International Journal of Applied Engineering Research ISSN 0973-4562 Volume 16, Number 3 (2021) pp. 214-219

An EOQ Model for Non-Instantaneous Deteriorating Items with Stock and Time Dependent Demand with Partial Backlogging

[1]Chhavi Gupta, [2*]Vipin Kumar, [2]S.R. Singh

[1]Research Scholar, Mewar University, Chittorgarh

[2]Dept. of Mathematics, BKBIET, Pilani

[3]Dept. of Mathematics, CCS University Meerut

Abstract

In this study, we consider a problem to determining the optimal replenishment policy for non-instantaneous deteriorating items with stock and time dependent demand. Rate of deterioration is following the two-parameter Weibull distribution. Shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment. The major objective is to determine the optimal replenishment cycle time and the order quantity simultaneously, such that the total profit is to be minimize. Finally, numerical example is given to demonstrate the developed model and the solution procedure.

Keywords: Non-instantaneously deteriorating items, Partial backlogging, Price and time dependent demand

1. INTRODUCTION

In order to keep any major business running smoothly and efficiently, it is very important to control the inventory. A number of mathematical models have been developed by Researchers to control inventory. In most of these, it is believed that items start to deteriorate as they come in the warehouse, whereas here it is not necessary. Many things are spoiled after some time like grains, pulses etc. Contributions of various Researchers are as follows. First of all **Ghera and Sachchradar (1962)** designed their inventory model with assuming exponential decay. **Kovart and Philippe(1973)** made changes to the model of **Ghera and Sachchradar(1962)** and developed there model with Weibull distribution deterioration. **Aggarwal and Jaggi (1989)** developed replenishment and pricing policy for exponentially deteriorating items. **Wu (2006)** establish the phenomena of non-instantaneous and developed the model with partial backlogging and stock dependent demand. **Liang-Yuh Ouyang et al. (2006)** considered permissible delay in payments for non-instantaneous deteriorating items.

Chih-Te Yang (2009) considered with price-dependent demand for non-instantaneous deteriorating items. **S.R. Singh et. Al. (2010)** discussed the inventory model with stock dependent demand under inflation. **Ashendra Kumar Saxena (2011)** developed EOQ model for non-instantaneous deteriorating items with permissible delay in payments. **Reza Maihami (2012)** assumed time and price dependent demand for non-instantaneous deteriorating items. **Krishna Prasad (2014)** developed an optimal policy under stock and time dependent demand. **Ali. Akbar. Shaikh et. Al. (2017)** has discussed an inventory model for non-instantaneous items with price and stock dependent demand**. Chandra K. Jaggi (2017)** applied the LIFO FIFO despatched policy for two warehouses. **K Rangarajan (2017)** developed inventory model with ramp type demand. **Udayakumar (2018)** considered two warehouse non-instantaneous deteriorating items with permissible delay in payments**. Vipin Kumar (2019)** assumed price and stock dependent demand for non-instantaneous items.

On the basis of real-life practical experiences, we can say that the rate of demand is dependent on the current stock level and time. Inspired by this, we have developed an inventory model for non-instantaneous deteriorating items, in which the demand rate is current stock level and time dependent. This is also a realistic situation that it is natural of the products to be deteriorate. Some items begin to deteriorate as soon as they come into stock, while some after a period of time. This study includes products that deteriorate after a time interval and the deterioration rate is assumed to follow the two-parameter Weibull distribution. Shortage is allowed and are partially backlogged. In last, a numerical example is presented to demonstrate the developed model and the solution procedure.

2. NOTATIONS AND ASSUMPTIONS

To develop the inventory model of deteriorating item the following notations and assumptions are used throughout the paper.

2.1. Notations:

$I_1(t)$ Inventory level at any time t in the interval $(0, t_d)$

$I_1(t)$ Inventory level at any time t in the interval (t_d, t_1)

$I_1(t)$ Backorder level at any time t in the interval (t_1, T)

t_d the length of time in which the product exhibits no deterioration

t_1 the length of time in which there is no inventory shortages

T the length of the replenishment cycle time.

S Maximum inventory level.

S_1 Shortage level.

Q Ordered quantity.

A Replenishment cost.

C_h Holding cost per unit per unit time.

