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Y-index of some graph operations

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Abstract
Topological indices have important role in theoretical chemistry. Among the all topological indices the Zagreb indices have been used more considerably than any other topological indices in chemical literature. In this study, the Y-index for some special graphs, and graph operations has been computed, that have been applied to compute the Y-index for Nano- tube and Nano-torus. Also the strong/good correlation coefficient between the Y-index and some physical and chemical properties as Acentric factor (Acenfac), Entropy (S),Enthalpy of vaporization (HVAP) and Standard enthalpy of vaporisation (DHVAP) have been Appeared.
Keywords: Zagreb indices, Hyper-Zagreb index, F-index, Y-index, graph operations.

1. INTRODUCTION

Chemical graph theory which is a fascinating branch of graph theory has many applications related to chemistry. A topological index which is a numerical quantity derived from the chemical graph of a molecule is used to modelling chemical and physical properties of molecules in QSPR/QSAR researches [4].

Throughout this paper, we consider a finite connected graph G that has no loops or multiple edges. The vertex and the edge sets of a graph G are denoted by $V(G)$ and $E(G)$, respectively. The degree of the vertex a is number of edges joined with this vertex denoted by $\delta(a)$. The distance between any two vertices a and b in $V(G)$ is denoted by $d(a,b)$ and it is defined as the number of edges in a shortest path, connecting the vertices a and b.
In practical applications, Zagreb Indices are among the best applications to recognize the physical properties, and chemical reactions. First Zagreb index $M_1(G)$, and Second Zagreb index $M_2(G)$ were firstly considered by I. Gutman and N.Trinajstic in 1972 [2, 6]. They are defined as:

$$M_1(G) = \sum_{v\in V(G)} {\delta_G}^2(v) = \sum_{uv\in E(G)} \delta_G(u)+\delta_G(v)$$

$$M_2(G) = \sum_{uv\in E(G)} \delta_G(u)\,\delta_G(v)$$

In 2005, Li and Zheng [1] introduced the first general Zagreb index as:

$$ {M_1}^{\alpha+1}(G) = \sum_{v\in V(G)} {\delta_G}^{\alpha+1}\delta_G(v) = \sum_{uv\in E(G)} {\delta_G}^{\alpha}(u)+{\delta_G}^{\alpha}(v)$$

In 2013, Shirdel et al [8, 9, 17]. introduced distance-based of Zagreb indices named Hyper-Zagreb index as:

$$HM(G) = \sum_{uv\in E(G)} (\delta_G(u)+\delta_G(v))^2$$

Furtula and Gutman in 2015 introduced forgotten index (F-index) [13, 15] which defined as:

$$F(G) = \sum_{uv\in E(G)} \left({\delta_G}^2(u)+{\delta_G}^2(v)\right)$$

In 2018, computed exact formulas for the Zagreb and Hyper-Zagreb indices of Some Molecular Graphs by S. Ghobadi and M. Ghorbaninejad. They defined a new distance-based of Zagreb indices named Forgotten topological index or hyper F-index defined as, [14]:

$$HF(G) = \sum_{uv\in E(G)} [{\delta_G}^2(u)+{\delta_G}^2(v)]^2$$

Also in 2018, Nilanjan De use modern index to calculate the F-index and coindex Of Some Derived Graphs [12], it's special of first general Zagreb index where $\alpha = 3$,

$${M_1}^4(G) = \sum_{uv\in E(G)} {\delta_G}^3(u)+{\delta_G}^3(v).$$

But this index for some special graphs or some graph operations and it's applications didn't study yet. Therefore, in this paper, we named this index 'Yemen index' or "Y-index". We will present some exact formulaes of the Y-index for some special graphs and some graph binary operations such as tensor product $G_1 \otimes G_2$, Cartesian product $G_1 \times G_2$, composition $G_1 \circ G_2$, strong product $G_1 * G_2$, disjunction $G_1 \vee G_2$, symmetric difference $G_1 \oplus G_2$, of graphs. We will compare some topological indices with the Y-index by using strong / good correlation coefficient acquired from the chemical graphs of octane isomers. Also we will apply some results to compute the Y-index for some classes of nano-structures such as nano-tube and nano-torus.

