Thermal Marangoni number for case (i) A-A:

Adiabatic- Adiabatic, both the boundaries of the composite layer are adiabatic

Solving equations (20) and (22) for the temperature distributions θ and θ_m using the following four temperature boundary conditions, where both the boundaries are adiabatic and the heat and heat flux are continuous at the interface, which are as follows

$$D\theta(1) = 0$$

$$\theta(0) = \hat{T}\theta_m(0)$$

$$D\theta(0) = D_m\theta_m(0)$$

$$D_m\theta_m(-1) = 0$$

And the temperature distributions for case (i) are obtained as

$$\theta(z) = A_1\left\{D_1 Cosh[az] + D_2 Sinh[az] - f(z)\right\} \tag{26}$$

$$\theta_m(z) = A_1\left\{D_3 Cosh[a_m z_m] + D_4 Sinh[a_m z_m] - f_m(z_m)\right\} \tag{27}$$

where $f(z) = \frac{z}{2a}\text{Sinh}[\text{az}] + \frac{z}{2a}\text{Cosh}[\text{az}]A_2 + \left(\frac{z^2}{4a}\text{Sinh}[\text{az}] - \frac{z}{4a^2}\text{Cosh}[\text{az}]\right)A_3 + \left(\frac{z^2}{4a}\text{Cosh}[\text{az}] - \frac{z}{4a^2}\text{Sinh}[\text{az}]\right)A_4 + \frac{\text{Cosh}[\delta\text{z}]}{\delta^2 - a^2}A_5 + \frac{\text{Sinh}[\delta\text{z}]}{\delta^2 - a^2}A_6$

and $f_m(z_m) = \frac{z_m}{a_m}\text{Sinh}[a_m z_m]A_{\text{m}1} + \frac{z_m}{a_m}\text{Cosh}[a_m z_m]A_{\text{m}2} + \frac{1}{\delta_m{}^2 - a_m{}^2}\text{Cosh}[\delta_m z_m]A_{\text{m}3} + \frac{1}{\delta_m{}^2 - a_m{}^2}\text{Sinh}[\delta_m z_m]A_{\text{m}4}$

where,

$$D_1 = \frac{\lambda_{76}}{\lambda_{75}} - \frac{a\text{Cosh}[a_m]}{\lambda_{75}}D_2$$

$$D_2 = \frac{\lambda_{78}}{\lambda_{77}}$$

$$\lambda_{75} = -\frac{1}{\hat{T}}a_m\text{Sinh}[a_m],\ \lambda_{76} = \text{Cosh}[a_m]\lambda_{69} - \frac{1}{\hat{T}}a_m\text{Sinh}[a_m]\lambda_{66} - \lambda_{74}$$

$$\lambda_{77} = a\text{Cosh}[a]\lambda_{75} - a^2\text{Sinh}[a]\text{Cosh}[a_m],\ \lambda_{78} = \lambda_{65}\lambda_{75} - \lambda_{76}\left(a\text{Sinh}[a]\right)$$

From the boundary condition (23), we have the thermal Marangoni number

$$M_t = \frac{-D^2W(1)}{a^2\theta(1)}$$

The thermal Marangoni number for case (i) which is M_{t1} is obtained as

$$M_{t1} = \frac{-1}{a^2}\left(\frac{a^2\text{Cosh}[a] + a^2\text{Sinh}[a]A_2 + J_1A_3 + J_2A_4 + \delta^2\text{Cosh}[\delta]A_5 + \delta^2\text{Sinh}[\delta]A_6}{D_1\text{Cosh}[a] + D_2\text{Sinh}[a] - \frac{1}{2a}\text{Sinh}[a] - \frac{1}{2a}\text{Cosh}[a]A_2 - J_3A_3 - J_4A_4 - J_5A_5 - J_6A_6}\right)$$

where

$$J_1 = 2a\text{Sinh}[a] + a^2\text{Cosh}[a],\ J_2 = 2a\text{Cosh}[a] + a^2\text{Sinh}[a]$$

$$J_3 = \frac{1}{4a}\text{Sinh}[a] - \frac{1}{4a^2}\text{Cosh}[a],\ J_4 = \frac{1}{4a}\text{Cosh}[a] - \frac{1}{4a^2}\text{Sinh}[a]$$

$$J_5 = \frac{\text{Cosh}[\delta]}{\delta^2 - a^2},\ J_6 = \frac{\text{Sinh}[\delta]}{\delta^2 - a^2}$$

Thermal Marangoni number for the case (ii) I-A:

Isothermal –Adiabatic, the lower boundary of the porous layer is isothermal and the upper boundary of the fluid layer is Adiabatic.

