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EOQ Model for both Ameliorating and Deteriorating Items with Cubic Demand and Shortages

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Abstract

It is widely observed that the items like fruits, flowers, green vegetables, dairy products etc are either kept in farms, in flower shops, in supermarkets or in cold storages. The demand of such items is very high and at the same time it is also decreased owing to spoilage or decay. So we cannot ignore the effect of amelioration and deterioration in the inventory management system. Here we developed an economic order quantity (EOQ) inventory model for both ameliorating and deteriorating items. The assumption of constant demand rate may not be always appropriate for many inventory goods like milk, vegetables etc., the age of inventory has negative impact on demand due to loss of consumer confidence on quality of such products. Here demand rate is considered as a cubic function of time and shortages are allowed which are fully backlogged. Finally the model is illustrated with the help of a numerical example, some particular cases are derived and a comparative study of the optimal solutions is furnished to illustrate different aspects of the inventory models.

Keywords: Inventory, EOQ, ameliorating deteriorating,, cubic demand and shortages.

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1. INTRODUCTION

It is natural that goods like fruits, flowers, green vegetables, dairy products, radioactive substances etc deteriorate over time. Normally goods deteriorate during storage period. Several researchers have addressed the importance of the deterioration phenomenon in their field of applications; as a result, many inventory models with deteriorating items have been developed. But due to lack of considering the influence of demand, the ameliorating items for the amount of inventory is increasing gradually. Amelioration is a natural phenomenon observing in much life stock models. A few researchers have focused on ameliorating system. Professionals did not give much attention for fast growing animals like broiler, ducks, pigs etc. in the poultry farm, highbred fishes in berry (pond) which are known as ameliorating items. When these items are in storage, the stock increases (in weight) due to growth of the items and also decrease due to death, various diseases or some other factors. At the point when these things are away, the stock increases (in weight) because of development of the things. Furthermore the stock diminishes because of death, different illnesses or due to some different components. Hwang [1997] developed an inventory model for ameliorating items only. Again Hwang [2004] added to a stock model for both ameliorating and deteriorating things independently. Mallick et al. [2018] has considered a creation inventory model for both ameliorating and deteriorating items. Many researchers like Moon et al[2005], Law et al [2006], L-Q ji[2008], Valliathal et al[2010], Chen [2011], Nodoust [2017] are few noteworthy. In this paper, effort is given to discuss on an economic order quantity (EOQ) inventory model for both ameliorating and deteriorating items where the environment of Amelioration followed by Weibull Distribution to describe the different life spans effectively by utilizing the changes of parameters.

Biswaranjan-Mandal [2010] analyzed the EOQ inventory model for Weibull distributed deteriorating items under ramp type demand and shortages. Sahoo et al. [2010] formulated an EOQ model for price dependent demand rate and time-varying holding cost. Hung [2011] have used generalized type demand, deterioration and backorder rates. Mishra and Singh [2011] find an inventory model for ramp type demand, and time dependent deteriorating items with salvage value and shortages. According to Mishra and Singh [2011] an inventory model for deteriorating items with uniform replenishment rate with power form demand, the rate of deterioration is cubic polynomial. Anil Kumar Sharma et.al. [2012] are considered an inventory model with time dependent holding cost. Babu Krishnaraj and Ramasamy [2012] find an inventory model with power demand pattern for Weibull deterioration rate without shortages. Mukesh kumar et al. [2012] are considered a deterministic inventory model for deteriorating items with price dependent used demand rate and time-varying holding cost under trade credit. Tripathy and Pradhan [2012] examined is used salvage value and developed an inventory model for three parameter Weibull distribution deterioration rate under permissible delay in payments. Amutha and Chandrasekaran [2013] studied on deteriorating items with price dependent demand, three parameter Weibull distribution deterioration rate. Pratibha Yadav [2013] have used cubic demand rate and production rate is variable with Weibull distribution. Sharma et al [2015], Biswaranjan Mandal [2020] and many others developed inventory models assuming demand rate as cubic function of time.

In this paper we consider an inventory model for deteriorating products having cubic demand with respect to time, infinite time horizon and fully backlogged shortages. Here the rate of deterioration is assumed as constant and amelioration rate followed by Weibull distribution. Finally the model is illustrated with the help of a numerical example, some particular cases are derived and a comparative study of the optimal solutions is furnished to illustrate different aspects of the inventory models.