Definition 1.1: The Y-index of a graph G define as;

$$Y(G) = \sum_{u \in V(G)} \delta_G^4(u) = \sum_{uv \in E(G)} [\delta_G^3(u) + \delta_G^3(v)]$$

.

Lemma 1.2: Let G_1 and G_2 be graphs and $|V(G_i)| = p_i$: $i = 1, 2$.

Then [10, 16]

1. $\delta_{G_1 \otimes G_2}(u, v) = \delta_{G_1}(u)\delta_{G_2}(v)$
2. $\delta_{G_1 \times G_2}(u, v) = \delta_{G_1}(u) + \delta_{G_2}(v)$
3. $\delta_{G_1 \circ G_2}(u, v) = p_2\delta_{G_1}(u) + \delta_{G_2}(v)$
4. $\delta_{G_1 * G_2}(u, v) = \delta_{G_1}(u) + \delta_{G_2}(v) + \delta_{G_1}(u)\delta_{G_2}(v)$
5. $\delta_{G_1 \vee G_2}(u, v) = p_2\delta_{G_1}(u) + p_2\delta_{G_2}(v) - \delta_{G_1}(u)\delta_{G_2}(v)$
6. $\delta_{G_1 \oplus G_2}(u, v) = p_2\delta_{G_1}(u) + p_2\delta_{G_2}(v) - 2\delta_{G_1}(u)\delta_{G_2}(v)$

Any unexplained terminology is standard, typically as in [2, 3, 5, 7, 11].

2. BASIC PROPERTIES OF THE Y-INDEX

In this part, we give the Y-index of some special graphs as: complete graph K_n, cycle C_n, path P_n, complete bipartite graph $K_{m,n}$, and conical graph $C_{m,n}$ (cf. Fig. 1).

1. $Y(K_n) = n(n-1)^4$
2. $Y(C_n) = 16n$
3. $Y(P_n) = 16n - 30$
4. $Y(K_{m,n}) = mn(m^3 + n^3)$
5. $Y(C_{m,n}) = n(n^3 + 128m - 47)$

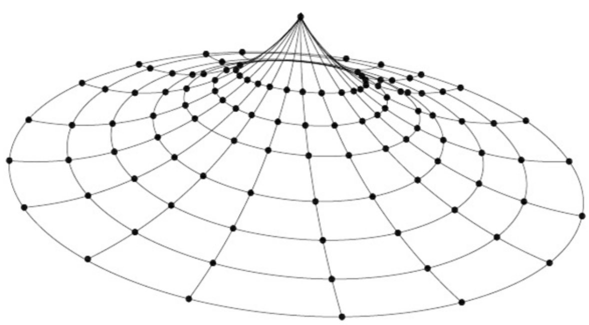

Figure 1: $C_{7,19}$

Example 2.1: Let G_1, G_2 be the four and six atoms of carbon from Chemical compounds: Methylcyclopropane and Methylcyclopentane respectively depicted in Figure 2. Thus,

