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Dynamic Quarantine Model for Rumor Propagation Control in Online Social Networks

Dipti Rashmi [a], Dr. Binod Kumar [b], Dr. Buddhadeo Mahato [c,*]
[a] Department of Computer Applications, Annada College, Hazaribag, India
[b] Department of Computer Applications, AISECT University, Hazaribag, India
[c] Department of Mathematics, University College of Engineering and Technology, Hazaribag, India
** Corresponding author.*
E-mail address: b.mahato12@gmail.com (B. Mahato).

Abstract
Spreading rumors is a common social interaction that is harmful to both society and individual lives. It is vital and significant to research how rumors propagate and are managed in online social networks. In this study, we explore how individuals respond to rumors in different ways based on their levels of understanding and personalities. We use the phrases susceptible(S) - infected (I) - quarantine (Q)-recovered(R) to group all human populations in social networks. Differential equations are used to describe how rumors spread in social networks. Then, using the Jacobian matrix and Next Generation matrix, the spreading threshold of the SIQR model in social networks is determined. Next, the stability and existence of the equilibria are examined. It has been determined that the rumor-free equilibrium E_0 is unstable if basic reproduction number R_0>1, indicating that fresh rumors are spreading across the population, and locally asymptotically stable if basic reproduction number R_0<1. Lastly, numerical simulations of the dynamic model are performed on the system to verify the analytical results.
Keywords: Epidemic model, Equilibrium point, Reproduction number, Stability, Online Social Networks.

1. Introduction

Rumors, like infectious diseases, propagate through social networks, influencing public perceptions, behaviors, and ultimately, the outcomes of epidemic situations. This statement underscores the complexity of infectious disease dynamics, which is crucial to understanding the context within which rumor propagation occurs during epidemic events. It highlights the interconnectedness of various factors that shape the spread of both pathogens and rumors, setting the stage for the development of mathematical models that capture these intricate dynamics. The dynamics of behaviour transmission in online social networks provide important new understandings of the mechanics behind digital ecosystems information diffusion [1]-[4]. Centola [5] investigated how social influence functions in online communities through a number of creative experiments, offering a fundamental framework for comprehending the propagation of rumors and false information in social networks. Musa [6] provided insight into the dynamics of information diffusion and behavioural contagion, which are pertinent to understanding how rumors propagate within the context of epidemic events. Emphasizing the critical role of network structure in shaping the spread of epidemic diseases, highlighting the significance of individual connections and network topology, it underscores the importance of considering network-based models in understanding and predicting epidemic dynamics, which can also inform the spread of rumors within social networks during epidemic events [7]-[11].

Social network theory, which emphasizes the significance of interpersonal interactions in influencing information dispersion, completely changed the study of rumor transmission. According to Granovetter's (1973) "strength of weak ties" theory [12], for example, people with a variety of social connections are essential in the propagation of rumors within various social circles. Network-based models of rumor dissemination, which take into account the dynamics of information flow, the topology of social networks, and the strength of relationships between individuals, were made possible by this viewpoint [13]-[14].

The first models of rumor transmission were frequently influenced by epidemiological models, which depict the transmission of contagious diseases across populations. Individuals are divided into three categories according to the traditional SIR (Susceptible-Infectious-Recovered) paradigm, which was first presented by Kermack and McKendrick [15] in 1927: susceptible, infectious, and recovered. In order to quantify the dynamics of information distribution and assess intervention efforts, this framework has been modified to simulate the propagation of rumors, where people move between states of ignorance, belief, and denial. Through differential equations, the SIR model elucidates the progression of an outbreak and the impact of interventions such as quarantine and vaccination. Existing methods for rumor detection have focused on text content, user profiles, and propagation patterns.

