

Analysis of Customer Induced Interruption in a Retrial Queuing System with Classical Retrial Policy

Varghese Jacob

Department of Mathematics, Government College, Kottayam, Kerala - 686013, India.

Abstract

In this paper we studied a retrial queuing system with customer induced interruption with classical retrial policy. We consider an infinite capacity queuing system with a single server to which customers arrive according to a Poisson process and the service time follows an exponential distribution. An arriving customer to an idle server obtains service immediately and customers who find server busy go directly to the orbit from where he retry for service. The inter-retrial time follows exponential distribution. The customer interruption while in service occurs according to a Poisson process and the interruption duration follows an exponential distribution. The customer whose service is got interrupted will enter into a finite buffer. Any interrupted customer, finding the buffer full, is considered lost. Those interrupted customers who complete their interruptions will be placed into another buffer of same size. The interrupted customers waiting for service are given non-preemptive priority over new customers. We analyse the steady state behavior of this queuing system. Several performance measures are obtained. Numerical illustrations of the system behaviour are also provided with example.

Keywords: Quasi-birth-and-death process, Matrix-geometric methods, customer induced interruption, Neuts-Rao truncation method, steady-state analysis.

AMS Subject Classification. 68K25

1. INTRODUCTION

Queuing models with repeated attempts, known as ' retrial queues ', have been widely used to model many problems in telecommunication and computer systems. The important feature of a retrial queue is that arriving customers who find all the server busy have to leave the service area and join a retrial group, called orbit, in order to try their luck again after some random time. For a detailed review of the literature on this topic the reader is referred to the survey paper by Artalejo *et al.* [1]. In the classical queuing systems, servers are always available to serve customers. But, there are situations where the servers may unavailable for a random period of time due to many reasons such as server interruptions, server vacations, removal of servers due to catastrophic or negative arrival of events, getting preempted due to the arrival of priority customers etc. A review of recent work on different type of server interruption, readers is referred to Krishnamoorthy *et*

al. [8]. So far in literature only a few papers studied customer induced interruptions such as customers leaving in the middle of a service due to not having enough information for completing a service. In many daily life situation where the customers are often leave the service area in the middle of a service due to not having enough information for completing a service. However, these customers are requesting their service after some random period of time. This type of interruption is known as customer induced interruption. A more commonly occurring example is the following :- In a doctors clinic, while patient is being examined, the Physician may find that one or more tests needed for prescription of medicine. Hence he/she is asked to undergo these and return to the clinic. Such patients can be regarded as interruption induced by the customer. As far our knowledge goes, the first work dealing queues with customer induced interruptions is [4] reported at 8th International Workshop on retrial queues in 2010 and the paper of Jacob *et al.* [5]. Subsequently, Krishnamoorthy and Jacob [6] extended the work to a multi-server $M/M/c$ model. Jacob and Krishnamoorthy [9] discussed $M/PH/1$ queuing system with customer induced interruption in the retrial set up with a finite orbit. Recently, Punalal and Babu [10] studied a retrial queuing model with self-generation of priorities and customer induced interruption. The purpose of this work is to introduce customer induced in a retrial queuing systems with classical retrial policy. In classical retrial policy, the rate of retrial of customers for service depends on the number of customers in the orbit.

The paper is organized as follows. In Section 2 the model under study is described. Section 3 provides the steady-state analysis of the model. Section 4 discuss the main performance measures of the system. Some illustrative examples are discussed in section 5.

2. MODEL DESCRIPTION

We consider an infinite capacity queuing system with a single sever to which customers arrive according to a Poisson process with rate λ . The service times are assumed to follow an exponential distribution with parameter μ . An arriving customer to the idle server obtains service immediately. Customers who find server busy go directly to the orbit from where he retry for service. The interval between two successive repeated attempts is exponentially distributed with rate $\sigma_j = j\sigma$, given that the number of customers in the orbit is j . Here a customer induced interruption while in service