Solving equations (20) and (22) for the temperature distributions θ and θ_m using the following temperature boundary conditions, where the lower boundary of the porous layer is isothermal and the upper boundary of the fluid layer is adiabatic and at the interface, heat and heat flux are continuous which are as follows

$$D\theta(1)=0$$

$$\theta(0)=\hat{T}\theta_m(0)$$

$$D\theta(0)=D_m\theta_m(0)$$

$$\theta_m(-1)=0$$

The temperature distributions for case (ii) are

$$\theta(z)=A_1\{D_1 Cosh[az]+D_2 Sinh[az]-f(z)\} \tag{26}$$

$$\theta_m(z) = A_1\{D_3 Cosh[a_m z_m] + D_4 Sinh[a_m z_m] - f_m(z_m)\} \tag{27}$$

where $f(z)=\frac{z}{2a}\text{Sinh}[\text{az}]+\frac{z}{2a}\text{Cosh}[\text{az}]A_2+\left(\frac{z^2}{4a}\text{Sinh}[\text{az}]-\frac{z}{4a^2}\text{Cosh}[\text{az}]\right)A_3+\left(\frac{z^2}{4a}\text{Cosh}[\text{az}]-\frac{z}{4a^2}\text{Sinh}[\text{az}]\right)A_4+\frac{\text{Cosh}[\delta z]}{\delta^2-a^2}A_5+\frac{\text{Sinh}[\delta z]}{\delta^2-a^2}A_6$

and $f_m(z_m)=\frac{z_m}{a_m}\text{Sinh}[a_m z_m]A_{m1}+\frac{z_m}{a_m}\text{Cosh}[a_m z_m]A_{m2}+\frac{1}{\delta_m{}^2-a_m{}^2}\text{Cosh}[\delta_m z_m]A_{m3}+\frac{1}{\delta_m{}^2-a_m{}^2}\text{Sinh}[\delta_m z_m]A_{m4}$

where,

$$D_1=\frac{\Delta_3}{\Delta_1}+\frac{\Delta_2}{\Delta_1}D_2$$

$$D_2=\frac{\Delta_4}{\Delta_5}$$

$$\Delta_1=\frac{1}{\hat{T}}\text{Cosh}[a_m],\ \Delta_2=\frac{a}{a_m}\text{Sinh}[a_m],\ \Delta_3=\lambda_{74}+\frac{1}{\hat{T}}\lambda_{66}\text{Cosh}[a_m]-\frac{\lambda_{69}}{a_m}\text{Sinh}[a_m]$$

$$\Delta_4=\lambda_{65}-a\text{Sinh}[a]\frac{\lambda_{77}}{\lambda_{75}},\ \Delta_5=a\text{Sinh}[a]\frac{\lambda_{76}}{\lambda_{75}}+a\text{Cosh}[a]$$

The thermal Marangoni number for case (ii) which is M_{t2} is obtained as

$$M_{t2}=\frac{-1}{a^2}\left(\frac{a^2\text{Cosh}[a]+a^2\text{Sinh}[a]A_2+J_1A_3+J_2A_4+\delta^2\text{Cosh}[\delta]A_5+\delta^2\text{Sinh}[\delta]A_6}{D_1\text{Cosh}[a]+D_2\text{Sinh}[a]-\frac{1}{2a}\text{Sinh}[a]-\frac{1}{2a}\text{Cosh}[a]A_2-J_3A_3-J_4A_4-J_5A_5-J_6A_6}\right)$$

where

$$J_1=2a\text{Sinh}[a]+a^2\text{Cosh}[a],\ J_2=2a\text{Cosh}[a]+a^2\text{Sinh}[a]$$

$$J_3=\frac{1}{4a}\text{Sinh}[a]-\frac{1}{4a^2}\text{Cosh}[a],\ J_4=\frac{1}{4a}\text{Cosh}[a]-\frac{1}{4a^2}\text{Sinh}[a]$$

$$J_5=\frac{\text{Cosh}[\delta]}{\delta^2-a^2},\ J_6=\frac{\text{Sinh}[\delta]}{\delta^2-a^2}$$