Yuan [16] introduced a global-local attention network (GLAN) for rumor detection that encodes both local semantic and global structural information. Zhu [17] proposed a delayed SIR epidemic-like rumor propagation model that considers forced silence functions, time delays, and network topology in both homogeneous and heterogeneous networks. Additionally, Hosni [18] extended the classical SIR model by incorporating the role of addicted individuals in the spreading process, showing a significant influence of online social network addiction on rumor propagation. Different models have been proposed to contain rumor spread in social networks. In the context of rumor source detection, Jin [19]

provided an overview of various diffusion models and source estimators used in online social networks. They highlighted the importance of diffusion models in modeling rumor propagation and discussed different schemes for source detection. Simsek [20] compared the performance of different SIR settings in estimating the expected influence capacities of nodes in complex networks, emphasizing the applications of epidemic modeling in various scenarios. Moreover, Qiang He [21] proposed a reinforcement learning-based rumor blocking approach in directed social networks. Their framework, CCSQ, involved community detection, candidate seed nodes, and a seeding algorithm using the Q-learning method. This approach aimed to select seed nodes effectively to block rumors in social networks. Overall, research in epidemic rumor propagation models on social networks continues to evolve, incorporating various factors such as network topology, addiction, and competitive diffusion to understand and contain the spread of rumors effectively.
In this paper, we have formulated a mathematical model of rumor transmission in social network by human population. We find basic reproduction number R_0 gives global dynamical behavior of the model. If $R_0 < 1$, the rumor -free equilibrium is globally stable, which means the rumor will die out and if $R_0 > 1$, the endemic equilibrium is globally stable. The basic reproduction number R_0 is also used in numerical simulations to discuss the effectiveness of control strategies.

The structure of the paper is as follows: Section 1 provides an introduction; Section 2 develops the model's fundamental assumptions and parameters as well as the epidemic model; Section 3 establishes the stability of the system produced; Section 4 provides numerical simulations; and Section 5 concludes.

2. Model Description and Formulation:

Schematic flow of this model is shown in figure 1 and the state variables and associated parameters of this model are given in Table 1.

Table 1: The state variables and associated parameters

$S(t)$:	Susceptible proportions of human in time t.
$I(t)$:	Infectious proportions i.e Human beings who are spreading the rumor in time t.
$Q(t)$:	Quarantine proportion i.e Human beings who are trying to stop the spread of rumor in time t.
$R(t)$:	Recovered proportion i.e. Human beings who are not spreading the rumor in time t.
$N(t)$:	Total population of humans at time t.
A :	Birth rate and immigration rate of humans.
β :	Transmission probability of rumor from human to human.
ρ :	Rate of transmission from infectious proportions to quarantine proportions.
σ :	Rate of transmission from quarantine proportions to recovered proportions.
μ :	Natural deactivation rate of rumors.
μ_c :	Deactivation rate of rumors due to certain steps taken.

2.1 Basic Assumptions of Model:

In this paper, our mathematical model is based on the following hypothesis:

1. (H1) We develop the dynamics of a pandemic model with vital dynamics having unequal birth and deactivation rates of rumors.
2. (H2) The arrival of people in social networks is at a constant influx rate $A > 0$.
3. (H3) Each susceptible individual is infected by rumor at constant rate $\beta > 0$.
4. (H4) Infectious population is subjected to transmit unproved rumors at a constant rate $\rho > 0$.
5. (H5) Quarantine population is trying to stop the spread of rumor at a constant rate $\sigma > 0$.
6. (H6) Deactivation rate of rumors after taking certain measures at a constant rate $\mu_c > 0$.
7. (H7) Normal deactivation rate of rumors is at constant rate $\mu > 0$.

