12)$$

Where θ is scale parameter, β is shape parameter and δ is location parameter are crisps in nature.

Fuzzy Reliability Function

Fuzzy reliability is based on the fuzzy set theory of Zadeh [12]. Fuzzy reliability (fuzzy survival) function $(\tilde{S}(t))$ is the fuzzy probability in which a unit survives beyond time t.

Let the random variable X denotes the lifetime of a system components, also let X has density function and fuzzy cumulative distribution function $\tilde{F}_X(t)=\tilde{P}(X\le t)$ where parameter $\tilde{\theta}$ is a fuzzy number, and then the fuzzy reliability function at time t is defined as:

$$\begin{aligned}\tilde{S}(t)&=\tilde{P}(X>t)\\&=1-\tilde{F}_X(t)\\&=\{[1-F_{\max}(x)[\alpha],1-F_{\min}(x)[\alpha]],\mu_{F(x)}=\alpha\},t>0\end{aligned}\tag{13}$$

Suppose that we want to calculate the reliability of a component, such that the lifetime random variable has fuzzy Weibull distribution. So we represent parameter $\tilde{\theta}$ with a trapezoidal fuzzy number as $\tilde{\theta}=(a_1,a_2,a_3,a_4)$ such that we can describe a membership function $\xi_{\tilde{\theta}}(x)$ in the following manner:

$$\xi_{\tilde{\theta}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & a_1\le x<a_2\\ 1, & a_2\le x<a_3\\ \dfrac{a_4-x}{a_4-a_3}, & a_3\le x<a_4\end{cases}\tag{14}$$

The α-cuts $\tilde{\theta}$ is denoted as follows:

$$\tilde{\theta}[\alpha]=[a_1+(a_2-a_1)\alpha,a_4-(a_4-a_3)\alpha]\tag{15}$$

So fuzzy reliability function of a component is as follows:

$$\begin{aligned}\tilde{S}(t)[\alpha]&=\left\{\int_c^d\frac{\beta}{\theta}\left(\frac{x-\delta}{\theta}\right)^{\beta-1}e^{-\left(\frac{x-\delta}{\theta}\right)^{\beta}}dx\mid\theta\in\tilde{\theta}[\alpha]\right\}\\&=\left\{e^{-\left(\frac{t-\delta}{\theta}\right)^{\beta}}\mid\theta\in\tilde{\theta}[\alpha]\right\}\end{aligned}\tag{16}$$

According to that the $e^{-\left(\frac{t-\delta}{\theta}\right)^{\beta}}$ increasing θ, then the α-cuts of fuzzy reliability function is as:

$$\tilde{S}(t)[\alpha]=\left[\exp\left\{-\left(\frac{t-\delta}{a_1+(a_2-a_1)\alpha}\right)^{\beta}\right\},\exp\left\{-\left(\frac{t-\delta}{a_4-(a_4-a_3)\alpha}\right)^{\beta}\right\}\right]\tag{17}$$

$\tilde{S}(t)[\alpha]$ is a two dimensional function in terms of α and t, where $(0\le\alpha\le1 \text{ and } t>1)$. For t_0, this is a trapezoidal fuzzy number and membership function of $\tilde{S}(t_0)$ is as follows:

$$\xi_{\tilde{S}(t_0)}(x)=\begin{cases}\dfrac{x-\exp\left\{-\left(\frac{t_0-\delta}{a_1}\right)^{\beta}\right\}}{\exp\left\{-\left(\frac{t_0-\delta}{a_2}\right)^{\beta}\right\}-\exp\left\{-\left(\frac{t_0-\delta}{a_1}\right)^{\beta}\right\}}, & \exp\left\{-\left(\frac{t_0-\delta}{a_1}\right)^{\beta}\right\}\le x<\exp\left\{-\left(\frac{t_0-\delta}{a_2}\right)^{\beta}\right\}\\ 1, & \exp\left\{-\left(\frac{t_0-\delta}{a_2}\right)^{\beta}\right\}\le x\le\exp\left\{-\left(\frac{t_0-\delta}{a_3}\right)^{\beta}\right\}\\ \dfrac{\exp\left\{-\left(\frac{t_0-\delta}{a_4}\right)^{\beta}\right\}-x}{\exp\left\{-\left(\frac{t_0-\delta}{a_4}\right)^{\beta}\right\}-\exp\left\{-\left(\frac{t_0-\delta}{a_3}\right)^{\beta}\right\}}, & \exp\left\{-\left(\frac{t_0-\delta}{a_3}\right)^{\beta}\right\}<x\le\exp\left\{-\left(\frac{t_0-\delta}{a_4}\right)^{\beta}\right\}\end{cases}\tag{18}$$

Fuzzy mean time to failure (FMTTF) is the expected time to failure.