The terms common in both the cases are as follows:

$$D_3 = \hat{T}D_1 - \lambda_{66}$$

$$D_4 = \frac{a}{a_m}D_2 - \frac{\lambda_{69}}{a_m}$$

$$\lambda_{59} = \frac{\text{Cosh}[a]}{2} + \frac{\text{Sinh}[a]}{2a},\ \lambda_{60} = \frac{\text{Sinh}[a]}{2} + \frac{1}{2a}\text{Cosh}[a]$$

$$\lambda_{61} = \frac{\text{Cosh}[a]}{4} + \frac{1}{4a}\text{Sinh}[a] - \frac{\text{Cosh}[a]}{4a^2},\ \lambda_{62} = \frac{\text{Sinh}[a]}{4} + \frac{1}{4a}\text{Cosh}[a] - \frac{\text{Sinh}[a]}{4a^2}$$

$$\lambda_{63} = \frac{\delta\text{Sinh}[\delta]}{\delta^2 - a^2},\ \lambda_{64} = \frac{\delta\text{Cosh}[\delta]}{\delta^2 - a^2}$$

$$\lambda_{65} = \lambda_{59} + A_2\lambda_{60} + A_3\lambda_{61} + A_4\lambda_{62} + A_5\lambda_{63} + A_6\lambda_{64}$$

$$\lambda_{66} = \frac{A_5}{\delta^2 - a^2} - \frac{\hat{T}A_{m3}}{\delta_m{}^2 - a_m{}^2},\ \lambda_{67} = \frac{A_2}{2a} - \frac{A_3}{4a^2} + \frac{A_6\delta}{\delta^2 - a^2}$$

$$\lambda_{68} = \frac{A_{m2}}{a_m} + \frac{A_{m4}\delta_m}{\delta_m{}^2 - a_m{}^2},\ \lambda_{69} = \lambda_{67} - \lambda_{68}$$

$$\lambda_{70} = \text{Cosh}[a_m] + \frac{1}{a_m}\text{Sinh}[a_m],\ \lambda_{71} = \text{Sinh}[a_m] + \frac{1}{a_m}\text{Cosh}[a_m]$$

$$\lambda_{72} = \frac{\delta_m\text{Sinh}[\delta_m]}{\delta_m{}^2 - a_m{}^2},\ \lambda_{73} = \frac{\delta_m\text{Cosh}[\delta_m]}{\delta_m{}^2 - a_m{}^2}$$

$$\lambda_{74} = A_{m1}\lambda_{70} - A_{m2}\lambda_{71} + A_{m3}\lambda_{72} - A_{m4}\lambda_{73}$$