From equation (5) we can obtain

$$x(0)Q_{10} + x(1)Q_{21} = 0$$

$$x(i-1)Q_0 + x(i)Q_i + x(i+1)Q_{2(i+1)} = 0, i = 1, \dots, N-2$$

$$x(N-2)Q_0 + x(N-1)Q_{1(N-1)} + x(N)Q_2 = 0$$

$$x(i-1)Q_0 + x(i)Q_1 + x(i+1)Q_2 = 0, i = N, N+1, \dots$$

We see that x , under the assumption that the stability condition (3) holds, is obtained as (see Neuts [3])

$$x(N+i) = x(N-1)R_N^{i+1}, i \geq 0 \quad (7)$$

where R_N is the minimal nonnegative solution to the matrix quadratic equation

$$R_N^2 Q_2 + R_N Q_1 + Q_0 = 0 \quad (8)$$

(see Neuts [3]. Again $xQ = 0$ gives us to

$$x(N-i) = x(N-i-1)R_{N-i}, i = 1, 2, \dots, (N-1), \quad (9)$$

where

$$R_{N-i} = -Q_0(Q_{1(N-i)} + R_{N-i+1}Q_{2(N-i+1)})^{-1}$$

Finally,

$$x(0)Q_{10} + x(1)Q_{21} = 0$$

We find $x(0)$ as the steady-state distribution of the finite Markov chain with the generator matrix $Q_{10} + R_1 Q_{21}$. Then using equations (7) and (9), we find $x(i)$, $i \geq 1$. Now x can be calculated by dividing each $x(i)$ with the normalizing constant $\sum_{i=0}^{\infty} x(i)e$

Once the rate matrix R_N is obtained, the vector can be computed using logarithmic reduction algorithm. For full details of the logarithmic reduction algorithm we refer the reader to [2].

4. SYSTEM PERFORMANCE MEASURES

In this section we present the main system performance measures to bring out the qualitative aspects of the model under study. These are listed below along with their formula for computation. We further partition the vectors,

$$(x(n) \ n \geq 0), \text{ into } x(n) = (x^*(n), x_1(n), x_2(n)),$$

$n \geq 0$ where

$$x^*(n) = (x_0^*(n), x_1^*(n), \dots, x_K^*(n)) ;$$

$$x_1(n) = (x_{1,0}(n), x_{1,1}(n), \dots, x_{1,K}(n)) ;$$

$$x_2(n) = (x_{2,0}(n), x_{2,1}(n), \dots, x_{2,K}(n))$$

Where each components, $x_{j,r}(n)$; $j = 1, 2$; $0 \leq r \leq K$ $n \geq 0$, is of dimension $K + 1 - r$.

- The probability that the server is idle : $P_{idle} =$

$$\sum_{n=0}^{\infty} x^*(n)e$$

- The probability that the server is busy with a primary/orbital customer

$$P_{bryo} =$$

$$\sum_{n=0}^{\infty} x_1(n)e = \sum_{n=0}^{N-1} x_1(n)e + x(N-1)(I-R)^{-1}e_2(3) \otimes e$$

- The probability that the server is busy with an interrupted customer:

$$P_{brys} =$$

$$\sum_{n=0}^{\infty} x_2(n)e = \sum_{n=0}^{N-1} x_2(n)e + x(N-1)(I-R)^{-1}e_3(3) \otimes e$$

- The probability that an interrupted customer is lost:

$$P_{loss} = \frac{\theta}{\theta + \mu} \sum_{n=0}^{\infty} x_{1,0,K}(n)$$

- The expected number of customers in the orbit :

$$E_{orbit} = \sum_{n=0}^{N-1} x(n)e + x(N-1)R(I-R)^{-2}e$$

- The expected number of interrupted customers in the BIC buffer :

$$E_{BIC} = \sum_{n=0}^{\infty} \sum_{i=1}^2 \sum_{j=0}^K \sum_{m=0}^{K-j} j x_{i,j,m}(n)$$

- The expected number of interrupted customers in the BIP buffer :

$$E_{BIP} = \sum_{n=0}^{\infty} \sum_{i=1}^2 \sum_{j=0}^K \sum_{m=0}^{K-j} m x_{i,j,m}(n)$$

- The rate at which the orbiting customer successfully reach the server is given by

$$\sigma_1^* = \sigma \sum_{n=0}^{\infty} x^*(n)e$$

- The overall rate of retrials at which the orbiting customers request service is given by

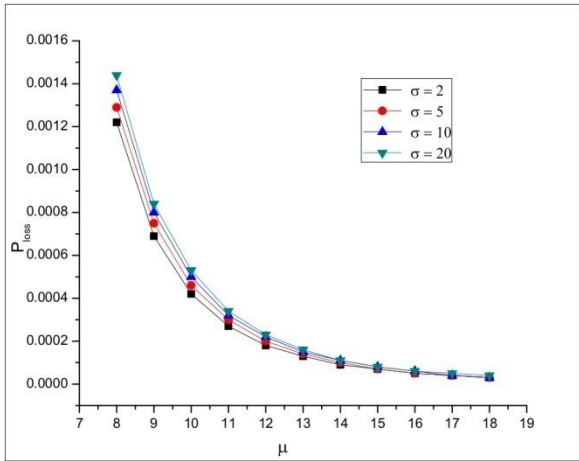
$$\sigma_2^* = \sigma \mu_{orbit}$$

- The fraction, FSR, of successful rate of retrials is given by $FSR = \sigma_1^* / \sigma_2^*$