2. NOTATIONS AND ASSUMPTIONS

The present inventory model is developed under the following notations and assumptions:

Notations:

(i) I(t) : On hand inventory at time t.

(ii) R(t) : Demand rate.

(iii) Q : On-hand inventory.

(iv) θ : The constant deterioration rate where $0 \le \theta < 1$

(v) A(t) : The ameliorating rate at time t.

(vi) T : The fixed length of each production cycle.

(vii) A_0 : The ordering cost per order during the cycle period.

(viii) p_c : The purchasing cost per unit item.

(ix) h_c : The holding cost per unit item.

(x) d_c : The deterioration cost per unit item.

(xi) a_c : The cost of amelioration per unit item.

(xii) c_s : The shortage cost per unit item.

(xiii) TC : Average total cost per unit time.

Assumptions:

(i) Lead time is zero.

(ii) Replenishment rate is infinite but size is finite.

(iii) The time horizon is finite.

(iv) There is no repair of deteriorated items occurring during the cycle.

(v) Amelioration and deterioration occur when the item is effectively in stock.

(vi) A(t) is the amelioration rate following Weibull distributed $A(t) = \alpha\beta t^{\beta-1}, 0 \le \alpha << 1, \beta \ge 1$, where α is the shape parameter and β is the scale parameter.

(vii) The demand rate is a time dependent cubic function

R(t) = a + bt + ct^2 + dt^3, a, b, c, d $\ge$ 0

where a is the initial demand rate, b is the initial rate of change of demand, c is the rate at which the demand rate increases and d is the rate at which the change in the demand rate itself increases.

(viii) Shortages are allowed and fully backlogged.

3. MATHEMATICAL FORMULATION AND SOLUTION

In this model, we consider an EOQ model starting with no shortage. Replenishment occurs at time t=0 and the inventory level attains its maximum. From t = 0 to t = t_1 the stock will be diminished due to the effect of amelioration, deterioration and demand, and ultimately falls to zero at t = t_1. The shortages occur during time period [t_1, T] which are fully backlogged. The behaviour of the model at any time t can be described by the following differential equations:

$$\frac{dI(t)}{dt} + \left(\theta - \alpha\beta t^{\beta-1}\right) I(t) = -(a + bt + ct^2 + dt^3), 0 \le t \le t_1 \tag{3.1}$$

And $$\frac{dI(t)}{dt} = -(a+bt+ct^2+dt^3), t_1 \le t \le T \tag{3.2}$$

The initial condition is $I(0)=Q$ and $I(t_1)=0$ (3.3)

The solutions of the equations (3.1) and (3.2) using (3.3) and neglecting second and higher order powers of $\theta(<1)$ and $\alpha(<1)$ (as $O(\theta^2)$, $O(\alpha^2)$ are very small) are given by the following

$$\text{I(t)} = Q(1-\theta t+\alpha t^{\beta}) - \{at + (\frac{b}{2}-\frac{a\theta}{2})t^2 + (\frac{c}{3}-\frac{b\theta}{6})t^3 + (\frac{d}{4}-\frac{c\theta}{12})t^4 - \frac{d\theta}{20}t^5$$
$$+ \frac{a\alpha\beta}{\beta+1}t^{\beta+1} + \frac{b\alpha\beta}{2(\beta+2)}t^{\beta+2} + \frac{c\alpha\beta}{3(\beta+3)}t^{\beta+3} + \frac{d\alpha\beta}{4(\beta+4)}t^{\beta+4}\}, 0 \le t \le t_1 \tag{3.4}$$

And $$\text{I(t)} = a(t_1-t) + \frac{b}{2}(t_1^{\,2}-t^2) + \frac{c}{3}(t_1^{\,3}-t^3) + \frac{d}{4}(t_1^{\,4}-t^4), t_1 \le t \le T \tag{3.5}$$

Since $I(t_1)=0$, we get the following expression of on-hand inventory from the equation (3.4)

$$Q = at_1 + (\frac{b}{2}+\frac{a\theta}{2})t_1^{\,2} + (\frac{c}{3}+\frac{b\theta}{3})t_1^{\,3} + (\frac{d}{4}+\frac{c\theta}{4})t_1^{\,4} + \frac{d\theta}{5}t_1^{\,5}$$
$$-\frac{a\alpha}{\beta+1}t_1^{\,\beta+1} - \frac{b\alpha}{\beta+2}t^{\beta+2} - \frac{c\alpha}{\beta+3}t_1^{\,\beta+3} - \frac{d\alpha}{\beta+4}t_1^{\,\beta+4} \tag{3.6}$$