5 INTERPRETATIONS

The eigenvalue thermal Marangoni number M_t is obtained as a function of parameters such as horizontal wave numbers 'a' for the fluid & 'a_m' for porous layer, porous parameter 'β', thermal ratio '$\hat{T}$', viscosity ratio '$\hat{\mu}$', couple stress parameters 'C_p' & 'C_{pm}' for fluid and porous layer respectively. A graph is drawn with thermal Marangoni number M_t versus depth ratio $\hat{d}$ for the fixed values of a = $\hat{\mu}$ = 1, C_p = C_{pm} = $\hat{T}$ = 0.5 & β =10. The effects of the variations of each of these parameters on thermal Marangoni number with all other parameters unaltered is displayed in figures 3,4,5,6,7 and 8 for the case (i) and case (ii). Figure 2 exhibits the comparison of the Thermal Marangoni numbers as a function of depth ratio, for case (i) A-A, both the boundaries are adiabatic and case (ii) I-A, the lower boundary of the porous layer is made isothermal. The pattern of the curves for case (i) and case(ii) differs, for case (i), when the value of depth ratio increases, the thermal Marangoni number increases, but for case (ii), as the value of depth ratio increases, the thermal Marangoni number decreases upto a certain value of depth ratio, then it increases for further increase in depth ratio. It is quite interesting to note that the Thermal Marangoni numbers for cases (i) and (ii) change only for smaller depth ratio $\hat{d}$ values. And the eigenvalue Thermal Marangoni number coincides for both case(i) and case(ii)for the values of depth ratio $\hat{d} = \frac{d_m}{d}$, $\hat{d}$ >4 that is for porous layer dominant composite system $d << d_m$. It is quite interesting to note that for smaller values of depth ratio, there is opposite effect of these cases (i) and (ii) on thermal Marangoni numbers and is physically reasonable. Also for a fixed depth ratio $\hat{d}$, the value of thermal Marangoni number is higher for case (ii), the difference in thermal Marangoni number reduces as the value of depth ratio $\hat{d}$ increases and as the value of $\hat{d}$ >4, they coincide.

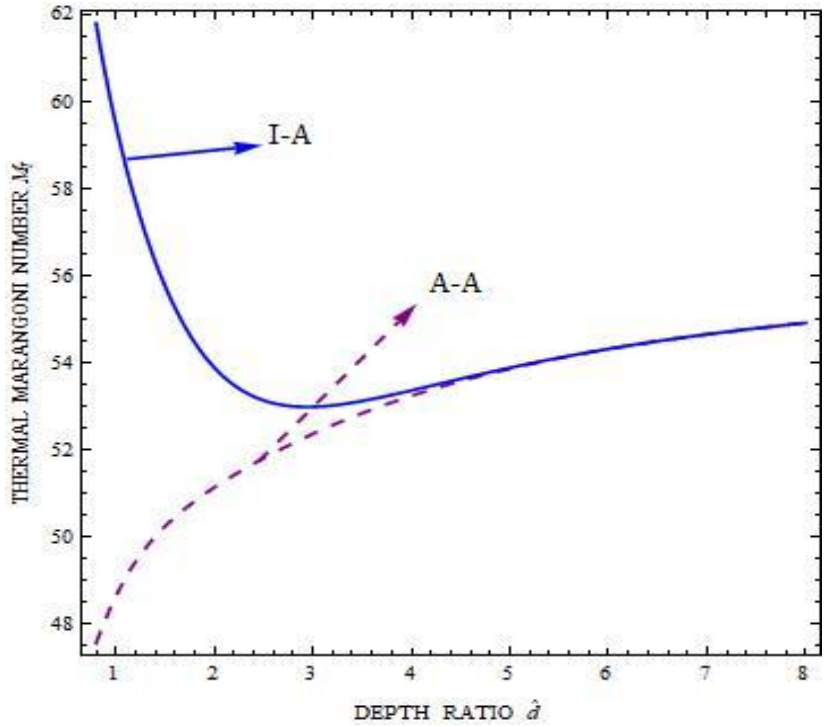


Figure 2: Comparison of Thermal Marangoni numbers for Case(i) & Case(ii)