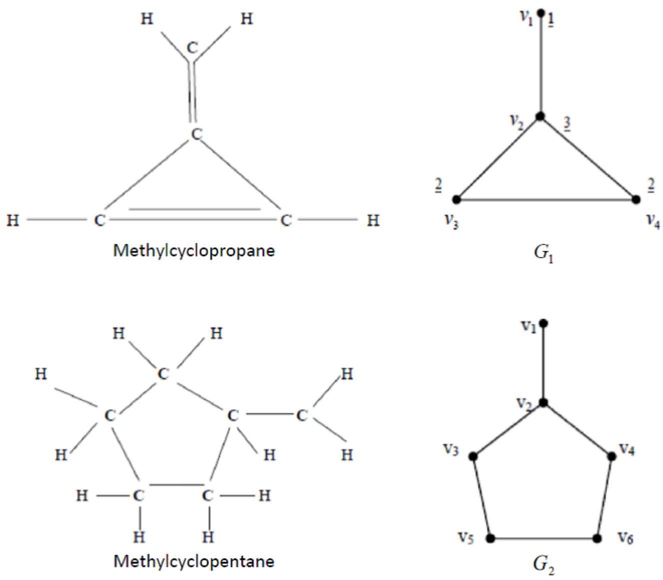

Figure 2

$$Y(G_1) = \sum_{v \in V(G_1)} \delta_{G_1}^4(v) = (1)^4 + (3)^4 + 2(2)^4 = 114,$$

$$Y(G_2) = \sum_{v \in V(G_2)} \delta_{G_2}^4(v) = (1)^4 + (3)^4 + 4(2)^4 = 146.$$

3. MOTIVATION

In this section we compare some topological indices with each other by using strong or good correlation coefficients acquired from the chemical graphs of octane isomers. The dataset of octane isomers (first six columns of Table 1) are taken from www.moleculardescriptors.eu and (the last four columns of Table 1) are computed from the definitions of $HM(G), F(G), HF(G)$ and $Y(G)$ respectively. The following physicochemical features have been modeled:

- Acentric factor (AcenFac)
- Entropy (S)
- Enthalpy of vaporization (HVAP))
- Standard enthalpy of vaporisation (DHVAP)

Table 1: Some physicochemical properties and topological indices of octane isomers

MolID	AcenFac	S	HVAP	DHVAP	M_1	M_2	F	HM	Y	HF
C_8-01	0.397898	111.67	73.19	9.915	26	24	50	98	98	370
C_8-02	0.377916	109.84	70.30	9.484	28	26	62	114	148	586
C_8-03	0.371002	111.26	71.3	9.521	28	27	62	116	148	616
C_8-04	0.371504	109.32	70.91	9.483	28	27	62	116	148	616
C_8-05	0.362472	109.43	71.7	9.476	28	28	62	118	148	646
C_8-06	0.339426	103.42	67.7	8.915	32	30	92	152	308	1372
C_8-07	0.348247	108.02	70.2	9.272	30	30	74	134	198	882
C_8-08	0.344223	106.98	68.5	9.029	30	29	74	132	198	832
C_8-09	0.35683	105.72	68.6	9.051	30	28	74	130	198	802
C_8-10	0.322596	104.74	68.5	8.973	32	32	92	156	308	1492
C_8-11	0.340345	106.59	70.2	9.316	30	31	74	136	198	912
C_8-12	0.332433	106.06	69.7	9.209	30	31	74	136	198	912
C_8-13	0.306899	101.48	69.3	9.081	32	34	92	160	308	1564
C_8-14	0.300816	101.31	67.3	8.826	34	35	104	174	358	1786
C_8-15	0.30537	104.09	64.87	8.402	34	32	104	168	358	1636
C_8-16	0.293177	102.06	68.1	8.897	34	36	104	176	358	1828
C_8-17	0.317422	102.39	68.37	9.014	32	33	86	152	248	1037
C_8-18	0.255294	93.06	66.2	8.41	38	40	134	214	518	2758

In Table 2. We find that the correlation coefficient between the Y-index and some topological indices, then the correlation coefficient between the Y-index $Y(G)$ and first Zagreb index $M_1(G)$, F-index $F(G)$, Hyper Zagreb index $HM_1(G)$, and Hyper F-index $HF(G)$ are highly correlated ($r \geqslant 0.95$), and then the correlation coefficient between Y-index and the second Zagreb index $M_2(G)$ is good correlated ($r \geqslant 0.90$).