2.2 Model Equations:

Based on our assumptions and the flow of rumor transmission in social networks as depicted in figure 1, we have the following system of equations:

$$\begin{aligned}
\frac{dS}{dt} &= A - \beta SI - \mu S \\
\frac{dI}{dt} &= \beta SI - (\mu + \mu_c + \rho)I \\
\frac{dQ}{dt} &= \rho I - (\mu + \mu_c + \sigma)Q \\
\frac{dR}{dt} &= \sigma Q - \mu R.
\end{aligned} \qquad \textbf{(1)}$$

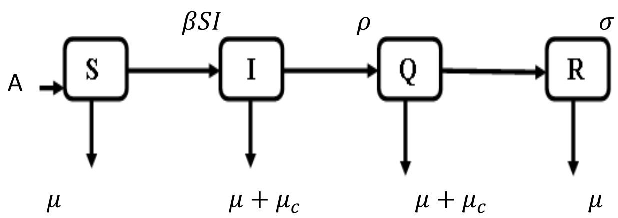

Figure 1: Rumor transmission diagram.

3. Stability of the Model

In this section, we find the equilibrium states and basic reproduction number of the model. We also prove that the model is locally and globally stable both for rumor-free-equilibrium and endemic equilibrium points.
Finding equilibrium states by setting the right hand side of all the model equations of system (1) equal to zero, we obtain two equilibrium states:
(i) Rumor free equilibrium state: $E_0 = (1,0,0,0)$
(ii) Endemic equilibrium state: $E_1 = (S^*, I^*, Q^*, R^*)$.
The system modeled is expected to show different kinds of behavior in the long term. The equilibrium points and the conditions for their existence are that they provide us mathematical conditions based on which the long-term behavior of the system can be predicted and classified into a finite number of possibilities represented by the equilibrium points.
Endemic Equilibrium points of the system (1):
From the equation two of the system (1) by equating to zero,

we get
$S^* = \frac{f(I)}{\mu+\mu_c+\rho}$.

Similarly, from the first, third and fourth equation of the system (1) by equating to zero and solving it, we get
$I^* = \frac{1}{\beta}\left[\frac{A(\mu+\mu_c+\rho)}{\beta} + \mu\right]$, $Q^* = \frac{\rho}{\beta(\mu+\mu_c+\sigma)}\left[\frac{A(\mu+\mu_c+\rho)}{\beta} + \mu\right]$, $R^* = \frac{\sigma\rho}{\mu\beta(\mu+\mu_c+\sigma)}\left[\frac{A(\mu+\mu_c+\rho)}{\beta} + \mu\right]$.

3.1 Basic Reproduction Number:
The average number of secondary infectious cases caused by a single infection in the entire susceptible population is the basic reproduction number for any epidemic model. The basic reproduction number R_0 is calculated by $\rho(FV^{-1})$, where ρ is called spectral radius of the matrix FV^{-1} and F & V are the matrices of new infected terms and the remaining transmission terms respectively .
From system (1), the matrices F and V are as follows:
$F = \begin{bmatrix} 0 & \beta \\ 0 & 0 \end{bmatrix}$ and $V = \begin{bmatrix} (\mu+\mu_c+\rho) & 0 \\ \rho & (\mu+\mu_c+\sigma) \end{bmatrix}$.
Hence, the basic reproduction number of the above model is
$R_0 = \frac{\rho\beta}{(\mu+\mu_c+\sigma)(\mu+\mu_c+\rho)}$.

Theorem 1: System (1) is locally asymptotically stable for rumor - free equilibrium, when $R_0 < 1$.
Proof: Jacobian matrix of the system (1) is as follows:

$$J = \begin{bmatrix} -\mu & -\beta & 0 & 0 \\ 0 & -(\mu+\mu_c+\rho) & 0 & 0 \\ 0 & \rho & -(\mu+\mu_c+\sigma) & 0 \\ 0 & 0 & \sigma & -\mu \end{bmatrix}$$

The eigenvalues of Jacobian matrix J are as follows:
$\lambda_1 = -\mu$, $\lambda_2 = -(\mu+\mu_c+\rho)$, $\lambda_3 = -(\mu+\mu_c+\sigma)$, $\lambda_4 = -\mu$
Hence all eigenvalues of Jacobian matrix J are negative when $R_0 < 1$.
This proves that our the system is locally asymptotically stable when $R_0 < 1$.