The total inventory holding during the time interval $[0, t_1]$ is given by

$$I_T = \int_0^{t_1} I(t)dt$$

Putting the value of I(t) from (3.4) and then integrating and neglecting second and higher order powers of $\theta(<1)$ and $\alpha(<1)$ (as $O(\theta^2)$, $O(\alpha^2)$ are very small), we get

$$I_T = \frac{a}{2}t_1^{\,2} + (\frac{b}{3}+\frac{a\theta}{6})t_1^{\,3} + (\frac{c}{4}+\frac{b\theta}{8})t_1^{\,4} + (\frac{d}{5}+\frac{c\theta}{10})t_1^{\,5} + \frac{d\theta}{12}t_1^{\,6}$$
$$-\frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\,\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\,\beta+3} - \frac{c\alpha\beta}{(\beta+1)(\beta+4)}t_1^{\,\beta+4} - \frac{d\alpha\beta}{(\beta+1)(\beta+5)}t_1^{\,\beta+5} \tag{3.7}$$

The total number of deteriorated units during the inventory cycle is given by

$$D_T = \theta\int_0^{t_1} I(t)dt$$

$$= \theta\{\frac{a}{2}t_1^{\,2} + (\frac{b}{3}+\frac{a\theta}{6})t_1^{\,3} + (\frac{c}{4}+\frac{b\theta}{8})t_1^{\,4} + (\frac{d}{5}+\frac{c\theta}{10})t_1^{\,5} + \frac{d\theta}{12}t_1^{\,6}$$
$$-\frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\,\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\,\beta+3} - \frac{c\alpha\beta}{(\beta+1)(\beta+4)}t_1^{\,\beta+4} - \frac{d\alpha\beta}{(\beta+1)(\beta+5)}t_1^{\,\beta+5}\} \tag{3.8}$$

The total number of ameliorating units during the inventory cycle is given by

$$A_T = \int_0^{t_1} \alpha\beta t^{\beta-1} I(t)dt = \frac{a\alpha}{\beta+1}t_1^{\beta+1} + \frac{b\alpha}{\beta+2}t^{\beta+2} + \frac{c\alpha}{\beta+3}t_1^{\beta+3} + \frac{d\alpha}{\beta+4}t_1^{\beta+4} \quad (3.9)$$

The total number of shortages during the period [t_1, T] is given by

$$S_T = \int_{t_1}^{T} (T-t)R(t)dt = \int_{t_1}^{T} (T-t)(a+bt+ct^2+dt^3)dt$$

$$= \{\frac{a}{2}(T^2 - 2Tt_1 + t_1^2) + \frac{b}{6}(T^3 - 3Tt_1^2 + 2t_1^3) + \frac{c}{12}(T^4 - 4Tt_1^3 + 3t_1^4) + \frac{d}{20}(T^5 - 5Tt_1^4 + 4t_1^5)\} \quad (3.10)$$

Cost Components:

The total cost over the period [0, T] consists of the following cost components:

(i). Ordering cost **(OC)** over the period [0,T]= A_0 (fixed)

(ii). Purchasing cost **(PC)** over the period [0,T] **=** p_c I(0) = p_c Q

$$= p_c [at_1 + (\frac{b}{2} + \frac{a\theta}{2})t_1^2 + (\frac{c}{3} + \frac{b\theta}{3})t_1^3 + (\frac{d}{4} + \frac{c\theta}{4})t_1^4 + \frac{d\theta}{5}t_1^5$$

$$- \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{b\alpha}{\beta+2}t_1^{\beta+2} - \frac{c\alpha}{\beta+3}t_1^{\beta+3} - \frac{d\alpha}{\beta+4}t_1^{\beta+4}]$$

(iii). Holding cost for carrying inventory **(HC)** over the period [0,T] = $h_c I_T$

$$= h_c[\frac{a}{2}t_1^2 + (\frac{b}{3} + \frac{a\theta}{6})t_1^3 + (\frac{c}{4} + \frac{b\theta}{8})t_1^4 + (\frac{d}{5} + \frac{c\theta}{10})t_1^5 + \frac{d\theta}{12}t_1^6$$

$$- \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3} - \frac{c\alpha\beta}{(\beta+1)(\beta+4)}t_1^{\beta+4} - \frac{d\alpha\beta}{(\beta+1)(\beta+5)}t_1^{\beta+5}]$$