Table 2: The correlation coefficients between Y-index and some topological indices of some physicochemical properties of octane isomers

r	M_1	M_2	F	HM	Y	HF
$Y - index$	0.98620	0.92469	0.99669	0.98899	1.00000	0.99353

In Table 3. We select those physicochemical properties of octane isomers for which give reasonably highly or good correlations, i.e., Table 3. is shown that the Y-index is highly correlated with the Acentric factor (AcenFac) ($|r| = 0.950848503$) and also with the entropy ($|r| = 0.944776767$) of octane isomers, and it is shown that the Y-index is good correlated with the Enthalpy of vaporization (HVAP) ($|r| = 0.851856551$) and also with the Standard enthalpy of vaporisation (DHVAP) ($|r| = 0.906174441$) of octane isomers. we can say that the Y-index is possible tools for QSPR researches.

Table 3: The correlation coefficients between topological indices and some physicochemical properties of octane isomers

Index	AcenFac	S	HVAP	DHVAP
M_1	-0.97306	-0.95429	-0.88602	-0.93614
M_2	-0.98642	-0.94169	-0.72815	-0.81182
F	-0.96505	-0.95272	-0.87157	-0.92402
HM_1	-0.98291	-0.96143	-0.84248	-0.90425
Y	-0.95085	-0.94478	-0.85186	-0.90617
HF	-0.9514	-0.94259	-0.86357	-0.86357

4. MAIN RESULTS

In the following section, we study the Y-index of some graph operations.

Theorem 4.1: Y-index of $(G_1 \otimes G_2)$ is given by:

$$Y(G_1 \otimes G_2) = Y(G_1)Y(G_2)$$

Proof. By definition of Y-index and Lemma 1.2, we have

$$\begin{aligned} Y(G_1 \otimes G_2) &= \sum_{(u,v)\in V(G_1\otimes G_2)} [\delta_{(G_1\otimes G_2)}(u,v)]^4 = \sum_{u\in V(G_1)} \sum_{v\in V(G_2)} [\delta_{G_1}(u)\delta_{G_2}(v)]^4 \\ &= \sum_{u\in V(G_1)} [\delta_{G_1}(u)]^4 \sum_{v\in V(G_2)} [\delta_{G_2}(v)]^4 = Y(G_1)Y(G_2). \quad \square \end{aligned}$$

Theorem 4.2: Y-index of $(G_1 \times G_2)$ is given by:

$$Y(G_1 \times G_2) = p_2Y(G_1) + p_1Y(G_2) + 8q_1F(G_2) + 8q_2F(G_1) + 6M_1(G_1)M_1(G_2)$$

Proof. By definition of Y-index and Lemma 1.2, we have

$$Y(G_1 \times G_2) = \sum_{(u,v)\in V(G_1\times G_2)} [\delta_{(G_1\times G_2)}(u,v)]^4 = \sum_{u\in V(G_1)} \sum_{v\in V(G_2)} [\delta_{G_1}(u) + \delta_{G_2}(v)]^4$$

$$\begin{aligned} &= \sum_{(u,v)\in V(G_1\times G_2)} [\delta^4_{G_1}(u) + \delta^4_{G_2}(v) + 4\delta^3_{G_1}(u)\delta_{G_2}(v) + 4\delta_{G_1}(u)\delta^3_{G_2}(v) + 6\delta^2_{G_1}(u)\delta^2_{G_2}(v)] \\ &= \sum_{u\in V(G_1)} [\delta_{G_1}(u)]^4 \sum_{v\in V(G_2)} [1] + \sum_{u\in V(G_1)} [1] \sum_{V\in V(G_2)} [\delta_{G_2}(V)]^4 + 4 \sum_{u\in V(G_1)} [\delta_{G_1}(u)]^3 \sum_{v\in V(G_2)} \delta_{G_2}(V) \\ &+ 4 \sum_{u\in V(G_1)} \delta_{G_1}(u) \sum_{v\in V(G_2)} [\delta_{G_2}(V)]^3 + 6 \sum_{u\in V(G_1)} [\delta_{G_1}(u)]^2 \sum_{v\in V(G_2)} [\delta_{G_2}(V)]^2 \\ &= p_2Y(G_1) + p_1Y(G_2) + 8q_1F(G_2) + 8q_2F(G_1) + 6M_1(G_1)M_1(G_2). \quad \square \end{aligned}$$