Theorem-2: The endemic equilibrium $E_1 = (S^*, I^*, Q^*, R^*)$ is locally asymptotically stable when $R_0 < 1$, otherwise it is unstable.
Proof: At the endemic equilibrium $E_1 = (S^*, I^*, Q^*, R^*)$, the Jacobian matrix is

$$J^* = \begin{bmatrix} -(\beta I^* + \mu) & -\beta S^* & 0 & 0 \\ \beta I^* & -(\mu+\mu_c+\rho)+\beta S^* & 0 & 0 \\ 0 & \rho & -(\mu+\mu_c+\sigma) & 0 \\ 0 & 0 & \sigma & -\mu \end{bmatrix}.$$

The eigenvalues of the above matrix are $\lambda_1 = -\mu$, $\lambda_2 = -(\mu+\mu_c+\sigma)$,
$\lambda_3 = -\frac{1}{2}\left[\beta I^* + 2\mu + \mu_c + \rho + \sqrt{(\beta I^* - \mu_c - \rho)^2 - 4\beta^2 S^* I^*}\right]$ and
$\lambda_4 = -\frac{1}{2}\left[\beta I^* + 2\mu + \mu_c + \rho - \sqrt{(\beta I^* - \mu_c - \rho)^2 - 4\beta^2 S^* I^*}\right]$.
All eigenvalues λ_1 , λ_2 , λ_3 and λ_4 have negative real values, when $R_0 < 1$.
This proves that The endemic equilibrium $E_1 = (S^*, I^*, Q^*, R^*)$ is locally asymptotically stable when $R_0 < 1$, otherwise it is unstable.

3.2 Global Stability for Endemic Equilibrium:
In this section, we prove global stability for endemic equilibrium. We adopt the geometrical approach for the mapping $g: L \subset R^n \to R^n$, where L is an open set, if its differential equations $y' = g(y)$ be such that its every solution $y(t)$ can be uniquely determined by its initial condition $y(t) = y_0$, then an equilibrium points $\bar{y} \in L$ and satisfies the following hypotheses H1: L is simply connected
H2: There exists a compact absorbing sub set K of L
$\bar{x}$ the only equilibrium point in L is globally stable, if it satisfies the additional Bendixson condition given by $\bar{q}_2 = lim_{t\to\infty}^{sup} \max_{x_0 \in K} \frac{1}{t}\int_0^t \varphi\left(B\left(y(s, y_0)\right)\right) ds < 0$, where $B = P_f P^{-1} + P_f J^{[2]} P^{-1}$ and P is a matrix valued function satisfying $\varphi(P_f P^{-1} + P_f J^{[2]} P^{-1}) < 0$. Further $J^{[2]}$ is the second compound additive matrix of order three and φ denote the Lozinskii measure defined as $\varphi(B) = \lim_{h\to 0} \frac{|I+hB|-I}{h}$.
The existence of a compact absorbing set which in the interior of region follows from the uniform persistence of the system as $\lim_{t\to\infty} inf\ S(t) > \epsilon$, $\lim_{t\to\infty} inf\ I(t) > \epsilon$, $\lim_{t\to\infty} inf\ Q(t) > \epsilon$, $\lim_{t\to\infty} inf\ R(t) > \epsilon$ for some $\epsilon > 0$. Based on this procedure system (1) is used to prove for Bendixson condition $\bar{q}_2 < 0$.