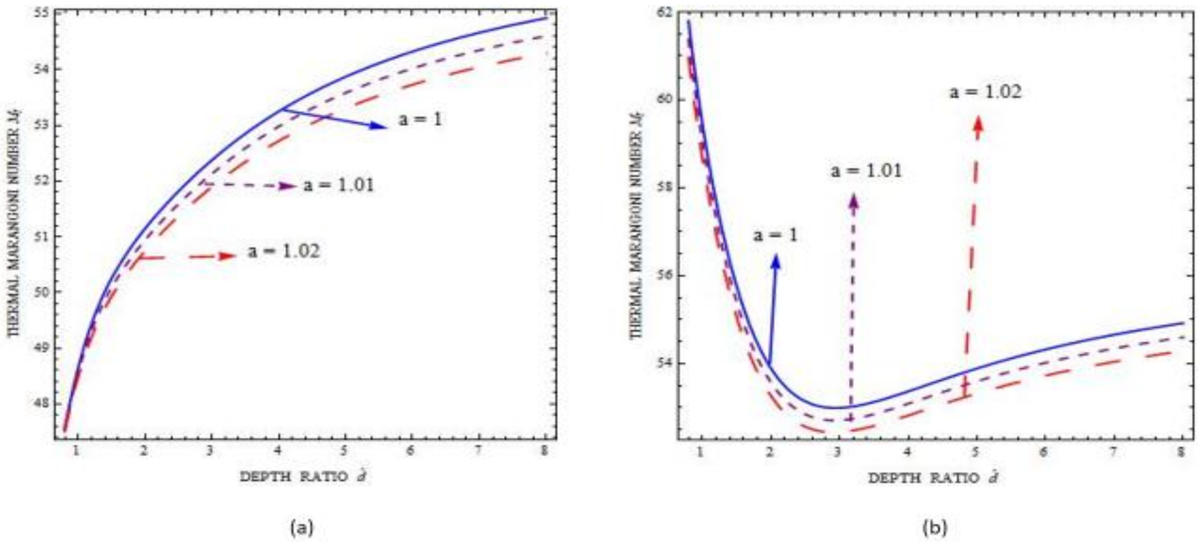
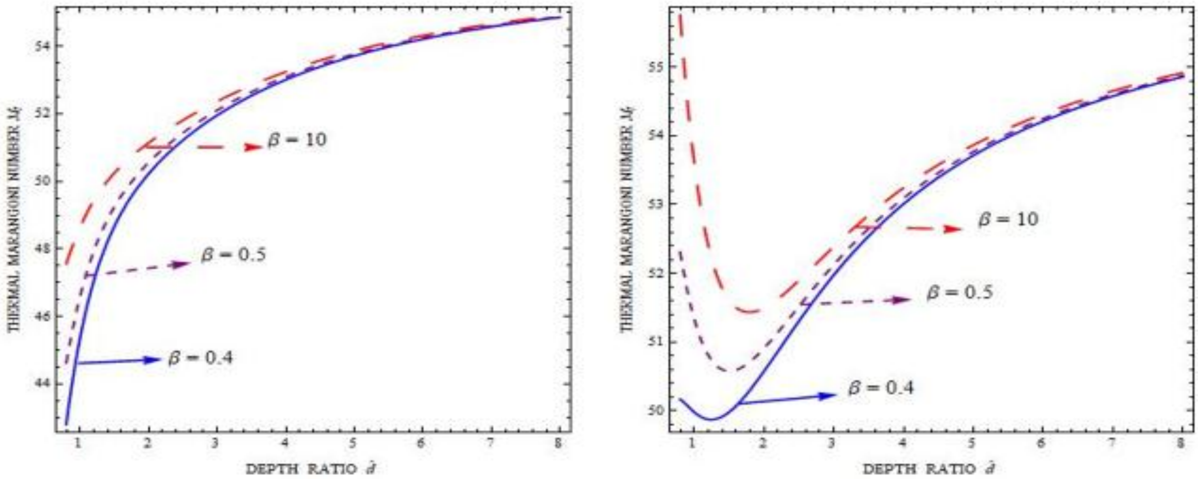


Figure3: Effects of horizontal wave number 'a'