Theorem 4.3: Y-index of $(G_1 \circ G_2)$ is given by:

$$Y(G_1 \circ G_2) = p_2^5Y(G_1) + p_1Y(G_2) + 8p_2^3q_2F(G_1) + 8p_2q_1F(G_2) + 6p_2^2M_1(G_1)M_1(G_2)$$

Proof. By definition of Y-index and Lemma 1.2, we have

$$Y(G_1 \circ G_2) = \sum_{(u,v)\in V(G_1\circ G_2)} [\delta_{(G_1\circ G_2)}(u,v)]^4 = \sum_{u\in V(G_1)} \sum_{v\in V(G_2)} [p_2\delta_{G_1}(u) + \delta_{G_2}(v)]^4$$

$$\begin{aligned} &= \sum_{(u,v)\in V(G_1\circ G_2)} [p_2^4\delta^4_{G_1}(u) + \delta^4_{G_2}(v) + 6p_2^2\delta^2_{G_1}(u)\delta^2_{G_2}(v) + 4p_2^3\delta^3_{G_1}(u)\delta_{G_2}(v) + 4p_2\delta_{G_1}(u)\delta^3_{G_2}(v)] \\ &= p_2^5Y(G_1) + p_1Y(G_2) + 8p_2^3q_2F(G_1) + 8p_2q_1F(G_2) + 6p_2^2M_1(G_1)M_1(G_2). \quad \square \end{aligned}$$

Theorem 4.4: Y-index of $(G_1 * G_2)$ is given by:

$$\begin{aligned} Y(G_1 * G_2) &= Y(G_1)[4F(G_2) + 6M_1(G_2) + 8q_2 + p_2] + 4F(G_1)[3M_1(G_2) + 2q_2] \\ &+ Y(G_2)[4F(G_1) + 6M_1(G_1) + 8q_1 + p_1] + 4F(G_2)[3M_1(G_1) + 2q_1] \\ &+ Y(G_1)Y(G_2) + 12F(G_1)F(G_2) + 6M_1(G_1)M_1(G_2). \end{aligned}$$

Proof. By definition of Y-index and Lemma 1.2, we have

$$Y(G_1 * G_2) = \sum_{(u,v)\in V(G_1*G_2)} [\delta_{(G_1*G_2)}(u,v)]^4 = \sum_{u\in V(G_1)} \sum_{v\in V(G_2)} [\delta_{G_1}(u) + \delta_{G_2}(v) + \delta_{G_1}(u)\delta_{G_2}(v)]^4$$

$$\begin{aligned}
&= \sum_{u\in V(G_1)} \sum_{v\in V(G_2)} [\delta^4_{G_1}(u) + \delta^4_{G_2}(v) + \delta^4_{G_1}(u)\delta^4_{G_2}(v) + 6\delta^2_{G_1}(u)\delta^2_{G_2}(v) + 12\delta^3_{G_1}(u)\delta^3_{G_2}(v)\\
&+ 6\delta^2_{G_1}(u)\delta^4_{G_2}(v) + 6\delta^4_{G_1}(u)\delta^2_{G_2}(v) + 12\delta^2_{G_1}(u)\delta^3_{G_2}(v) + 12\delta^3_{G_1}(u)\delta^2_{G_2}(v) + 4\delta^4_{G_1}(u)\delta^3_{G_2}(v)\\
&+ 4\delta^3_{G_1}(u)\delta^4_{G_2}(v) + 4\delta^3_{G_1}(u)\delta_{G_2}(v) + 4\delta_{G_1}(u)\delta^3_{G_2}(v) + 4\delta_{G_1}(u)\delta^4_{G_2}(v) + 4\delta^4_{G_1}(u)\delta_{G_2}(v)]\\
&= p_2Y(G_1) + p_1Y(G_2 + Y(G_1)Y(G_2) + 12F(G_1)F(G_2) + 6Y(G_1)M_1(G_2)\\
&+ 6M_1(G_1)Y(G_2) + 6M_1(G_2)Y(G_1) + 12M_1(G_1)F(G_2) + 12M_1(G_2)F(G_1) + 4Y(G_1)F(G_2)\\
&+ 4Y(G_2)F(G_1) + 4F(G_1)(2q_2) + 4F(G_2)(2q_1) + 4Y(G_1)(2q_2) + 4Y(G_2)(2q_1)\\
&= Y(G_1)[4F(G_2) + 6M_1(G_2) + 8q_2 + p_2] + 4F(G_1)[3M_1(G_2) + 2q_2]\\
&+ Y(G_2)[4F(G_1) + 6M_1(G_1) + 8q_1 + p_1] + 4F(G_2)[3M_1(G_1) + 2q_1]\\
&+ Y(G_1)Y(G_2) + 12F(G_1)F(G_2) + 6M_1(G_1)M_1(G_2). \quad \square
\end{aligned}$$