(iv). Cost due to deterioration **(CD)** over the period [0, T]= $d_c D_T$

$$= d_c\theta[\frac{a}{2}t_1^2 + (\frac{b}{3} + \frac{a\theta}{6})t_1^3 + (\frac{c}{4} + \frac{b\theta}{8})t_1^4 + (\frac{d}{5} + \frac{c\theta}{10})t_1^5 + \frac{d\theta}{12}t_1^6$$

$$- \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3} - \frac{c\alpha\beta}{(\beta+1)(\beta+4)}t_1^{\beta+4} - \frac{d\alpha\beta}{(\beta+1)(\beta+5)}t_1^{\beta+5}]$$

(v).The amelioration cost **(AMC)** over the period [0,T]= $a_c A_T$

$$= a_c[\frac{a\alpha}{\beta+1}t_1^{\beta+1} + \frac{b\alpha}{\beta+2}t^{\beta+2} + \frac{c\alpha}{\beta+3}t_1^{\beta+3} + \frac{d\alpha}{\beta+4}t_1^{\beta+4}]$$

(vi). Cost due to shortage **(CS)** over the period [0,T]= $c_s S_T$

$$= c_s[\frac{a}{2}(T^2 - 2Tt_1 + t_1^2) + \frac{b}{6}(T^3 - 3Tt_1^2 + 2t_1^3) + \frac{c}{12}(T^4 - 4Tt_1^3 + 3t_1^4) + \frac{d}{20}(T^5 - 5Tt_1^4 + 4t_1^5)]$$

The average total cost per unit time of the system during the cycle [0,T] will be

TC(t_1) = $\frac{1}{T}$ **[OC +PC+HC+CD+AMC+CS]**

$$= \frac{1}{T}[A_0 + p_c[at_1 + (\frac{b}{2} + \frac{a\theta}{2})t_1^2 + (\frac{c}{3} + \frac{b\theta}{3})t_1^3 + (\frac{d}{4} + \frac{c\theta}{4})t_1^4 + \frac{d\theta}{5}t_1^5$$

$$-\frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{b\alpha}{\beta+2}t^{\beta+2} - \frac{c\alpha}{\beta+3}t_1^{\beta+3} - \frac{d\alpha}{\beta+4}t_1^{\beta+4}]$$

$$+(h_c + \theta d_c)[\frac{a}{2}t_1^2 + (\frac{b}{3} + \frac{a\theta}{6})t_1^3 + (\frac{c}{4} + \frac{b\theta}{8})t_1^4 + (\frac{d}{5} + \frac{c\theta}{10})t_1^5 + \frac{d\theta}{12}t_1^6$$

$$-\frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3} - \frac{c\alpha\beta}{(\beta+1)(\beta+4)}t_1^{\beta+4} - \frac{d\alpha\beta}{(\beta+1)(\beta+5)}t_1^{\beta+5}]$$

$$+ a_c[\frac{a\alpha}{\beta+1}t_1^{\beta+1} + \frac{b\alpha}{\beta+2}t^{\beta+2} + \frac{c\alpha}{\beta+3}t_1^{\beta+3} + \frac{d\alpha}{\beta+4}t_1^{\beta+4}] + c_s[\frac{a}{2}(T^2 - 2Tt_1 + t_1^2) + \frac{b}{6}(T^3 - 3Tt_1^2 + 2t_1^3)$$

$$+\frac{c}{12}(T^4 - 4Tt_1^3 + 3t_1^4) + \frac{d}{20}(T^5 - 5Tt_1^4 + 4t_1^5)]] \tag{3.11}$$