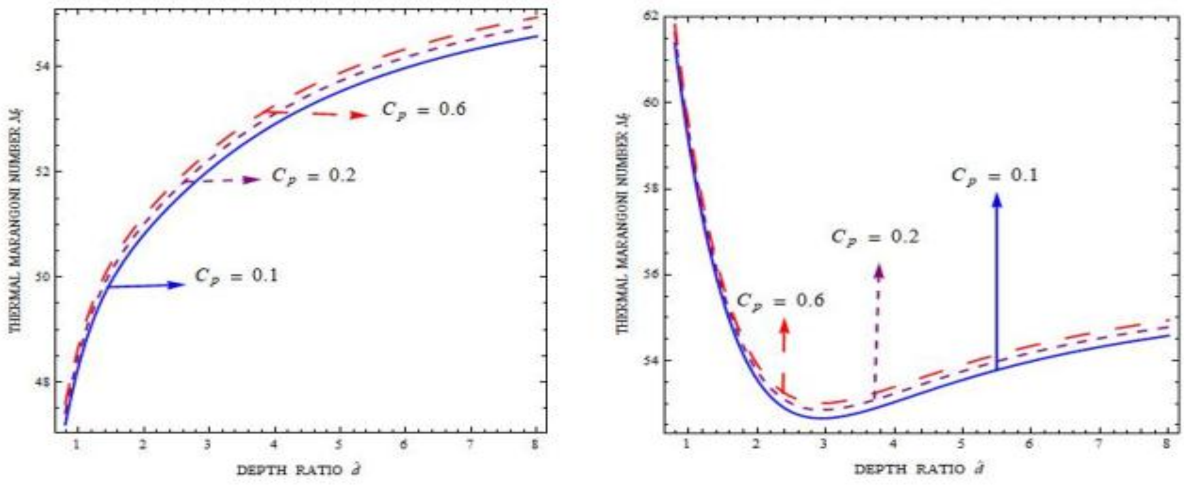
The effects of $'a'$, the horizontal wave number for the fluid layer on the thermal Marangoni number are shown in figure 3a and 3b for the case(i) and case(ii) respectvely. It is seen that the curves are diverging with assigned values $a = 1$, 1.01, 1.02 which proves that variation effect is prominent for the composite layer with $d << d_m$. It is also evident from the figure that increase in $'a'$ results into decrease in the value of thermal Marangoni number and hence the system can be destabilized. As a result, onset of single component Darcy - Benard Marangoni convection is faster. The graph indicates that increasing values of $'a'$ will have effect only for larger values of the depth ratio $\hat{d} = \frac{d_m}{d}$, that is for porous layer dominant composite systems. The nature of the effect of $'a'$ remains the same for both the cases.



(a) (b)

Figure4: Effects of porous parameter 'β'

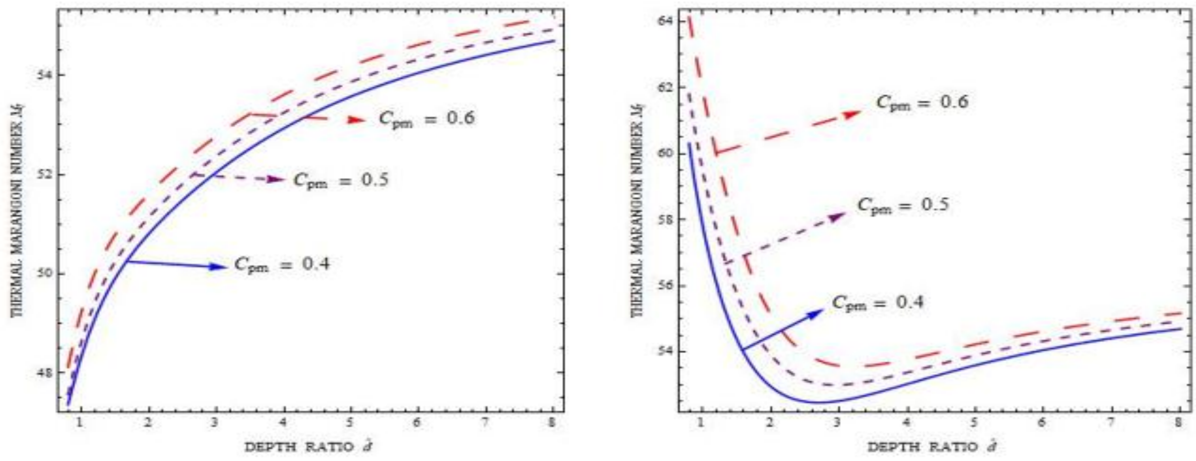
The effects of porous parameter 'β' on the thermal Marangoni number are shown in figures 4a &4b for case(i) and case (ii) respectively. In this case the curves are converging for values of β = 0.4, 0.5 and 10 and variation effect is prominent for the composite layer with $d >> d_m$. It is noticed that thermal Marangoni number increases when 'β' increases and thereby the system can be stabilised. That is, single component Darcy - Benard Marangoni convection gets delayed. Also, it is clear from the graph that increasing values of 'β' will have effect only for smaller values of the depth ratio $\hat{d} = \frac{d_m}{d}$, that is for fluid layer dominant composite systems. Similar effects of 'β' on single component Marangoni convection for both the cases.



