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Characterizations of $(\alpha p)^*$- R_0 Spaces

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Abstract

In this paper we introduce $(\alpha p)^*$-R_0- spaces and we study some characterization of $(\alpha p)^*$-R_0 Spaces. We analyse the relation between $(\alpha p)^*$- closed sets with already existing closed sets.

Keywords: $(\alpha p)^*$- closed sets, $(\alpha p)^*$- open sets, $(\alpha p)^*$-closure, $(\alpha p)^*$- R_o spaces.

Introduction

Levine [7] introduced generalized closed sets (briefly g-closed sets) in topological spaces and studied their basic properties. O. Njastad[9] defined α - closed in 1965. N.Levine [6] introduced the class of semi-closed and semi-open sets in 1963. A. S. Mashhour [7] defined preopen and pre closed sets in 1982. L.Elvina Mary, R.Saranya [6] introduced $(\alpha p)^*$ - closed sets in 2017. The aim of this paper is to introduce a $(\alpha p)^*$ - R_o spaces and we investigate some characterization of $(\alpha p)^*$ - R_o - spaces.

Preliminaries

Definition 2.1: A subset A of a topological space (X,τ) is called

(i) A semi-open set if A ⊆ cl(int(A)) and a semi-closed set if int(cl(A)) ⊆A,

(ii) A preopen set if A ⊆ int(cl(A)) and a preclosed set if cl(int(A)) ⊆ A,

(iii) An α- open set if A ⊆ int(cl(int(A))) and an α -closed set if cl(int(cl(A))) ⊆ A,

(iv) A semi-preopen set if A⊆ cl(int(cl(A))) and a semi-preclosed set if int(cl(int(A))) ⊆A

(v) A regular open set if A=int(cl(A))and a regular closed set if cl(int(A))=A.

The semi-closure (resp.preclosure , semi-preclosure) of a subset A of a space (X,τ) is the intersection of all semi-closed(resp. preclosed , α-closed, semi-preclosed) sets that contain A and is denoted by scl(A) (resp.pcl(A), Acl(A), spcl(A)).

Definition 2.2: A subset A of a space (X,τ) is called

(i) A generalized closed (briefly g-closed) set[10] if cl(A)⊆U whenever A⊆U and U is open in (X,τ); the compliment of a g-closed set is called a g-open set,

(ii) A semi-generalized closed (briefly sg-closed) set[2] if scl(A)⊆U whenever A⊆U and U is semi-open in(X,τ); the compliment of sg-closed set is called a sg-open set,

(iii) A generalized semi-closed (briefly gs-closed) set if scl(A)⊆U whenever A⊆U and U is open in (X,τ)

(iv) An α-generalized closed (briefly αg-closed) set[3] if αcl(A)⊆U whenever A⊆U and U is α-open in (X,τ),

(v) A generalized α-closed (briefly gα-closed) set [3] if αcl(A)⊆U whenever A⊆U and U is α-open in (X,τ)

(vi) A g*- closed set [10] if cl(A)⊆U whenever A⊆U and U is g-open in (X,τ),

(vii) A g^{**}-closed set [8] if cl(A)⊆U whenever A⊆U and U is g*-open in (X,τ),

(viii) A generalized preclosed(briefly gp-closed) setif pcl(A)⊆U whenever A⊆U and U is open in (X,τ),

(ix) A generalized semi-preclosed (briefly gsp-closed) set [5] if spcl(A)⊆U whenever A⊆U and U is open in(X,τ)

(x) A generalized pre regular closed (briefly gpr-closed) set [6] if pcl(A)⊆U whenever A⊆U and U is regular open in (X,τ),

(xi) A $g^{\#}$-closed set [11] if cl(A)⊆U whenever A⊆U and U is αg-open in (X,τ),

(xii) A generalized α**-closed (briefly gα**-closed) set [3] if αcl(A)⊆int(cl(U)) whenever A⊆U and U is α-open in (X,τ),

(xiii) A μ*-closed set [4] if cl(A)⊆U whenever A⊆U and U is gα**-open in (X,τ),

(xiv) A g*s- closed set [9] if scl(A)⊆U whenever A⊆U and U is gs-open in (X,τ).

The compliment of the above mentioned sets are called their respective open sets.