For minimum, the necessary condition is $\frac{dTC(t_1)}{dt_1} = 0$

This gives

$$p_c[a + (b + a\theta)t_1 + (c + b\theta)t_1^2 + (d + c\theta)t_1^3 + d\theta t_1^4 - a\alpha t_1^{\beta} - b\alpha t_1^{\beta+1} - c\alpha t_1^{\beta+2} - d\alpha t_1^{\beta+3}]$$

$$+(h_c + \theta d_c)[at_1 + (b + \frac{a\theta}{2})t_1^2 + (c + \frac{b\theta}{2})t_1^3 + (d + \frac{c\theta}{2})t_1^4 + \frac{d\theta}{2}t_1^5$$

$$-\frac{a\alpha\beta}{\beta+1}t_1^{\beta+1} - \frac{b\alpha\beta}{\beta+1}t_1^{\beta+2} - \frac{c\alpha\beta}{\beta+1}t_1^{\beta+3} - \frac{d\alpha\beta}{\beta+1}t_1^{\beta+4}] + a_c[a\alpha t_1^{\beta} + b\alpha t^{\beta+1} + c\alpha t_1^{\beta+2} + d\alpha t_1^{\beta+3}]$$

$$+c_s[a(t_1 - T) + bt_1(t_1 - T) + ct_1^2(t_1 - T) + dt_1^3(t_1 - T)] = 0 \tag{3.12}$$

For minimum the sufficient condition $\frac{d^2TC(t_1)}{dt_1^2}$ >0 would be satisfied.

Let $t_1 = t_1^*$ be the optimum value of t_1.

The optimal values Q^* of Q and TC^* of TC are obtained by putting the value $t_1 = t_1^*$ from the expressions (3.6) and (3.11).

4. SOME PARTICULAR CASES

(a). Absence of deterioration :

If the deterioration of items is ignored i.e. $\theta = 0$, then the expressions (3.6) and (3.11) of on-hand inventory (Q) and average total cost per unit time (TC (t_1)) during the period [0,T] become

$$Q = at_1 + \frac{b}{2}t_1^2 + \frac{c}{3}t_1^3 + \frac{d}{4}t_1^4 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{b\alpha}{\beta+2}t^{\beta+2} - \frac{c\alpha}{\beta+3}t_1^{\beta+3} - \frac{d\alpha}{\beta+4}t_1^{\beta+4} \qquad (4.1)$$

$$\text{TC}(t_1) = \frac{1}{T}[A_0 + p_c[at_1 + \frac{b}{2}t_1^2 + \frac{c}{3}t_1^3 + \frac{d}{4}t_1^4 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{b\alpha}{\beta+2}t^{\beta+2} - \frac{c\alpha}{\beta+3}t_1^{\beta+3} - \frac{d\alpha}{\beta+4}t_1^{\beta+4}]$$

$$+h_c[\frac{a}{2}t_1^2 + \frac{b}{3}t_1^3 + \frac{c}{4}t_1^4 + \frac{d}{5}t_1^5 - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3}$$

$$-\frac{c\alpha\beta}{(\beta+1)(\beta+4)}t_1^{\beta+4} - \frac{d\alpha\beta}{(\beta+1)(\beta+5)}t_1^{\beta+5}] + a_c[\frac{a\alpha}{\beta+1}t_1^{\beta+1} + \frac{b\alpha}{\beta+2}t^{\beta+2} + \frac{c\alpha}{\beta+3}t_1^{\beta+3} + \frac{d\alpha}{\beta+4}t_1^{\beta+4}]$$

$$+ c_s[\frac{a}{2}(T^2 - 2Tt_1 + t_1^2) + \frac{b}{6}(T^3 - 3Tt_1^2 + 2t_1^3) + \frac{c}{12}(T^4 - 4Tt_1^3 + 3t_1^4) + \frac{d}{20}(T^5 - 5Tt_1^4 + 4t_1^5)]] \qquad (4.2)$$

The equation (3.12) becomes

$$p_c[a + bt_1 + ct_1^2 + dt_1^3 - a\alpha t_1^{\beta} - b\alpha t_1^{\beta+1} - c\alpha t_1^{\beta+2} - d\alpha t_1^{\beta+3}]$$

$$+h_c[at_1 + bt_1^2 + ct_1^3 + dt_1^4 - \frac{a\alpha\beta}{\beta+1}t_1^{\beta+1} - \frac{b\alpha\beta}{\beta+1}t_1^{\beta+2} - \frac{c\alpha\beta}{\beta+1}t_1^{\beta+3} - \frac{d\alpha\beta}{\beta+1}t_1^{\beta+4}]$$

$$+ a_c[a\alpha t_1^{\beta} + b\alpha t^{\beta+1} + c\alpha t_1^{\beta+2} + d\alpha t_1^{\beta+3}] + c_s[a(t_1 - T) + bt_1(t_1 - T) + ct_1^2(t_1 - T) + dt_1^3(t_1 - T)] = 0 \qquad (4.3)$$

This gives the optimum value of t_1.