C_s Shortage cost per unit per unit time.

C_d Cost of a deteriorated unit.

C Total cost per unit time of the system.

$C(t_1, T)$ cost function per unit time

2.2. Assumptions

(i) Items are non-instantaneous.

(ii) Demand rate is defined as the function of stock and time as

$$D(I(t), t) = a + bt^{\beta-1}I(t).$$

(iii) The deteriorating rate is defined as two parameter Weibull distribution

$$\theta\,(t) = \alpha\,\beta\,t^{\beta-1}, \text{ where } 0 < \alpha < 1.$$

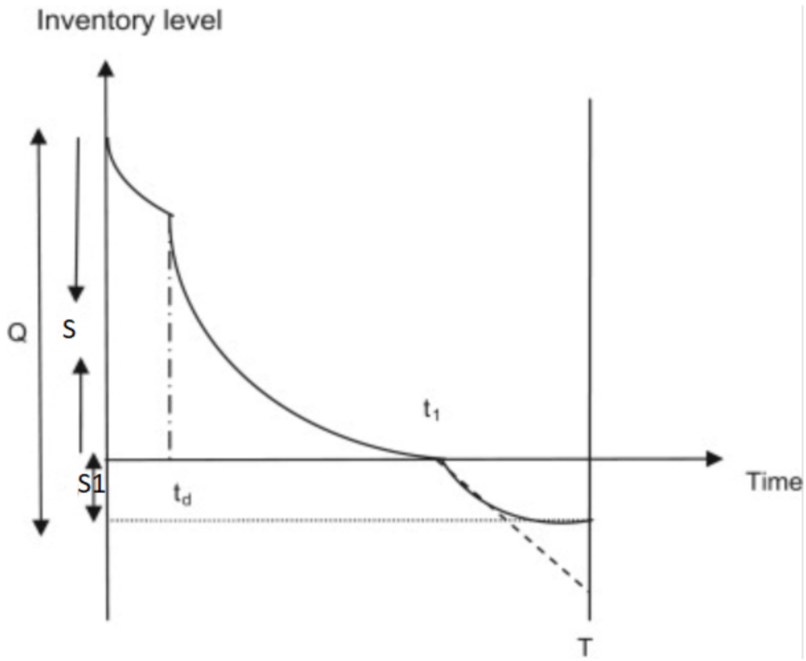
(iv) Shortages are allowed and partially backlogged with the rate $B(x) = e^{-\delta x}$.

(v) Lead time is zero and Replenishment is instantaneous.

3. PROPOSED MODEL

At the beginning time $t = 0$, let the inventory level S units are hold of the inventory system.

During the time interval $[0, t_d]$, the inventory decreases due to demand only. Because products discussed here follow non-instantaneous nature. Subsequently, the inventory level drop to zero due to both demand and deterioration during the time interval $[t_d, t_1]$. Finally, a shortage occurs due to demand and partial backlogging during the time interval $[t_1, T]$.



$$\frac{dI_1(t)}{dt} = -D(I(t),t), \qquad 0 \le t \le t_d \tag{1}$$

$$\frac{dI_2(t)}{dt} + \theta(t)I_2(t) = -D(I(t),t), \qquad t_d \le t \le t_1 \tag{2}$$

$$\frac{dI_3(t)}{dt} = -D(I(t),t)B(T-t), \qquad t_1 \le t \le T \tag{3}$$

With boundary conditions $I_1(0) = S$, $I_2(t_1) = 0$, $I_3(t_1) = 0$

The solution of above differential equations are as follows with the conditions

$$I_1(t) = \left\{S - a\left[t + \frac{k_1}{(\beta+1)}t^{\beta+1} + \frac{k_1^2}{2(2\beta+1)}t^{2\beta+1}\right]\right\}e^{-k_1 t^\beta} \qquad 0 \le t \le t_d \tag{4}$$

$$I_2(t) = a\left[(t_1 - t) + \frac{k_2}{(\beta+1)}\left(t_1^{\beta+1} - t^{\beta+1}\right) + \frac{k_2^2}{2(2\beta+1)}\left(t_1^{2\beta+1} - t^{2\beta+1}\right)\right]e^{-k_2 t^\beta} t_d \le t \le t_1 \tag{5}$$

$$I_3(t) = \frac{a}{\delta}\left[e^{-\delta(T-t_1)} - e^{-\delta(T-t)}\right] \qquad t_1 \le t \le T \tag{6}$$