According to the definition of Buckley [13] FMTTF of this fuzzy system is a fuzzy number and can be calculated as follows:

$$\begin{aligned}M\tilde{T}TF[\alpha]&=\left\{\int_0^{\infty}xf(x)dx\mid\theta\in\tilde{\theta}[\alpha]\right\}\\&=\left\{\int_0^{\infty}S(t)dt\mid\theta\in\tilde{\theta}[\alpha]\right\}\end{aligned}\tag{19}$$

When the failure random variable has fuzzy Weibull distributed then:

$$\begin{aligned}M\tilde{T}TF[\alpha]&=\{\theta\Gamma(1+\beta^{-1})\mid\theta\in\tilde{\theta}[\alpha]\}\\&=[(a_1+(a_2-a_1)\alpha)\Gamma(1+\beta^{-1}),(a_4-(a_4-a_3)\alpha)\Gamma(1+\beta^{-1})]\end{aligned}\tag{20}$$

Accordingly (20), the following membership function is obtained:

$$\xi_{\tilde{S}(t_0)}(x)=\begin{cases}\dfrac{x-a_1\Gamma(1+\beta^{-1})}{(a_2-a_1)\Gamma(1+\beta^{-1})}, & a_1\Gamma(1+\beta^{-1})\le x<a_2\Gamma(1+\beta^{-1})\\ 1, & a_2\Gamma(1+\beta^{-1})\le x\le a_3\Gamma(1+\beta^{-1})\\ \dfrac{a_1\Gamma(1+\beta^{-1})-x}{(a_4-a_3)\Gamma(1+\beta^{-1})}, & a_3\Gamma(1+\beta^{-1})\le x<a_4\Gamma(1+\beta^{-1})\end{cases} \tag{21}$$

Fuzzy Hazard Function

In fuzzy reliability theory, the fuzzy hazard function play very attractive role. We will propose the concept of a fuzzy hazard function based on the fuzzy probability measure and call its α-cuts. The fuzzy hazard function $\tilde{h}(t)$ is the fuzzy conditional probability of an item failing in the interval t to $(t+dt)$ given that it has not failed by time t. Hazard function is also known as the instantaneous failure rate. Mathematically, we would define the fuzzy hazard function as:

$$\begin{aligned}\tilde{h}(t)[\alpha]&=\lim_{\Delta t\to 0}\frac{\tilde{P}(t<X<t+\Delta t\mid X>t)}{\Delta t}\\ &=\left\{\lim_{\Delta t\to 0}\frac{S(t)-S(t+\Delta t)}{\Delta t S(t)}\mid \theta\in\tilde{\theta}[\alpha]\right\}\\ &=\left\{\frac{-S'(t)}{S(t)}\mid \theta\in\tilde{\theta}[\alpha]\right\}\\ &=\left\{\frac{f(t)}{S(t)}\mid \theta\in\tilde{\theta}[\alpha]\right\}\end{aligned} \tag{22}$$

The fuzzy Weibull distributed with $\delta=0$ has fuzzy hazard function is as follows:

$$\tilde{h}(t)[\alpha]=\left\{\frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}\mid \theta\in\tilde{\theta}[\alpha]\right\} \tag{23}$$

$\tilde{h}(t)[\alpha]$ is a two dimensional function in terms of α and t $(0\le\alpha\le 1$ and $t>1)$.