III. $(\alpha p)^*$ - R_0 Spaces

Definition 3.1. Let (X, τ) be a topological space and $A \subset X$. Then the $(\alpha p)^*$-kernel of A, denoted by $(\alpha p)^*$-Ker(A) is defined to be the set $(\alpha p)^*$ Ker(A)=∩{U∈$(\alpha p)^*$ O(X,τ)| A⊂ U}.

Lemma 3.1: Let (X, τ) be a topological space and $x \in X$. Then, $y \in (\alpha p)^*$ Ker({x}) if and only if $x \in (\alpha p)^*$-Cl({y}).

Proof: Assume that $y \notin$ Ker({x}) Then there exist a $(\alpha p)^*$-open set containing x such that $y \notin V$ .Therefore, we have $x \notin (\alpha p)^*$-Cl(y). The converse is similarly shown.

Lemma 3.2. Let (X, τ) be a topological space and A a subset of X. Then, $(\alpha p)^*$- Ker(A) = {x ∈ X $(\alpha p)^*$-Cl({x}) ∩ A ≠ ∅}.

Proof: Let $x \in (\alpha p)^*$-Ker(A) and $(\alpha p)^*$-Cl({x}) ∩ A = ∅. Therefore, $x \notin X$- $(\alpha p)^*$-Cl({x}) which is a $(\alpha p)^*$-open set containing A. But this is impossible, since $x \in (\alpha p)^*$-Ker(A). Consequently, $(\alpha p)^*$- Cl({x}) ∩ A ≠ ∅. Now, let $x \in X$ such that $(\alpha p)^*$- Cl({x}) ∩ A ≠ ∅. Suppose that $x \notin (\alpha p)^*$Ker(A). Then, there exists a $(\alpha p)^*$-open set U containing A and $x \notin U$. Let $y \in (\alpha p)^*$-Cl({x}) ∩ A. Thus, U is a $(\alpha p)^*$-neigbourhood of y such that $x \notin U$. By this contradiction $x \in (\alpha p)^*$- Ker(A).

Lemma 3.3: The following statements are equivalent for any points x and y in a topolog ical space (X, τ) : (1) $(\alpha p)^*$-Ker({x}) ≠ $(\alpha p)^*$-Ker({y}); (2) $(\alpha p)^*$-Cl({x}) ≠ $(\alpha p)^*$-Cl({y}).

Proof. (1) ⇒ (2): Let $(\alpha p)^*$-Ker({x})≠ $(\alpha p)^*$-Ker({y}). Then there exists a point z in X such that $z \in (\alpha p)^*$-Ker({x}) and $z \notin (\alpha p)^*$-Ker({y}). From $z \in (\alpha p)^*$-Ker({x}) it follows that {x} ∩$(\alpha p)^*$-Ker({z}) ≠ ∅ which implies $x \in (\alpha p)^*$-Cl({z}). By $z \notin (\alpha p)^*$-Ker({y}), we have {y} ∩ $(\alpha p)^*$-Cl({z}) = ∅. Since x∈$(\alpha p)^*$-Cl({z}, $(\alpha p)^*$-Cl({x})⊂$(\alpha p)^*$-Cl({z}) and {y} ∩ $(\alpha p)^*$-Cl({x}) = ∅. Therefore it follows that $(\alpha p)^*$-Cl({x}) ≠ $(\alpha p)^*$-Cl({y})Now $(\alpha p)^*$-Ker({x}) ≠ $(\alpha p)^*$-Ker({y}) implies that $(\alpha p)^*$-Cl({x}) ≠ $(\alpha p)^*$-Cl({y}). **(2) ⇒ (1)**: Suppose that $(\alpha p)^*$-Cl({x}) ≠ $(\alpha p)^*$-Cl({y}). Then there exists a point z in X such that $z \in (\alpha p)^*$-Cl({x}) and $z \notin (\alpha p)^*$-Cl({y}). It means that there exists a $(\alpha p)^*$-open set containing z and therefore x but not y, i.e., $y \notin (\alpha p)^*$-Ker({x}) and hence $(\alpha p)^*$-Ker({x}) ≠ $(\alpha p)^*$-Ker({y}).

Definition 3.2. A topological space (X, τ) is said to be a $(\alpha p)^*$-R_0 space if every $(\alpha p)^*$-open set contains the $(\alpha p)^*$-closure of each of its singletons.