Where $k_1 = \frac{b}{\beta}$ and $k_2 = \frac{\alpha\beta+b}{\beta}$

Considering the continuity of $I_1(t_d) = I_2(t_d)$ by equation (4) and (5), we get

$$\left\{S - a\left[t_d + \frac{k_1}{(\beta+1)}{t_d}^{\beta+1} + \frac{k_1^2}{2(2\beta+1)}{t_d}^{2\beta+1}\right]\right\}e^{-k_1 {t_d}^\beta}$$

$$= a\left[(t_1 - t_d) + \frac{k_2}{(\beta+1)}\left(t_1^{\beta+1} - {t_d}^{\beta+1}\right) + \frac{k_2^2}{2(2\beta+1)}\left(t_1^{2\beta+1} - {t_d}^{2\beta+1}\right)\right]e^{-k_2 {t_d}^\beta}$$

OR

$$S = a\left[t_d + \frac{k_1}{(\beta+1)}{t_d}^{\beta+1} + \frac{k_1^2}{2(2\beta+1)}{t_d}^{2\beta+1}\right]$$

$$+a\left[(t_1 - t_d) + \frac{k_2}{(\beta+1)}\left(t_1^{\beta+1} - {t_d}^{\beta+1}\right) + \frac{k_2^2}{2(2\beta+1)}\left(t_1^{2\beta+1} - {t_d}^{2\beta+1}\right)\right]e^{(k_1-k_2){t_d}^\beta} \tag{7}$$

Substituting (7) in (4), we get

$$I_1(t) = \left\{a\left[t_d + \frac{k_1}{(\beta+1)}{t_d}^{\beta+1} + \frac{k_1^2}{2(2\beta+1)}{t_d}^{2\beta+1}\right] + a\left[(t_1 - t_d) + \frac{k_2}{(\beta+1)}\left(t_1^{\beta+1} - {t_d}^{\beta+1}\right) + \frac{k_2^2}{2(2\beta+1)}\left(t_1^{2\beta+1} - {t_d}^{2\beta+1}\right)\right]e^{(k_1-k_2){t_d}^\beta} - a\left[t + \frac{k_1}{(\beta+1)}t^{\beta+1} + \frac{k_1^2}{2(2\beta+1)}t^{2\beta+1}\right]\right\}e^{-k_1 t^\beta} \tag{8}$$

The maximum backorder is obtained by putting $t = T$ in (6), we get

$$-S_1 = \frac{a}{\delta}\left[e^{-\delta(T-t_1)} - 1\right] \tag{9}$$

The ordering quantity per cycle is

$$Q = S + S_1$$

$$Q = a\left[t_d + \frac{k_1}{(\beta+1)}{t_d}^{\beta+1} + \frac{k_1^2}{2(2\beta+1)}{t_d}^{2\beta+1}\right]$$

$$+a\left[(t_1 - t_d) + \frac{k_2}{(\beta+1)}\left(t_1^{\beta+1} - {t_d}^{\beta+1}\right) + \frac{k_2^2}{2(2\beta+1)}\left(t_1^{2\beta+1} - {t_d}^{2\beta+1}\right)\right]e^{(k_1-k_2){t_d}^\beta}$$

$$-\frac{a}{\delta}\left[e^{-\delta(T-t_1)} - 1\right] \tag{10}$$

Total deteriorating units of item in the interval $(0, t_1)$ is given by

$$D_T = S - \left[\int_0^{t_d} \left(a + bt^{\beta-1} I_1(t)\right) dt + \int_{t_d}^{t_1} \left(a + bt^{\beta-1} I_2(t)\right) dt\right]$$