(a) (b)

Figure5: Effects of couple stress parameter in the fluid layer 'C_p'

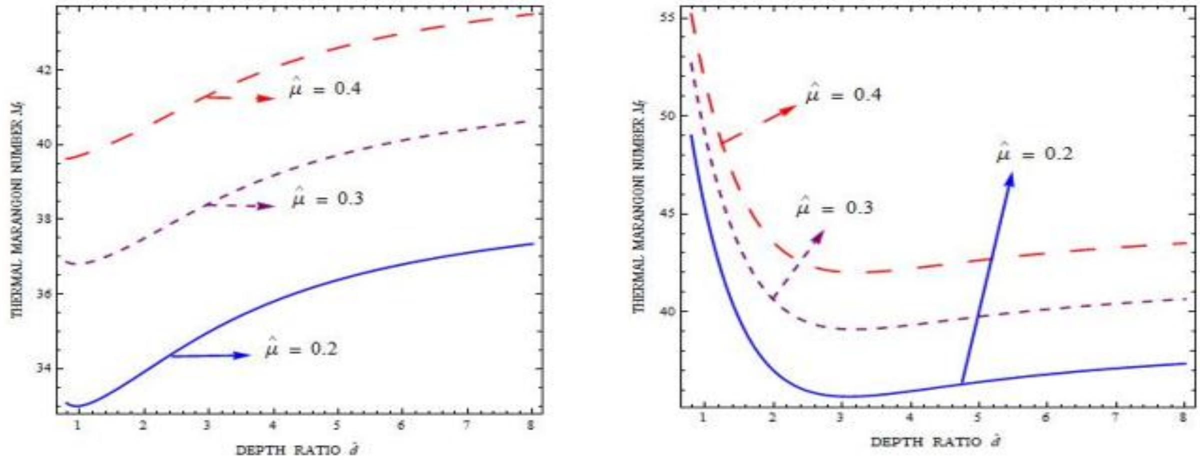
The effects of couple stress parameter in the fluid layer 'C_p' on the thermal Marangoni number are shown in figure 5a &5b for the values of C_p = 0.1, 0.2, 0.6. Here, the curves are diverging and its effect is prominent for the composite layer with $d << d_m$. It is evident that thermal Marangoni number increases when 'C_p' increases. Hence the system can be stabilised. In otherwords, the onset of single component Darcy – Benard Marangoni-convection is delayed. The graph represents that increasing values of 'C_p' will have effect only for larger values of the depth ratio $\hat{d} = \frac{d_m}{d}$, that is for porous layer dominant composite systems. The impact of 'C_p' is analogus for the both the cases.



(a) (b)

Figure6: Effects of couple stress parameter in the porous layer 'C_{pm}'

The effects of couple stress parameter in the porous layer 'C_{pm}' on the thermal Marangoni number are shown in figure 6a & 6b for case(i) and case(ii) respectively for C_{pm} = 0.4, 0.5 and 0.6. The curves are diverging under the condition when both the boundaries of the composite layer are adiabatic and curves found converging slightly at both the ends while lower boundary of the porous layer is isothermal and the upper boundary of the fluid layer is Adiabatic. The study reveals that the thermal Marangoni number increases when 'C_{pm}' increases. Hence the system can be stabilized by delaying the onset of single component Darcy – Benard Marangoni convection.