Proposition 3.1: For a topological space (X, τ), the following properties are equivalent:

(i) (X, τ) is$(\alpha p)^*$-R_0 space;

(2)For any $F \in (\alpha p)^*C(X, \tau)$, $x \notin F$ implies $F \subset U$ and $x \notin U$ for some U∈$(\alpha p)^*$O(X, τ); (3)For any $F \in (\alpha p)^*C(X, \tau)$, $x \notin F$ implies F ∩$(\alpha p)^*$Cl({x}) = ∅; (4)For any distinct points x and y of X, either $(\alpha p)^*$-Cl({x}) = $(\alpha p)^*$-Cl({y}) or $(\alpha p)^*$-Cl({x})∩$(\alpha p)^*$-Cl({y}) = ∅.

Proof: **(1) ⇒ (2)**: Let $F \in (\alpha p)^*C(X, \tau)$ and $x \notin F$. Then by (1) $(\alpha p)^*$Cl({x}) ⊂ X− F . Set U = X − $(\alpha p)^*$Cl({x}), then U ∈ $(\alpha p)^*$O(X, τ), F ⊂ U and $x \notin U$ **(2) => (3):** Let Let $F \in (\alpha p)^*$ C(X, τ) and $x \notin F$. There exists U ∈ $(\alpha p)^*$O(X, τ) such that F ⊂ U and $x \notin U$. Since U ∈ $(\alpha p)^*$O(X, τ)∩ $(\alpha p)^*$Cl({x}) = ∅and F ∩ $(\alpha p)^*$Cl({x})= ∅. **(3) => (4) :** Let $(\alpha p)^*$Cl({x}) ≠ $(\alpha p)^*$Cl({y}) for distinct points x,y ∈ X. so there exists z ∈ $(\alpha p)^*$Cl(x) such that z∉ $(\alpha p)^*$Cl(y) (or z ∈$(\alpha p)^*$Cl(y). such that z ∉ $(\alpha p)^*$Cl(x)). There exists V∈ $(\alpha p)^*$O(X, τ). such that y ∉ V and z ∈ V ; hence x∈ V.Hence,we have x∉ $(\alpha p)^*$Cl(y).By(3), we obtain $(\alpha p)^*$Cl(x)∩ $(\alpha p)^*$Cl({y}) = ∅.The proof for otherwise is similar **(4) => (1) :** Let V ∈ $(\alpha p)^*$O(X, τ) and x ∈ V. For each y ∉ v, x ≠ y and x∉$(\alpha p)^*$Cl({y}.This shows that $(\alpha p)^*$Cl({x}))≠ $(\alpha p)^*$Cl({y}). By (4) $(\alpha p)^*$Cl({x}))∩ $(\alpha p)^*$Cl({y}) = ∅ for each y ∈ X – V and hence $(\alpha p)^*$Cl({x}))∩ ($\cup_{y \in X-V}$$(\alpha p)^*$Cl({y})= ∅.On the other hand ,since V ∈ $(\alpha p)^*$O(X, τ) and y ∈ X–V,we have $(\alpha p)^*$Cl({y}) ⊂X – V and hence X– V=$\cup_{y \in X-V}$$(\alpha p)^*$**Cl({y}**.Therefore we obtain (X – V) ∩ $(\alpha p)^*$Cl({x}) = ∅.and $(\alpha p)^*$Cl({x}⊂V . This shows that (X, τ) is a $(\alpha p)^*$-R_0 space.

Corollary 3.1. A topological space (X, τ) is a $(\alpha p)^*$-R_0 space if and only if for any x and y in X, $(\alpha p)^*$Cl({x}) ≠ $(\alpha p)^*$Cl({y}) => $(\alpha p)^*$Cl({x}) ∩ $(\alpha p)^*$Cl({y})= ∅

Proof. It follows from Proposition 3.1.

Theorem 3.1. A topological space (X, τ) is a $(\alpha p)^*$- R_0 space if and only if for any points x and y in X, $(\alpha p)^*$Ker({x}) ≠ $(\alpha p)^*$Ker({y}) => $(\alpha p)^*$Ker({x}) ∩ $(\alpha p)^*$Ker({y}) = ∅.