$$= S - a\nu - \frac{1}{12} ab\nu^{\beta} \times$$

$$\left(\begin{array}{c} -\frac{\nu^{1+3\beta}k_1^3}{\beta+4\beta^2} + \frac{\nu^{1+4\beta}k_1^4}{\beta+5\beta^2} \\ -\frac{6e^{-\nu^{\beta}k_2}}{\beta(1+\beta)(1+2\beta)} \left(\begin{array}{c} 2(1+2\beta)\left\{\left(-e^{\nu^{\beta}k_2}\nu + e^{\nu^{\beta}k_1}(1+\beta)(\nu-\tau)\right) + e^{\nu^{\beta}k_1}\left(\nu^{1+\beta} - \tau^{1+\beta}\right)k_2\right\} \\ +e^{\nu^{\beta}k_1}(1+\beta)\left(\nu^{1+2\beta} - \tau^{1+2\beta}\right)k_2^2 \end{array}\right) \\ +\frac{3e^{-\nu^{\beta}k_2}\nu^{\beta}k_1}{\beta(1+\beta)(1+2\beta)} \left(\begin{array}{c} 2(1+\beta)\left(e^{\nu^{\beta}k_2}\nu + e^{\nu^{\beta}k_1}(1+2\beta)(\nu-\tau)\right) \\ +2e^{\nu^{\beta}k_1}(1+2\beta)\left(\nu^{1+\beta} - \tau^{1+\beta}\right)k_2 \\ +e^{\nu^{\beta}k_1}(1+\beta)\left(\nu^{1+2\beta} - \tau^{1+2\beta}\right)k_2^2 \end{array}\right) \\ -\frac{e^{-\nu^{\beta}k_2}\nu^{2\beta}k_1^2}{\beta(1+\beta)(1+2\beta)(1+3\beta)} \left(\begin{array}{c} 2(1+3\beta+2\beta^2)\left(-e^{\nu^{\beta}k_2}\nu + e^{\nu^{\beta}k_1}(1+3\beta)(\nu-\tau)\right) \\ +2e^{\nu^{\beta}k_1}(1+5\beta+6\beta^2)\left(\nu^{1+\beta} - \tau^{1+\beta}\right)k_2 \\ +e^{\nu^{\beta}k_1}(1+4\beta+3\beta^2)\left(\nu^{1+2\beta} - \tau^{1+2\beta}\right)k_2^2 \end{array}\right) \end{array}\right)$$

$$+a\left(-\frac{1}{8}\nu\left\{\begin{array}{c} -4\nu - \frac{4\beta\nu^{1+3\beta}k_2^3}{2+9\beta+13\beta^2+6\beta^3} - \frac{\nu^{1+4\beta}k_2^4}{(1+2\beta)^2} \\ +\frac{4\tau\left(2+6\beta+4\beta^2+2(1+2\beta)\tau^{\beta}k_2+(1+\beta)\tau^{2\beta}k_2^2\right)}{(1+\beta)(1+2\beta)} \\ +\frac{2\nu^{2\beta}k_2^2\left(-2(1+2\beta)\left(\beta^2\nu-(1+\beta)^2\tau\right)+2\left(1+3\beta+2\beta^2\right)\tau^{1+\beta}k_2+(1+\beta)^2\tau^{1+2\beta}k_2^2\right)}{(1+\beta)^2(1+2\beta)^2} \\ +\frac{4\nu^{\beta}k_2}{(1+\beta)^2(2+\beta)(1+2\beta)}\left(\begin{array}{c} -2(1+3\beta+2\beta^2)(\beta\nu-(2+\beta)\tau) \\ +2(2+5\beta+2\beta^2)\tau^{1+\beta}k_2 + (2+3\beta+\beta^2)\tau^{1+2\beta}k_2^2 \end{array}\right) \end{array}\right\}\right)$$

$$+\frac{1}{8}\tau\left(\begin{array}{c} -4\tau - \frac{4\beta\tau^{1+3\beta}k_2^3}{2+9\beta+13\beta^2+6\beta^3} - \frac{\tau^{1+4\beta}k_2^4}{(1+2\beta)^2} + \frac{4\tau\left(2+6\beta+4\beta^2+2(1+2\beta)\tau^{\beta}k_2+(1+\beta)\tau^{2\beta}k_2^2\right)}{(1+\beta)(1+2\beta)} \\ +\frac{2\tau^{2\beta}k_2^2\left(-2(1+2\beta)\left(\beta^2\tau-(1+\beta)^2\tau\right)+2\left(1+3\beta+2\beta^2\right)\tau^{1+\beta}k_2+(1+\beta)^2\tau^{1+2\beta}k_2^2\right)}{(1+\beta)^2(1+2\beta)^2} \\ -\frac{4\tau^{\beta}k_2(-2(1+3\beta+2\beta^2)(\beta\tau-(2+\beta)\tau)+2(2+5\beta+2\beta^2)\tau^{1+\beta}k_2+(2+3\beta+\beta^2)\tau^{1+2\beta}k_2^2)}{(1+\beta)^2(2+\beta)(1+2\beta)} \end{array}\right)$$