Theorem 3: The unique endemic equilibrium point E_1 is globally asymptotically stable if $R_0 > 1$.
Proof: For the general solution $\left(S(t), I(t), Q(t)\right)$ of system (1), the Jacobian matrix is

$$J = \begin{bmatrix} -\beta I - \mu & -\beta S & 0 \\ \beta I & -(\mu+\mu_c+\rho)+\beta S & 0 \\ 0 & \rho & -(\mu+\mu_c+\sigma) \end{bmatrix}.$$

The matrix $J^{[2]}$, the 2nd additive compound Jacobian matrix for n=3, is defined as

$$J^{[2]} = \begin{bmatrix} j_{11}+j_{22} & j_{23} & -j_{13} \\ j_{32} & j_{11}+j_{33} & j_{12} \\ -j_{31} & j_{21} & j_{22}+j_{33} \end{bmatrix}.$$

So, its 2nd additive compound matrix $J^{[2]}$ is

$$J^{[2]} = \begin{bmatrix} x & 0 & 0 \\ \rho & y & -\beta S \\ 0 & \beta I & -(\mu+\mu_c+\rho)+\beta S-(\mu+\mu_c+\sigma) \end{bmatrix},$$

where $x = -\beta I - \mu - (\mu+\mu_c+\rho) + \beta S$ and
$y = -\beta I - \mu - (\mu+\mu_c+\sigma)$.
Let the function $P = P(S, I, Q)$ be defined as

$$P = P(S,I,Q) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{I}{Q} & 0 \\ 0 & 0 & \frac{I}{Q} \end{bmatrix} = diag\left\{1, \frac{I}{Q}, \frac{I}{Q}\right\}$$

Then, $$P_f P^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{I'}{I} - \frac{Q'}{Q} & 0 \\ 0 & 0 & \frac{I'}{I} - \frac{Q'}{Q} \end{bmatrix} \quad \textbf{(3.1)}$$

where P_f matrix is obtained by replacing each elements of P by its derivative in the direction of f.
$P_f J^{[2]} P^{-1} =$

$$\begin{bmatrix} x & 0 & 0 \\ \rho\frac{I}{Q} & y+\frac{I'}{I}-\frac{Q'}{Q} & -\beta S \\ 0 & \beta I & -(\mu+\mu_c+\rho)+\beta S-(\mu+\mu_c+\sigma)+\frac{I'}{I}-\frac{Q'}{Q} \end{bmatrix},$$

where $x=-\beta I-\mu-(\mu+\mu_c+\rho)+\beta S$ and
$y=-\beta I-\mu-(\mu+\mu_c+\sigma)$.

$B=P_fP^{-1}+P_fJ^{[2]}P^{-1}=\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22}\end{bmatrix}$,

where $B_{11}=[-\beta I-\mu-(\mu+\mu_c+\rho)+\beta S]$, $B_{12}=[0 \quad 0]$, $B_{21}=\begin{bmatrix}\rho\frac{I}{Q} \\ 0\end{bmatrix}$ and

$B_{22}=$

$$\begin{bmatrix} -\beta I-\mu-(\mu+\mu_c+\sigma)+\frac{I'}{I}-\frac{Q'}{Q} & -\beta S \\ \beta I & -(\mu+\mu_c+\rho)+\beta S-(\mu+\mu_c+\sigma)+\frac{I'}{I}-\frac{Q'}{Q} \end{bmatrix}.$$

Now, for a vector (u,v,w) in $\boldsymbol{R}^3$, we select a norm as $|(u,v,w)|=max\{|u|,\ |v+w|\}$ and denote $\varphi(B)$ the Lozinskii measure for this norm.

From (3.1), it follows that $\varphi(B)\le \sup\{k_1,k_2\}$ **(3.2)**

where k_1 and k_2 are specified as follows:

$k_1=B_{11}+|B_{12}|$ and $k_2=\mu_1(B_{22})+|B_{21}|$, where $|B_{12}|$ and $|B_{21}|$ are matrix norms with respect to the vector norm L^1 and φ_1 denotes the Lozinskii measure with respect to the vector norm L^1. So, we have

$$k_1=B_{11}+|B_{12}|$$

$$=-\beta I-\mu-(\mu+\mu_c+\rho)+\beta S+Sup\{0,0\}$$

$$k_1=-\beta I-\mu-(\mu+\mu_c+\rho)+\beta S \quad \textbf{(3.3)}$$

Similarly, $k_2=\varphi_1(B_{22})+|B_{21}|$

$$=-(\mu+\mu_c+\rho)-(\mu+\mu_c+\sigma)+\frac{I'}{I}-\frac{Q'}{Q}+\rho\frac{I}{Q} \quad \textbf{(3.4)}$$