Proof. Suppose that (X, τ) is a $(\alpha p)^*$-R_0 space, for any points x and y in X if $(\alpha p)^*$Ker({x}) ≠ $(\alpha p)^*$Ker({y}) then $(\alpha p)^*$Cl({x}) ≠ $(\alpha p)^*$Cl({y}).We prove that $(\alpha p)^*$Ker({x}) ∩ $(\alpha p)^*$Ker({y}) = ∅.Let z ∈ $(\alpha p)^*$Ker({x}) $(\alpha p)^*$Ker({y}). By z ∈ $(\alpha p)^*$ Ker({x}), it follows that x ∈ $(\alpha p)^*$Cl({z}). Since x ∈ $(\alpha p)^*$Cl({x}), $(\alpha p)^*$Cl({x}) = $(\alpha p)^*$Cl({z}).Similarly, we have $(\alpha p)^*$Cl({y}) =

$(\alpha p)^*$Cl({z}) = $(\alpha p)^*$Cl({x}). This is a contradiction and therefore,we have $(\alpha p)^*$Ker({x}) ∩ $(\alpha p)^*$Ker({y}) =∅ Conversely , let for any points x and y in X $(\alpha p)^*$Ker({x}) ≠ $(\alpha p)^*$Ker({y}) implies $(\alpha p)^*$Ker({x}) ∩ $(\alpha p)^*$Ker({y}) = ∅.$(\alpha p)^*$Cl({x}) ≠ $(\alpha p)^*$Cl({y}), then ,$(\alpha p)^*$Ker({x}) ≠ $(\alpha p)^*$Ker({y}).Therefore $(\alpha p)^*$Ker({x}) ∩ $(\alpha p)^*$Ker({y}) = ∅ which implies $(\alpha p)^*$Cl({x}) ∩ $(\alpha p)^*$Cl({y}) = ∅.Since z ∈ $(\alpha p)^*$Cl({x}) implies that x ∈ $(\alpha p)^*$Ker({z}) and therefore $(\alpha p)^*$Ker({x}) ∩ $(\alpha p)^*$Ker({z}) ≠ ∅. By hypothesis, we have $(\alpha p)^*$Ker({x}) = $(\alpha p)^*$Ker({z}). Then z ∈ $(\alpha p)^*$Cl({x})∩$(\alpha p)^*$Cl({y})implies that $(\alpha p)^*$Ker({x}) = $(\alpha p)^*$Ker({z}) = $(\alpha p)^*$Ker({y}). But this is a contradiction. Therefore, $(\alpha p)^*$Cl({x}) ∩ $(\alpha p)^*$Cl({y}) = ∅ and (X, τ) is a $(\alpha p)^*$-R_0 space.

Theorem 3.2. For a topological space (X, τ), the following properties are equivalent:

(1) (X, τ) is a $(\alpha p)^*$-R_0 space;

(2) For any nonempty set A and G ∈ $(\alpha p)^*$o(X, τ) such that A ∩ G ≠ ∅, there exists F ∈ $(\alpha p)^*$o(X, τ) such that A ∩ F ≠ ∅ and F ⊂ G;

(3) Any G ∈ $(\alpha p)^*$ -o(X, τ), G = ∪{F ∈ G ∈ $(\alpha p)^*$-o(X, τ)| F ⊂ G};

(4) Any F ∈ $(\alpha p)^*$-o(X, τ), F = ∩{G ∈ $(\alpha p)^*$-o(X, τ), | F ⊂ G };

(5) For any x ∈ X, $(\alpha p)^*$-Cl({x}) ⊂ $(\alpha p)^*$-Ker({x})

Proof. (1) ⇒ (2): Let A be a nonempty set of X and G ∈ $(\alpha p)^*$-o(X, τ), such that A ∩ G ≠ ∅. There exists x ∈ A ∩ G. Since x ∈ G ∈ $(\alpha p)^*$-o(X, τ), μ^{**}-Cl({x}) ⊂ G. Set F = $(\alpha p)^*$-Cl({x}), then F ∈ $(\alpha p)^*$ -o(X, τ), F ⊂ G and A ∩ F ≠ ∅.