$$P_{loss}, E_{orbit}, P_{bsyo}, E_{BIC}, E_{BIP}, P_{idle}, P_{BIC}$$

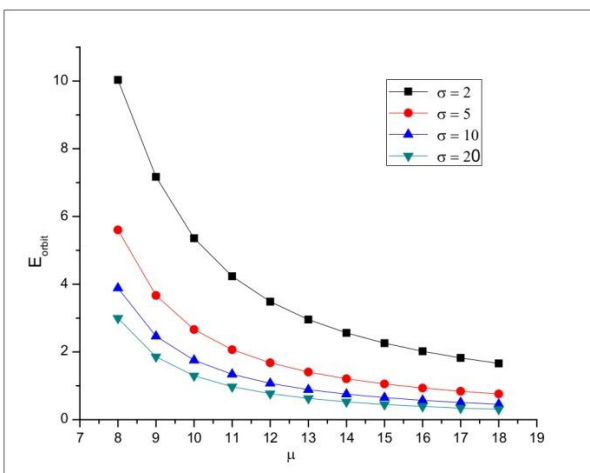
5. NUMERICAL ILLUSTRATIONS

In this section we present some numerical examples to show the effect of various parameters of the system when other parameters are fixed.



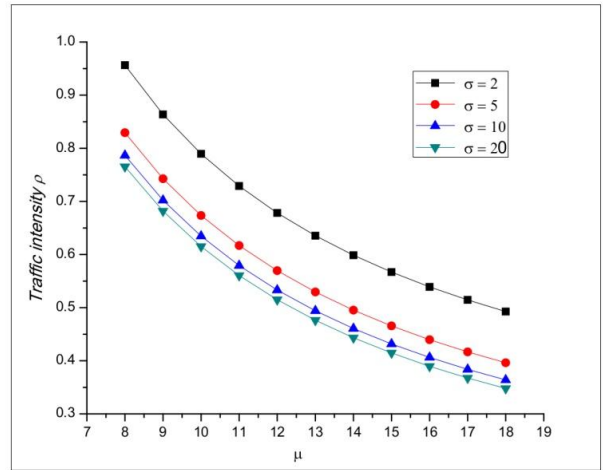
The effect of μ versus P_{loss}

Figure 1: Fixing the parameters $(K, \lambda, \theta, \eta) = (5, 6, 3, 2)$



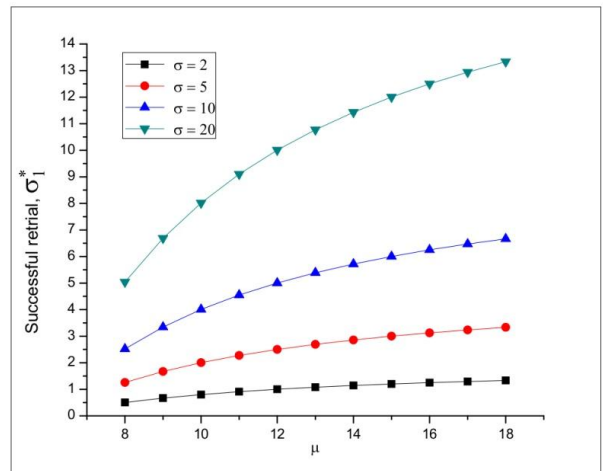
The effect of μ versus E_{orbit}

Figure 2: Fixing the parameters $(K, \lambda, \theta, \eta) = (5, 6, 3, 2)$



The effect of μ versus traffic intensity ρ

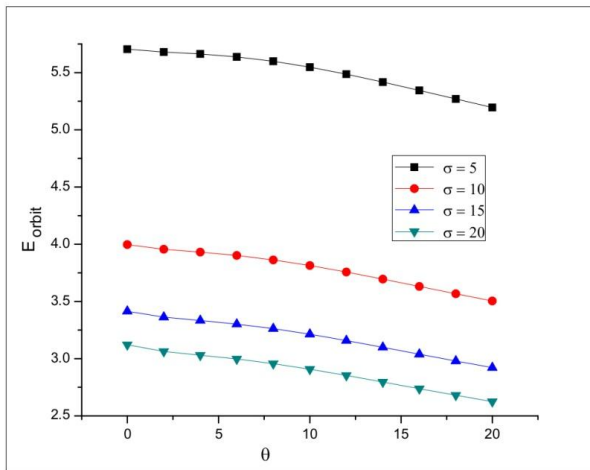
Figure 3: Fixing the parameters $(K, \lambda, \theta, \eta) = (5, 6, 3, 2)$