From second and third equations of system (1), we can rewrite as

$$\frac{I'}{I}=\beta S-(\mu+\mu_c+\rho) \quad \textbf{(3.5)}$$

$$\frac{Q'}{Q}=\alpha\frac{I}{Q}-(\mu+\mu_c+\sigma). \quad \textbf{(3.6)}$$

Putting (3.6) and (3.5) in (3.4) and (3.3) respectively, we get

$$k_1=-\beta I-\mu-(\mu+\mu_c+\rho)+\beta S\le \beta S-(\mu+\mu_c+\rho)$$

$$k_2=-2(\mu+\mu_c+\rho)+\beta S-\alpha\frac{I}{Q}+\rho\frac{I}{Q}\le \beta S-(\mu+\mu_c+\rho).$$

Hence, from (3.2)

$\varphi(B)\le \beta S-(\mu+\mu_c+\rho)$ and so, $\frac{1}{t}\int_0^t\varphi(B)ds\le\frac{1}{t}\log_e(\beta S-(\mu+\mu_c+\rho))$.

So, $\bar{q}_2<0$, and hence the Bendixson criteria is also satisfied, which thus proves the global stability of the endemic equilibrium.

4. Numerical Simulations and Effect of Parameteric Values:

In this section, a large number of numerical simulations have been carried out to verify the analytical results of local and global stability obtained in section 3. Some examples of numerical simulations for $R_0<1$ and $R_0>1$ are mentioned below.

Example 1: Figure 2 shows the local stability of the rumor-free equilibrium point for R0=0.8539, which has been quantitatively simulated. It is clearly observed that the rumor-free equilibrium point turns out to be stable when $R_0=0.8539<1$.

Example 2: The local stability of the endemic equilibrium point when $R_0=2.5385$ has been numerically simulated and depicted in Figures 3. It is clearly observed that the endemic equilibrium point turns out to be stable when $R_0=2.5385>1$.

Example 3: The global stability of endemic equilibrium of system (1) when $R_0=2.5385$ has been numerically simulated and depicted in Figures 4 on the plane formed by the susceptible population(S) and infectious population(I). From figure 4, it is observed that all resulting trajectories in the (S - I) plane are anticlockwise steady spiral and all trajectories converge to a unique equilibrium point (S^*,I^*). This example also numerically shows that a unique endemic equilibrium point is globally asymptotically stable if $R_0=2.5385>1$.

Example 4: The periodic behavior of system (1) also studied for different values of basic reproduction number (R_0) in (S, I) plane. Figures 5, 6 and 7 for $R_0=0.8539$, $R_0=1.1282$ and $R_0=2.5385$ respectively reflects that as the value of reproduction number increases, the oscillation of periodicity also increases in (S, I) plane.

Example 5: The behavior of system (1) is also studied for different values of R_0 by considering quarantine population-recovered population plane. Figure 8 and 9 for $R_0=0.8539$ and $R_0=2.5385$ respectively reflects that as the quarantine population increases, the recovered population also increases.

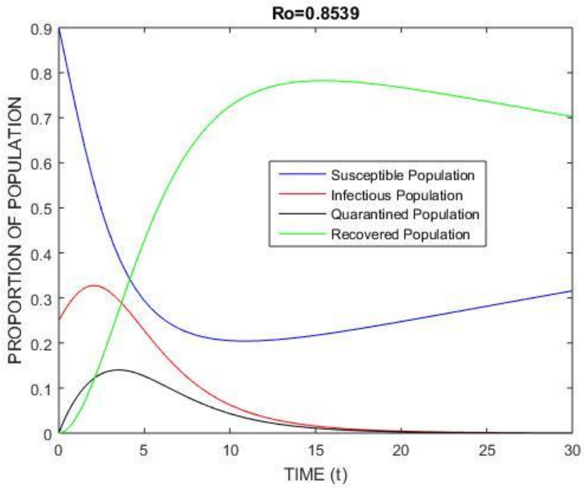

Figure 2: Proportions of different classes w.r.t. time when $R_0=0.8539$.