(2) ⇒ (3): Let G ∈ $(\alpha p)^*$ -o(X, τ), then G ⊃ ∪{F ∈ $(\alpha p)^*$-o(X, τ),) | F ⊂ G}. Let x be any point of G. There exists F ∈ $(\alpha p)^*$-o(X, τ), such that x ∈ F and F ⊂ G. Hence, we have x ∈ F ⊂ ∪{F ∈ $(\alpha p)^*$-o(X, τ), | F ⊂ G}hence G = ∪{F ∈ $(\alpha p)^*$-o(X, τ), | F ⊂ G}.

(3) ⇒ (4): Straightforward.

(4) ⇒ (5): Let x be any point of X and y /∈ $(\alpha p)^*$Ker({x}). There exists V ∈ $(\alpha p)^*$-o(X, τ), such that x ∈ V and y ∉V ; hence $(\alpha p)^*$-Cl({y}) ∩ V = ∅. By (4) (∩{G ∈ $(\alpha p)^*$-o(X, τ),| $(\alpha p)^*$-Cl({y})⊂G})∩ V = ∅ and there exists G ∈ $(\alpha p)^*$-o(X, τ),such that x ∉ G and $(\alpha p)^*$-Cl({y}) ⊂ G. Therefore, $(\alpha p)^*$-Cl({x}) ∩ G = ∅ and y /∈ **v** -Cl({x}). Consequently, we obtain

$(\alpha p)^*$-Cl({x}) ⊂ $(\alpha p)^*$-Ker({x}). (5) ⇒ (1) : Let G ∈ $(\alpha p)^*$-o(X, τ), and x ∈ G.Let y∈ $(\alpha p)^*$-Ker({x}),then x ∈ $(\alpha p)^*$Cl({y}) and y ∈ G. This implies that Ker({x}) ⊂ G. Therefore, we obtain x ∈ $(\alpha p)^*$- Cl({x}) ⊂ $(\alpha p)^*$-Ker({x}) ⊂ G. This shows that (X, τ) is a $(\alpha p)^*$-R_0 space

Corollary 3.2. For a topological space (X, τ), the following properties are equivalent : (1) (X, τ) is a $(\alpha p)^*$-R_0 space; (2) $(\alpha p)^*$- Cl({x}) = $(\alpha p)^*$-Ker({x}) for all x ∈ X.

Proof. (1) ⇒ (2) : Let (X, τ) be a $(\alpha p)^*$-R_0 space. It follows that $(\alpha p)^*$- Cl({x}) ⊂$(\alpha p)^*$-Ker({x}) for each x ∈ X. Suppose y ∈$(\alpha p)^*$-Ker({x}), then x ∈ $(\alpha p)^*$-Cl({y}) and $(\alpha p)^*$-Cl({x}) = $(\alpha p)^*$-Cl({y}).Therefore, y ∈ $(\alpha p)^*$-Cl({x}) and hence $(\alpha p)^*$-Ker({x}) ⊂ $(\alpha p)^*$-Cl({x}).This shows that $(\alpha p)^*$-Cl({x}) = $(\alpha p)^*$-Ker({x}). (2) ⇒ (1).

Theorem 3.3. For a topological space (X, τ), the following properties are equivalent :

(1) (X, τ) is a $(\alpha p)^*$-R_0 space;

(2) If F is $(\alpha p)^*$-closed, then F = $(\alpha p)^*$-Ker(F);

(3) If F is $(\alpha p)^*$-closed and x ∈ F, then $(\alpha p)^*$-Ker({x}) ⊂ F;

(4) If x ∈ X, then $(\alpha p)^*$-Ker({x}) ⊂ $(\alpha p)^*$-Cl({x}).

Proof. (1) ⇒ (2): Suppose that F is $(\alpha p)^*$-closed and x /∈ F. Thus X − F is $(\alpha p)^*$-open and x ∈ X − F. Since (X, τ) is $(\alpha p)^*$ − R0 μ^{**}-Cl({x}) ⊂ X − F. Thus $(\alpha p)^*$-Cl({x}) ∩ F = ∅ and x ∉ $(\alpha p)^*$-Ker(F). Therefore $(\alpha p)^*$-Ker(F) = F.

(2) ⇒ (3) : In general, A ⊂ B implies $(\alpha p)^*$-Ker(A) ⊂ $(\alpha p)^*$-Ker(B). Therefore, it follows from (2) that $(\alpha p)^*$-Ker({x}) ⊂ $(\alpha p)^*$-Ker(F) = F.