Total holding units of items in the interval $(0, t_1)$ is given by

$$H_T = \int_0^{t_d} \left(a + bt^{\beta-1} I_1(t)\right) dt + \int_{t_d}^{t_1} \left(a + bt^{\beta-1} I_2(t)\right) dt$$

$$= a\nu + \frac{1}{12} ab\nu^{\beta} \times$$

$$\left(\begin{array}{c} -\frac{\nu^{1+3\beta}k_1^3}{\beta+4\beta^2}+\frac{\nu^{1+4\beta}k_1^4}{\beta+5\beta^2} \\ -\frac{6e^{-\nu^\beta k_2}}{\beta(1+\beta)(1+2\beta)}\left(\begin{array}{c} 2(1+2\beta)\left\{\left(-e^{\nu^\beta k_2}\nu+e^{\nu^\beta k_1}(1+\beta)(\nu-\tau)\right)+e^{\nu^\beta k_1}\left(\nu^{1+\beta}-\tau^{1+\beta}\right)k_2\right\} \\ +e^{\nu^\beta k_1}(1+\beta)\left(\nu^{1+2\beta}-\tau^{1+2\beta}\right)k_2^2 \end{array}\right) \\ +\frac{3e^{-\nu^\beta k_2}\nu^\beta k_1}{\beta(1+\beta)(1+2\beta)}\left(\begin{array}{c} 2(1+\beta)\left(e^{\nu^\beta k_2}\nu+e^{\nu^\beta k_1}(1+2\beta)(\nu-\tau)\right) \\ +2e^{\nu^\beta k_1}(1+2\beta)\left(\nu^{1+\beta}-\tau^{1+\beta}\right)k_2 \\ +e^{\nu^\beta k_1}(1+\beta)\left(\nu^{1+2\beta}-\tau^{1+2\beta}\right)k_2^2 \end{array}\right) \\ -\frac{e^{-\nu^\beta k_2}\nu^{2\beta}k_1^2}{\beta(1+\beta)(1+2\beta)(1+3\beta)}\left(\begin{array}{c} 2(1+3\beta+2\beta^2)\left(-e^{\nu^\beta k_2}\nu+e^{\nu^\beta k_1}(1+3\beta)(\nu-\tau)\right) \\ +2e^{\nu^\beta k_1}(1+5\beta+6\beta^2)\left(\nu^{1+\beta}-\tau^{1+\beta}\right)k_2 \\ +e^{\nu^\beta k_1}(1+4\beta+3\beta^2)\left(\nu^{1+2\beta}-\tau^{1+2\beta}\right)k_2^2 \end{array}\right) \end{array}\right)$$

$$-a\left(-\frac{1}{8}\nu\left\{\begin{array}{c} -4\nu-\frac{4\beta\nu^{1+3\beta}k_2^3}{2+9\beta+13\beta^2+6\beta^3}-\frac{\nu^{1+4\beta}k_2^4}{(1+2\beta)^2} \\ +\frac{4\tau\left(2+6\beta+4\beta^2+2(1+2\beta)\tau^\beta k_2+(1+\beta)\tau^{2\beta}k_2^2\right)}{(1+\beta)(1+2\beta)} \\ +\frac{2\nu^{2\beta}k_2^2\left(-2(1+2\beta)\left(\beta^2\nu-(1+\beta)^2\tau\right)+2\left(1+3\beta+2\beta^2\right)\tau^{1+\beta}k_2+(1+\beta)^2\tau^{1+2\beta}k_2^2\right)}{(1+\beta)^2(1+2\beta)^2} \\ +\frac{4\nu^\beta k_2}{(1+\beta)^2(2+\beta)(1+2\beta)}\left(\begin{array}{c} -2(1+3\beta+2\beta^2)(\beta\nu-(2+\beta)\tau) \\ +2(2+5\beta+2\beta^2)\tau^{1+\beta}k_2+(2+3\beta+\beta^2)\tau^{1+2\beta}k_2^2 \end{array}\right) \end{array}\right\}\right)$$