The effect of μ versus successful retrial σ_1^*

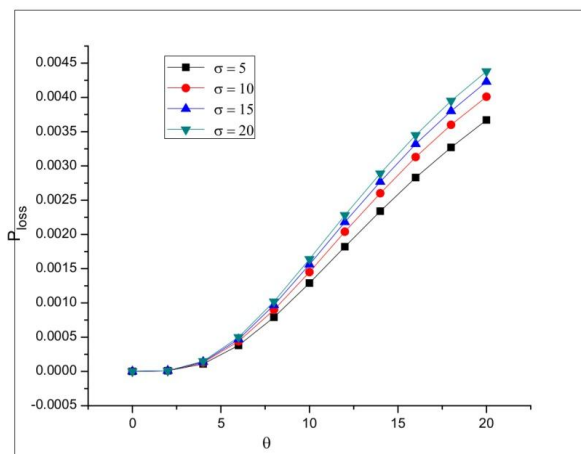
Figure 4: Fixing the parameters $(K, \lambda, \theta, \eta) = (5, 6, 3, 2)$

Looking to the Fig. 1 to 4, we can see the influence of the parameter μ on various measures. As μ increases the measures P_{loss}, E_{orbit} and the traffic intensity ρ are decreases. This is because increase of service rates results in fast clearance of customers from the system. From Fig. 4, as μ increases, the overall successful retrial rate of orbital customers also increases. The reason is that clearing of the customers at faster rate from the system result in increase of idle probability, so the retrial customers can get into service.



The effect of θ versus E_{orbit}

Figure 5 : Fixing the parameters $(K, \lambda, \mu, \eta) = (5, 6, 8, 5)$



The effect of θ versus loss probability P_{loss}

Figure 6 : Fixing the parameters $(K, \lambda, \mu, \eta) = (5, 6, 8, 5)$

From Fig 5, we observe that, E_{orbit} is a non-increasing function of θ and from Fig 6, P_{loss} is a non-decreasing function of θ for every values of σ . From Fig 5, for a fixed θ , E_{orbit} is getting reduced as the retrial rate σ increased. The measure P_{loss} is increases as θ increases because for a fixed K , the interrupted customers getting lost due to BIP being full.

Looking at the Table 1, we summarize the following.

- The measure ρ , is a non-increasing function of θ for every values of η . This is due to the fact that an increase in θ will cause more customers to be interrupted leading to an increase in the number of customers leaving the system without getting services.

- The measures P_{bsyo} and σ_2^* are decreases as θ increases whereas the measures E_{BIC} , E_{BIP} , P_{idle} , P_{BIC} and σ_1^* are increasing when θ is increasing for every values of η . This is due to the fact that beyond a certain point for θ , any further increase in θ will only result in the server being busy with interrupted customers,

we see that the phenomenon of increasing-decreasing of this measures can be justified.

- The rate of increase/decrease is low for higher values of η . This is because for smaller values of η , the BIP buffer remains full. So the number of customer leaving the system without getting service increases for smaller values of η . For large values of η , the rate of increase negligible as θ increases. The number of customers in the BIP buffer completes their interruption and moved to BIC buffer. So the probability that the server is busy with BIC customers increases.