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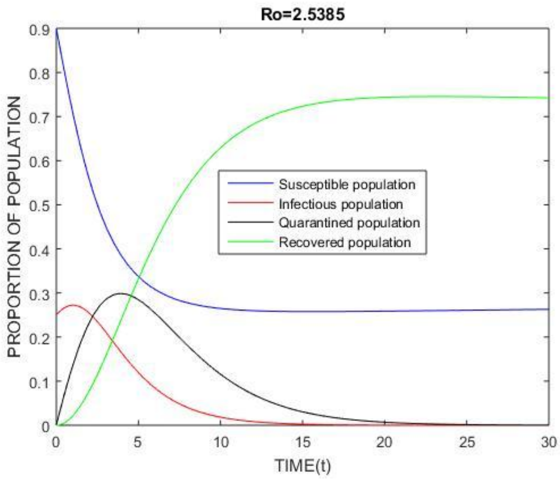

Figure 3: Transmission proportions of different classes when $R_0 = 2.5385$.

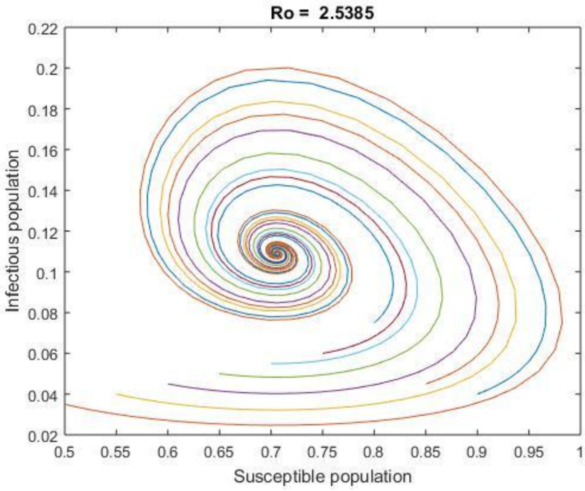

Figure 4: Global stability of endemic equilibrium in social networks when $R_0 = 2.5385$.

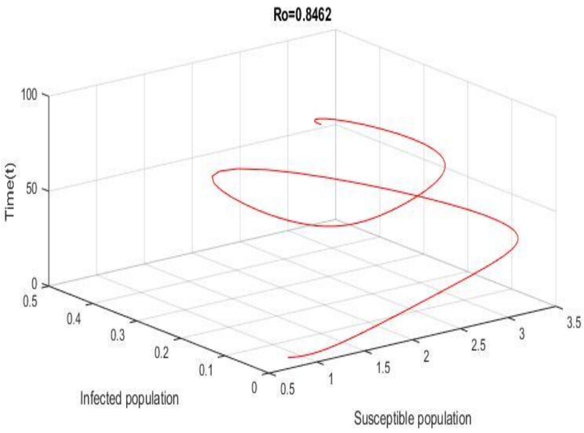

Figure 5: Susceptible versus infected population in social networks when $R_0 = 0.8462$.

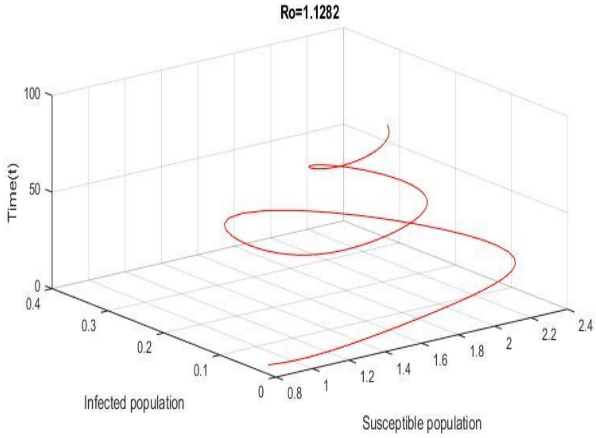

Figure 6: Susceptible versus infected population in social networks when$R_0 = 1.1282$.