(3)<=> (4) : Since x ∈$(\alpha p)^*$-Cl({x}) and $(\alpha p)^*$Cl({x}) is $(\alpha p)^*$-closed, by (3) $(\alpha p)^*$-Ker({x}) ⊂$(\alpha p)^*$-Cl({x}).

(4) <=> (1) : Let x ∈$(\alpha p)^*$-Cl({y}). Then y ∈ $(\alpha p)^*$-Ker({x}).

Since x ∈ $(\alpha p)^*$-Cl({x}) and $(\alpha p)^*$-Cl({x}) is $(\alpha p)^*$-closed, by (4) we obtain y ∈ $(\alpha p)^*$-Ker({x}) ⊂ $(\alpha p)^*$-Cl({x}). Therefore x ∈$(\alpha p)^*$Cl({y}) implies y ∈ $(\alpha p)^*$-Cl({x}). The converse is obvious and (X, τ) is $(\alpha p)^*$-space.

Definition 3.3. A topological space (X, τ) is $(\alpha p)^*$-symmetric if for x and y in X, x ∈ $(\alpha p)^*$-Cl({y}) implies y ∈ $(\alpha p)^*$-Cl({x}).

Definition 3.4. A subset A of a topological space (X, τ) is called a ($(\alpha p)^*$, $(\alpha p)^*$)closed set (briefly ($(\alpha p)^*$, $(\alpha p)^*$)closed) if $(\alpha p)^*$-Cl(A) ⊂ U whenever A ⊂ U and U is $(\alpha p)^*$-open in (X, τ).

Lemma 3.4. Every μ^{**}-closed set is ($(\alpha p)^*$, $(\alpha p)^*$) - closed.

Theorem 3.4. A topological space (X, τ) is $(\alpha p)^*$-symmetric if and only if {x} is ($(\alpha p)^*$, $(\alpha p)^*$) - closed for each x ∈ X.

Proof: Assume that x ∈ $(\alpha p)^*$-Cl({y}) but y ∉ $(\alpha p)^*$-Cl({x}). This means that the complement of $(\alpha p)^*$-Cl({x}) contains y. Therefore the set {y} is a subset of the complement of $(\alpha p)^*$-Cl({x}). This implies that $(\alpha p)^*$Cl({y}) is a subset of the complement of $(\alpha p)^*$-Cl({x}). Now the complement of $(\alpha p)^*$-Cl({x}) contains x which is a contradiction. Conversely, suppose that {x} ⊂ E ∈ $(\alpha p)^*$-O(X, τ) but $(\alpha p)^*$-Cl({x}) is not a subset of E. This means that $(\alpha p)^*$-Cl({x}) and the complement of E are not disjoint.

Let y ∈ ($(\alpha p)^*$-Cl({x}) ∩ Ec). Now we have x ∈ $(\alpha p)^*$-Cl({y}) ⊂ E^c and x ∉ E.

But this is a contradiction.

Definition 3.5. A topological space (X, τ) is called $(\alpha p)^*$-T_1 if for any distinct pair of points x and y in X, there is a $(\alpha p)^*$-open U in X containing x but not y and a $(\alpha p)^*$-open set V in X containing y but not x.

Theorem 3.5. A topological space (X, τ) is $(\alpha p)^*$-T_1 if and only if the singletons are $(\alpha p)^*$-closed sets.

Proof. Suppose that (X, τ) is $(\alpha p)^*$-T_1 and x ∈ X. Let y ∈ $\{x\}^c$. Then x ≠ y and so there exists a $(\alpha p)^*$-open set U_y such that y ∈ Uy but x ∉ U_y. Consequently y ∈ U_y ⊂ $\{x\}^c$ i.e., $\{x\}^c$ = ∪{ U_y /y ∈ $\{x\}^c$} which is $(\alpha p)^*$-open. Conversely. Suppose that {p} is $(\alpha p)^*$-closed for every p ∈ X. Let x, y ∈ X with x ≠ y . Now x ≠y implies y ∈ $\{x\}^c$. Hence$\{x\}^c$is a$(\alpha p)^*$-open set containing y but not x.