$$-\frac{1}{8}\tau\left(\begin{array}{c} -4\tau-\frac{4\beta\tau^{1+3\beta}k_2^3}{2+9\beta+13\beta^2+6\beta^3}-\frac{\tau^{1+4\beta}k_2^4}{(1+2\beta)^2}+\frac{4\tau\left(2+6\beta+4\beta^2+2(1+2\beta)\tau^\beta k_2+(1+\beta)\tau^{2\beta}k_2^2\right)}{(1+\beta)(1+2\beta)} \\ +\frac{2\tau^{2\beta}k_2^2\left(-2(1+2\beta)\left(\beta^2\tau-(1+\beta)^2\tau\right)+2\left(1+3\beta+2\beta^2\right)\tau^{1+\beta}k_2+(1+\beta)^2\tau^{1+2\beta}k_2^2\right)}{(1+\beta)^2(1+2\beta)^2} \\ -\frac{4\tau^\beta k_2(-2(1+3\beta+2\beta^2)(\beta\tau-(2+\beta)\tau)+2(2+5\beta+2\beta^2)\tau^{1+\beta}k_2+(2+3\beta+\beta^2)\tau^{1+2\beta}k_2^2)}{(1+\beta)^2(2+\beta)(1+2\beta)} \end{array}\right)$$

Shortage over the period (t_1, T)

$$B_T = -\int_{t_1}^{T} I_3 dt = \frac{a}{\delta}(1-e^{-\delta(T-\tau)})(T-\nu)$$

Optimality of the cost function

The cost per unit time of the system

$$C(t_1,T) = \frac{A}{T} + C_d\frac{D_T}{T} + C_h\frac{H_T}{T} + C_s\frac{B_T}{T}$$

Differentiating the cost function with respect to t_1 and T respectively as

$$\frac{\partial C}{\partial t_1} = C_d\frac{1}{T}\frac{\partial D_T}{\partial t_1} + C_h\frac{1}{T}\frac{\partial H_T}{\partial t_1} + C_s\frac{1}{T}\frac{\partial B_T}{\partial t_1},$$

$$\frac{\partial C}{\partial T} = -\frac{A}{T^2} - C_d\frac{D_T}{T^2} - C_h\frac{H_T}{T^2} - C_s\frac{B_T}{T^2}$$

The optimal values of t_1 and T as t_1^* and T^* can be obtained by satisfying the necessary condition for minimization of the cost function

$$\frac{\partial C}{\partial t_1} = 0, \frac{\partial C}{\partial T} = 0$$

provided the following sufficient conditions are satisfied

$$\frac{\partial^2 C}{\partial t_1^2} > 0, \frac{\partial^2 C}{\partial T^2} > 0$$

$$\left(\frac{\partial^2 C}{\partial t_1^2}\right)\left(\frac{\partial^2 C}{\partial T^2}\right) > \left(\frac{\partial^2 C}{\partial t_1 \partial T}\right)$$

Numerical Example:

The parameter values are given as follows

$$A = \$\,500\ per\ order, C_h = \$\,3\ per\ Unit, C_s = \$\,3\ per\ Unit, C_d = \$2\ per\ Unit,$$

$$a = 200\ Units\ /year, b = 0.4, \alpha = .005, \beta = 2, \delta = 0.02, t_d = 0.4\ Year$$

The computational results are as follows

$T^{*} = 0.9186212148724066\, Year$,

$\boldsymbol{t_1^*} = 0.7166887533790196\, Year,$

$TC^{*} = \$\, 328.415,$

$Q^{*} = 878.861 Units$

CONCLUSION

The present paper develops a mathematical model of an inventory system in which demand depending upon stock level and time. We have solved the model for two parameter Weibull distribution and rate of holding cost is constant. Furthermore, shortage occurs and partial backlogged. Numerical example is given to validate the model

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