(a) (b)

Figure7: Effects of viscosity ratio $\hat{\mu}$

The effects of viscosity ratio '$\hat{\mu}$' on the thermal Marangoni number are shown in figure 7a and 7b for case (i) and case(ii) respectively for the values of $\hat{\mu}$ = 0.2, 0.3 and 0.4. Variation effect of '$\hat{\mu}$' found unaltered for all the values of depth ratios for the case (i) and the curves are diverging that the the variation effect of '$\hat{\mu}$' is becoming drastic as the value of depth ratio increases. It is evident that thermal Marangoni number increases when '$\hat{\mu}$' increases. Hence the system can be stabilized by delaying the the onset of single component Darcy – Benard Marangoni convection for larger values of

'$\hat{\mu}$'. In other words when the effective viscosity of the porous medium 'μ_m' is made larger than the fluid viscosity 'μ', the Marangoni convection in the fluid layer can be delayed.

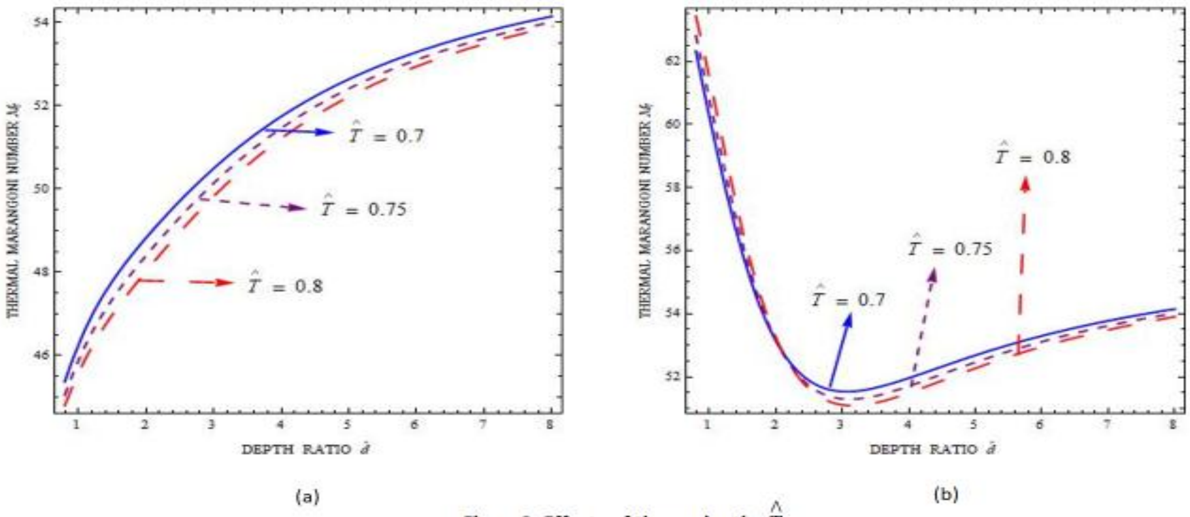


Figure8: Effects of thermal ratio $\hat{T}$

The effects of the thermal ratio '$\hat{T}$' on the thermal Marangoni number are shown in figure 8a and 8b for case (i) and case(ii) respectively for $\hat{T}$ = 0.7, 0.75 and 0.8. It is seen from the graph that the curves are converging at both the ends for case(ii) and this parameter has dual effect on the thermal Marangoni number. For $\hat{d} \leq 2.2$, the curves are converging and the increase in '$\hat{T}$' increases the thermal Marangoni number whereas for the values of the depth ratio $2.2 \leq \hat{d} \leq 8.0$, the curves are diverging and increase in '$\hat{T}$' causes decrease in the value of thermal Marangoni number. For case(i), there is uniform effect of '$\hat{T}$' on the thermal Marangoni number and for a fixed value of depth ratio, the increase in the value of thermal ratio '$\hat{T}$' decreases the thermal Marangoni number, hence the increase in the value of '$\hat{T}$' makes the system unstable.

6. CONCLUSION

1) For smaller values of depth ratio, there is opposite effect of these cases (i) and (ii) on thermal Marangoni numbers and is physically reasonable.

2) The effect of horizontal wavenumber and couple stress parameter of the fluid in the fluid layer is prominent for porous layer dominant composite layer where as the effect of the porous parameter is prominent for fluid layer dominant composite layer and other parameters are prominent for all composite layers.

3) Single component Darcy – Benard Marangoni convection can be postponed or preponed by choosing appropriate values for the physical parameters.

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