Similarly $\{y\}^c$ is a $(\alpha p)^*$-open set containing x but not y. Accordingly X is a $(\alpha p)^*$-T_1 space.

Theorem 3.6. For a topological space (X, τ) the following are equivalent: (1) (X, τ) is $(\alpha p)^*$-R_0 ; (2) (X, τ) is $(\alpha p)^*$-symmetric.

Proof. (1) ⇒ (2). If x ∉ $(\alpha p)^*$-Cl({y}). Then there exist a $(\alpha p)^*$-open set U containing x such that y ∉ U. Hence y ∉ $(\alpha p)^*$-Cl(U). The converse is similarly shown.

(2) ⇒ (1): Let U be a $(\alpha p)^*$-open set and x ∈ U. If y ∉ U, then x ∉$(\alpha p)^*$-Cl({y}) and hence y∉ $(\alpha p)^*$-Cl({x}) This implies that $(\alpha p)^*$-Cl({x}) ⊂ U. Hence (X, τ) is $(\alpha p)^*$-R_0.

Definition 3.6. A filterbase F is called $(\alpha p)^*$-convergent to a point x in X, if for any $(\alpha p)^*$-open set U of X containing x, there exists B in F such that B is a subset of U.

Lemma 3.5. Let (X, τ) be a topological space and let x and y be any two points in X such that every net in X $(\alpha p)^*$-converging to y μ^{**}-converges to x. Then x ∈ $(\alpha p)^*$-Cl({y}).

Proof. Suppose that x_n = y for each n ∈ N. Then $\{x_n\}_n$ ∈N is a net in $(\alpha p)^*$-Cl({y}). Since $\{x_n\}_n$ ∈N $(\alpha p)^*$-converges to y, then $\{x_n\}_n$ ∈N $(\alpha p)^*$-converges to x and this implies that x ∈ $(\alpha p)^*$-Cl({y}).

Theorem 3.7. For a topological space (X, τ), the following statements are equivalent: (1) (X, τ) is a $(\alpha p)^*$-R_0 space; (2) If x, y ∈ X, then y ∈ $(\alpha p)^*$-Cl({x}) if and only if every net in X ge-converging to y $(\alpha p)^*$-converges to x.

Proof. (1)→(2): Let x, y ∈ X such that y ∈ $(\alpha p)^*$-Cl({x}). Suppose that $\{x_\alpha\}_{\alpha\in\wedge}$ be a net in such that $\{x_\alpha\}_{\alpha\in\wedge}$ $(\alpha p)^*$-converges to y. Since y ∈ $(\alpha p)^*$-Cl({x}), we have $(\alpha p)^*$-Cl({x}) = $(\alpha p)^*$-Cl({y}). Therefore x ∈ $(\alpha p)^*$-Cl({y}). This means that $\{x_\alpha\}_{\alpha\in\wedge}$ $(\alpha p)^*$-converges to x. Conversely, let x, y ∈ X such that every net in X $(\alpha p)^*$-converging to y $(\alpha p)^*$-converges to x. Then x ∈ $(\alpha p)^*$-Cl({y}) by Lemma l3.2. By Theorem 3.5, we have $(\alpha p)^*$-Cl({x}) = $(\alpha p)^*$-Cl({y}). Therefore y ∈ $(\alpha p)^*$-Cl({x}).

(2)→(1): Assume that x and y are any two points of X such that $(\alpha p)^*$-Cl({x}) ∩ $(\alpha p)^*$-Cl({y}) 6= ∅. Let z ∈ $(\alpha p)^*$-Cl({x}) ∩ $(\alpha p)^*$-Cl({y}). So there exists a net $\{x_\alpha\}_{\alpha\in\wedge}$ in geCl({x}) such that {xα}α∈∧ ge-converges to z. Since z ∈ $(\alpha p)^*$-Cl({y}), then $\{x_\alpha\}_{\alpha\in\wedge}$ $(\alpha p)^*$converges to y. It follows that y ∈ $(\alpha p)^*$-Cl({x}). By the same taken we obtain x ∈ $(\alpha p)^*$-Cl({y}). Therefore $(\alpha p)^*$Cl({x}) = $(\alpha p)^*$Cl({y}) and (X, τ) is $(\alpha p)^*$-R_0

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