Numerical Analysis

Let lifetime of the component is modeled by a Weibull distribution with fuzzy parameter $\tilde{\theta}$ and $\delta=0$ that $\tilde{\theta}$ = (1.25, 1.30, 1.45, 1.60). Then α-cuts of fuzzy reliability function, fuzzy hazard function and fuzzy mean time to failure are given by

(i) Fuzzy reliability is:

$\tilde{\theta}[\alpha]=(1.25+0.05\alpha,\ 1.60-0.15\alpha),$

$$\tilde{S}(t)[\alpha]=\left[\exp\left\{-\frac{t}{1.25+.005\alpha}\right\}^{\beta},\exp\left\{-\frac{t}{1.60-.002\alpha}\right\}^{\beta}\right] \tag{24}$$

$$\tilde{S}(t)[\alpha]=\left[\exp\left\{-\left(\frac{t-\delta}{1.25+0.05\alpha}\right)^{\beta}\right\},\ \exp\left\{-\left(\frac{t-\delta}{1.60-0.15\alpha}\right)^{\beta}\right\}\right] \tag{25}$$

For all α

(1) If t = 0.5

$$\xi_{\tilde{S}(0.5)}(x)=\begin{cases}\dfrac{x-\exp\{-(0.4)^{\beta}\}}{\exp\{-(0.3846)^{\beta}\}-\exp\{-(0.4)^{\beta}\}}, & \exp\{-(0.4)^{\beta}\}\le x<\exp\{-(0.3846)^{\beta}\}\\ 1, & \exp\{-(0.3846)^{\beta}\}\le x\le\exp\{-(0.3125)^{\beta}\}\\ \dfrac{\exp\{-(0.3125)^{\beta}\}-x}{\exp\{-(0.3125)^{\beta}\}-\exp\{-(0.3448)^{\beta}\}}, & \exp\{-(0.3448)^{\beta}\}<x\le\exp\{-(0.3125)^{\beta}\}\end{cases} \tag{26}$$

(2) If α = 0 then reliability is as:

$$\tilde{S}(t)[0]=\left[\exp\{-(0.8)^{\beta}\},\ \exp\{-(0.625t)^{\beta}\}\right] \tag{27}$$

(ii) Fuzzy hazard function is:

$$\tilde{h}(t)[\alpha]=\left\{\frac{\beta}{1.60-0.15\alpha}\left(\frac{t}{1.60-0.15\alpha}\right)^{\beta-1},\ \frac{\beta}{1.25+0.05\alpha}\left(\frac{t}{1.25+0.05\alpha}\right)^{\beta-1}\right\} \tag{28}$$

(iii) Fuzzy mean time to failure:

$$\begin{aligned}M\tilde{T}TF[\alpha](\beta)&=\left\{\theta\Gamma(1+\beta^{-1})\mid\theta\in\tilde{\theta}[\alpha]\right\}\\ &=[(1.125+0.005\alpha)\Gamma(1+\beta^{-1}),\ (1.60-0.15\alpha)\Gamma(1+\beta^{-1})]\end{aligned} \tag{29}$$

According to the behavior of the gamma function, the value of FMTTF at β = 2.166 has a minimum value:

$$M\widetilde{TTF}[\alpha](2.166) \\ = [(1.1070 + 0.0443\alpha),\ (1.4170 - 0.0443\alpha)] \quad (30)$$

Accordingly the following membership function is obtained:

$$\xi_{\tilde{S}(t_0)}(x) = \begin{cases} \dfrac{x - 1.1070}{0.0443} & , \quad 1.1070 \le x < 1.1513 \\ 1 & , \quad 1.1513 \le x \le 1.2841 \\ \dfrac{1.4170 - x}{0.1328} & , \quad 1.2841 \le x < 1.4170 \end{cases} \quad (31)$$

Conclusion

Weibull distribution, fuzzy reliability function, fuzzy hazard function and their α -cuts have been successfully investigated in this paper. Whenever the lifetimes of a components and parameters contain randomness and fuzziness respectively, the approach of reliability theory based on traditional statistical analysis may be inappropriate. Fuzzy system reliability is based on the concept of fuzzy set and fuzzy probability theory in our method. In this paper, the scale parameter was considered as fuzzy trapezoidal number. In further research, the shape and location parameters can be considered fuzzy separately or combined. Also in the further research is required to study some important topics in fuzzy reliability theory such as mean residual life.

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