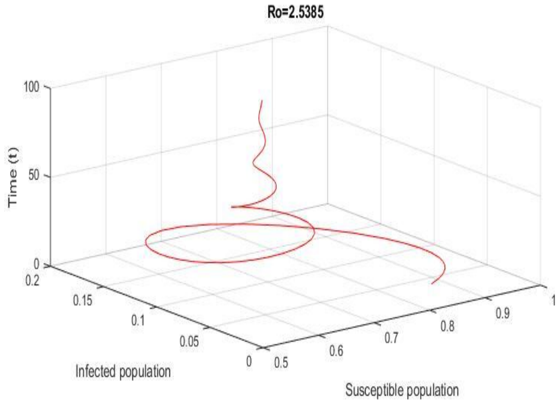

Figure 7: Susceptible versus infected population in social networks when $R_0 = 2.5385$.

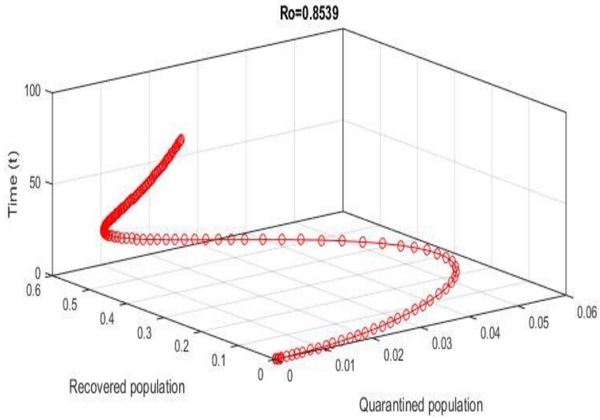

Figure 8: Quarantined versus recovered population in social networks when $R_0 = 0.8539$.

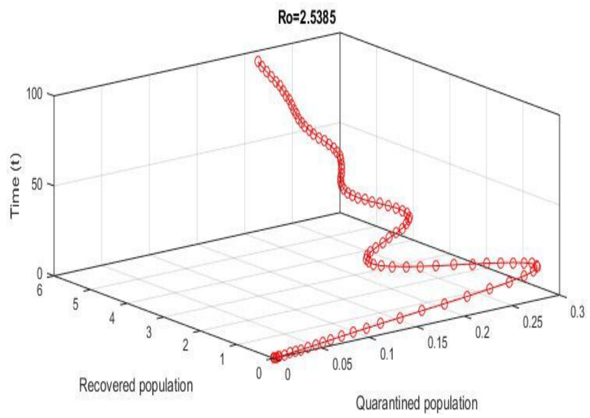

Figure 9: Quarantined versus recovered population in social networks when $R_0 = 2.5385$.

Discussion and Conclusion:

A mathematical model *SIQR* (Susceptible-Infected-Quaratined-Recovered) is developed for transmission dynamics of rumor in social networks. The basic reproduction number for the model is obtained and the condition for local and global asymptotic stability is well established. The endemic equilibrium in the S-I phase plane of the model is globally asymptotically stable when the basic reproduction number is larger than one. The periodicity and effect of four very important pivot parameters of the model due to behavior change and awareness in quarantine population is well analyzed. From the numerical simulations it is very clear that the more we have awareness about the facts, the less we have the chances to be infected and the rumor to be transmitted in the population. From the simulation results it is also evident from Figures (2-9) that population is rumor free when the transmission of the rumor is reduced and the rate of recovery also increases.

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