Theorem 4.5: Y-index of $(G_1 \vee G_2)$ is given by:

$$\begin{aligned}
Y(G_1 \vee G_2) &= p_1Y(G_2)[p_1^4 + 6p_1M_1(G_1) - 8p_1^2q_1 - 4F(G_1)] + 4p_1^2p_2F(G_2)[2p_1q_1 - 3M_1(G_1)]\\
&+ p_2Y(G_1)[p_2^4 + 6p_2M_1(G_2) - 8p_2^2q_2 - 4F(G_2)] + 4p_2^2p_1F(G_1)[2p_2q_2 - 3M_1(G_2)]\\
&+ Y(G_1)Y(G_2) + 12p_1p_2F(G_1)F(G_2) + 6p_1^2p_2^2M_1(G_1)M_1(G_2).
\end{aligned}$$

Proof. By definition of Y-index and Lemma 1.2, we have

$$Y(G_1 \vee G_2) = \sum_{(u,v)\in V(G_1\vee G_2)} [\delta_{(G_1\vee G_2)}(u,v)]^4 = \sum_{u\in V(G_1)} \sum_{v\in V(G_2)} [p_2\delta_{G_1}(u) + p_1\delta_{G_2}(v) - \delta_{G_1}(u)\delta_{G_2}(v)]^4$$

$$\begin{aligned}
&= \sum_{(u,v)\in V(G_1\vee G_2)} [p_1^4\delta^4_{G_2}(v) + p_2^4\delta^4_{G_1}(u) + 6p_1^2p_2^2\delta^2_{G_1}(u)\delta^2_{G_2}(v) + 12p_1p_2\delta^3_{G_1}(u)\delta^3_{G_2}(v)\\
&+ 6p_1^2\delta^2_{G_1}(u)\delta^4_{G_2}(v) + 6p_2^2\delta^4_{G_1}(u)\delta^2_{G_2}(v) + 4p_1^3p_2\delta_{G_1}(u)\delta^3_{G_2}(v) + 4p_1p_2^3\delta^3_{G_1}(u)\delta_{G_2}(v)\\
&- 4p_1^3\delta_{G_1}(u)\delta^4_{G_2}(v) - 4p_2^3\delta^4_{G_1}(u)\delta_{G_2}(v) - 12p_1^2p_2\delta^2_{G_1}(u)\delta^3_{G_2}(v) - 12p_1p_2^2\delta^3_{G_1}(u)\delta^2_{G_2}(v)\\
&+ \delta^4_{G_1}(u)\delta^4_{G_2}(v) - 4p_1\delta^3_{G_1}(u)\delta^4_{G_2}(v) - 4p_2\delta^4_{G_1}(u)\delta^3_{G_2}(v)]\\
&= p_1^5Y(G_2) + p_2^5Y(G_1) + 6p_1^2p_2^2M_1(G_1)M_1(G_2) + 12p_1p_2F(G_1)F(G_2)\\
&+ 6p_1^2M_1(G_1)Y(G_2) + 6p_2^2M_1(G_2)Y(G_1) + 4p_1^3p_2(2q_1)F(G_2) + 4p_1p_2^3(2q_2)F(G_1)\\
&- 4p_1^3(2q_1)Y(G_2) - 4p_2^3(2q_2)Y(G_1) - 12p_1^2p_2M_1(G_1)F(G_2) - 12p_1p_2^2M_1(G_2)F(G_1)\\
&+ Y(G_1)Y(G_2) - 4p_1F(G_1)Y(G_2) - 4p_2F(G_2)Y(G_1)\\
&= p_1Y(G_2)[p_1^4 + 6p_1M_1(G_1) - 8p_1^2q_1 - 4F(G_1)] + 4p_1^2p_2F(G_2)[2p_1q_1 - 3M_1(G_1)]\\
&+ p_2Y(G_1)[p_2^4 + 6p_2M_1(G_2) - 8p_2^2q_2 - 4F(G_2)] + 4p_2^2p_1F(G_1)[2p_2q_2 - 3M_1(G_2)]\\
&+ Y(G_1)Y(G_2) + 12p_1p_2F(G_1)F(G_2) + 6p_1^2p_2^2M_1(G_1)M_1(G_2). \quad \square
\end{aligned}$$