Table 1. Fixing the parameters $(K, \lambda, \mu, \sigma) = (5, 6, 8, 2)$

θ	ρ	E_{BIC}	E_{BIP}	P_{idle}	P_{bsyo}	P_{BIC}	σ_1^*	σ_2^*
$\eta = 1$								
1	0.9593	0.0587	0.6641	0.2503	0.6667	0.0830	0.5007	20.1667
2	0.9524	0.0999	1.1679	0.2540	0.6000	0.1460	0.5080	19.8938
5	0.8857	0.1470	1.9766	0.2914	0.4615	0.2471	0.5828	17.1116
10	0.7523	0.1465	2.4212	0.3640	0.3333	0.3027	0.7280	12.4403
15	0.6527	0.1370	2.5818	0.4164	0.2609	0.3227	0.8328	9.9465
20	0.5815	0.1298	2.6598	0.4532	0.2143	0.3325	0.9065	8.5472
$\eta = 3$								
1	0.9600	0.0553	0.2222	0.2500	0.6667	0.0833	0.5000	20.1950
2	0.9598	0.0970	0.3998	0.2501	0.6000	0.1499	0.5002	20.1845
5	0.9550	0.1812	0.7636	0.2521	0.4615	0.2863	0.5042	20.0332
10	0.9304	0.2535	1.0770	0.2628	0.3333	0.4039	0.5256	19.2457
15	0.8954	0.2864	1.2320	0.2772	0.2609	0.4620	0.5543	18.1501
20	0.8594	0.3025	1.3193	0.2910	0.2143	0.4947	0.5820	17.0921
$\eta = 10$								
1	0.9600	0.0462	0.0667	0.2500	0.6667	0.0833	0.5000	20.1986
2	0.9600	0.0799	0.1200	0.2500	0.6000	0.1500	0.5000	20.1956
5	0.9600	0.1457	0.2308	0.2500	0.4615	0.2885	0.5000	20.1899
10	0.9594	0.2106	0.3332	0.2502	0.3333	0.4165	0.5003	20.1771
15	0.9582	0.2537	0.3909	0.2506	0.2609	0.4886	0.5011	20.1505
20	0.9561	0.2855	0.4276	0.2512	0.2143	0.5346	0.5023	20.1090
$\eta = 20$								
1	0.9600	0.0383	0.0333	0.2500	0.6667	0.0833	0.5000	20.2000
2	0.9600	0.0659	0.0600	0.2500	0.6000	0.1500	0.5000	20.1979
5	0.9600	0.1181	0.1154	0.2500	0.4615	0.2885	0.5000	20.1944
10	0.9600	0.1659	0.1667	0.2500	0.3333	0.4167	0.5000	20.1919
15	0.9599	0.1963	0.1956	0.2500	0.2609	0.4891	0.5001	20.1900
20	0.9598	0.2186	0.2143	0.2501	0.2143	0.5357	0.5001	20.1875

CONCLUSION

In this paper, we analysed an $M/M/1$ retrial queuing model with classical retrial policy and customer induced interruption. All underlying distributions are assumed to be exponential that are independent of each other. There is a finite buffer for self interrupted customers to wait for completion of interruption and another buffer of the same capacity for those who have completed their interruptions. The system and steady state are analyzed by using matrix geometric method. Several performance measures are derived. The effect of various parameters on the system performance is also investigated with the numerical examples.

REFERENCES

- [1] J. R. Artalejo Accessible bibliography on retrial queues: Progress in 2000- 2009. *Mathematical and Computer Modelling*, 51, (2010), 1071--1081.
- [2] G. Latouche, and V. Ramaswami, A logarithmic reduction algorithm for quasi-birth-and-death processes. *Journal of Applied Probability*, 30, (1993), 650--674.
- [3] M. F. Neuts, *Matrix-Geometric Solutions in Stochastic Models – An Algorithmic Approach*, 2nd ed., Dover Publications, Inc., New York (1994).
- [4] V. Jacob, S. R. Chakravarthy and A. Krishnamoorthy, On a Customer induced interruption in a service system. Abstract of International Workshop on Retrial Queues (8th WRQ), July 27-29, (2010), Beijing, China.
- [5] V. Jacob, S. R. Chakravarthy, and A. Krishnamoorthy, On a Customer Induced Interruption in a service system. *Stochastic Analysis and Applications*, 30, (2012), 949--962.
- [6] A. Krishnamoorthy and V. Jacob, Analysis of Customer induced interruption in a multi server system. *Neural, Parallel, and Scientific Computations*, 20, (2012), 153-172.
- [7] M.F. Neuts, B.M. Rao, Numerical investigation of a multiserver retrial model, *Queue. Syst.* 7, (1990), 169--190.
- [8] A. Krishnamoorthy, P. K. Pramod and S. R. Chakravarthy, A survey on queues with interruptions. *TOP- Spanish journal of Statistics & Operations Research*, 22 : 1, (2014), 290-320.
- [9] V. Jacob and A. Krishnamoorthy, Analysis of Customer-Induced Interruption and Retrial of Interrupted Customers. *American Journal of Mathematical and Management Sciences*, 34: 4, (2015), 343-366.
- [10] J. Punalal and S. Babu, A single server queueing model with selfgeneration of priorities, customer induced interruption and retrial of customers. *International Journal of Research in Advent Technology*, 7: 3, (2019), 1299--1303.