Theorem 4.6: Y-index of $(G_1 \oplus G_2)$ is given by:

$$\begin{aligned}
Y(G_1 \oplus G_2) &= p_1Y(G_2)[p_1^4 + 24p_1M_1(G_1) - 16p_1^2q_1 - 32F(G_1)] + 8p_1^2p_2F(G_2)[p_1q_1 - 3M_1(G_1)]\\
&+ p_2Y(G_1)[p_2^4 + 24p_2M_1(G_2) - 16p_2^2q_2 - 32F(G_2)] + 8p_2^2p_1F(G_1)[p_2q_2 - 3M_1(G_2)]\\
&+ 16Y(G_1)Y(G_2) + 48p_1p_2F(G_1)F(G_2) + 6p_1^2p_2^2M_1(G_1)M_1(G_2).
\end{aligned}$$

Proof. By using a similar method, one can prove the exact formula for the Y-index of symmetric difference of graphs.

Example 4.7: Khalifeh, M. H., et al. [18] computed the Y-index of C4 nanotubes and nanotori. In this example, we compute these molecular graphs. Suppose R and S denote a C_4 nanotube and nanotorus, respectively. Then $R = P_n \times C_m$ and $S = C_n \times C_m$. by Theorem 4.2:

$$Y(R) = Y(P_n \times C_m) = 224nm + 16n - 304m - 30$$

where

$$G_1 \equiv P_n, \quad p_1 = n, q_1 = n-1, M_1(P_n) = 4n-6, F(P_n) = 8n-14, Y(P_n) = 16n-30,$$
$$G_1 \equiv C_m, \quad p_2 = q_2 = m, M_1(C_m) = 4m, F(C_m) = 8m, Y(C_m) = 16m$$

Similar we have

$$Y(S) = Y(C_n \times C_m) = 192nm + 16n + 16m$$

Example4.8: Let P_2, P_3 denote a paths with 2 and 3 vertices, respectively. By theorem 4.1-6. we have The Y-index of some graph operations of P_2, P_3 in table 4.

Table 4: The Y-index of some graph operations

G	$P_2 \otimes P_3$	$P_2 \times P_3$	$P_2 \circ P_3$	$P_2 * P_3$	$P_2 \oplus P_3$	$P_2 \vee P_3$
$Y(G)$	96	226	2274	1574	486	2274

5. CONCLUDING REMARKS

In this paper, a new index was introduced from the Zagreb index family, named that Y-index. it has investigated the basic mathematical properties of the Y-index and obtained explicit formula for their values under several graph operations. The strong correlation coefficient between Y-index and some physico-chemical properties as Acentric factor has been Appeared. Here we mention some possible directions for future research as multiplicative Y-index.

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