(b). Absence of amelioration:

If the environment of amelioration is vanished i.e. $\alpha = 0$, then the expressions (3.6) and (3.11) of on-hand inventory(Q) and average total cost per unit time (TC(t_1)) during the period [0,T] become

$$Q = at_1 + (\frac{b}{2} + \frac{a\theta}{2})t_1^2 + (\frac{c}{3} + \frac{b\theta}{3})t_1^3 + (\frac{d}{4} + \frac{c\theta}{4})t_1^4 + \frac{d\theta}{5}t_1^5 \qquad (4.4)$$

$$\text{And TC}(t_1) = \frac{1}{T}[A_0 + p_c[at_1 + (\frac{b}{2} + \frac{a\theta}{2})t_1^2 + (\frac{c}{3} + \frac{b\theta}{3})t_1^3 + (\frac{d}{4} + \frac{c\theta}{4})t_1^4 + \frac{d\theta}{5}t_1^5]$$

$$+(h_c + \theta d_c)[\frac{a}{2}t_1^2 + (\frac{b}{3} + \frac{a\theta}{6})t_1^3 + (\frac{c}{4} + \frac{b\theta}{8})t_1^4 + (\frac{d}{5} + \frac{c\theta}{10})t_1^5 + \frac{d\theta}{12}t_1^6]$$

$$+c_s[\frac{a}{2}(T^2-2Tt_1+t_1^2)+\frac{b}{6}(T^3-3Tt_1^2+2t_1^3)+\frac{c}{12}(T^4-4Tt_1^3+3t_1^4)+\frac{d}{20}(T^5-5Tt_1^4+4t_1^5)]] \quad (4.5)$$

The optimum value of t_1 is calculated from the following equation

$$p_c[a+(b+a\theta)t_1+(c+b\theta)t_1^2+(d+c\theta)t_1^3+d\theta t_1^4]$$

$$+(h_c+\theta d_c)[at_1+(b+\frac{a\theta}{2})t_1^2+(c+\frac{b\theta}{2})t_1^3+(d+\frac{c\theta}{2})t_1^4+\frac{d\theta}{2}t_1^5]$$

$$+c_s[a(t_1-T)+bt_1(t_1-T)+ct_1^2(t_1-T)+dt_1^3(t_1-T)]=0 \quad (4.6)$$

(c). If the demand rate is quadratic function of time then $d=0$

From (3.6), the total amount of inventory Q becomes

$$Q=at_1+(\frac{b}{2}+\frac{a\theta}{2})t_1^2+(\frac{c}{3}+\frac{b\theta}{3})t_1^3+\frac{c\theta}{4}t_1^4-\frac{a\alpha}{\beta+1}t_1^{\beta+1}-\frac{b\alpha}{\beta+2}t^{\beta+2}-\frac{c\alpha}{\beta+3}t_1^{\beta+3} \quad (4.7)$$

From (3.11), the average total cost per unit time of the system during the cycle [0,T] becomes

$$\text{TC}(t_1)=\frac{1}{T}[A_0+p_c[at_1+(\frac{b}{2}+\frac{a\theta}{2})t_1^2+(\frac{c}{3}+\frac{b\theta}{3})t_1^3+\frac{c\theta}{4}t_1^4$$

$$-\frac{a\alpha}{\beta+1}t_1^{\beta+1}-\frac{b\alpha}{\beta+2}t^{\beta+2}-\frac{c\alpha}{\beta+3}t_1^{\beta+3}]+(h_c+\theta d_c)[\frac{a}{2}t_1^2+(\frac{b}{3}+\frac{a\theta}{6})t_1^3+(\frac{c}{4}+\frac{b\theta}{8})t_1^4+\frac{c\theta}{10}t_1^5$$

$$-\frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}-\frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3}-\frac{c\alpha\beta}{(\beta+1)(\beta+4)}t_1^{\beta+4}]+a_c[\frac{a\alpha}{\beta+1}t_1^{\beta+1}+\frac{b\alpha}{\beta+2}t^{\beta+2}+\frac{c\alpha}{\beta+3}t_1^{\beta+3}]$$

$$+c_s[\frac{a}{2}(T^2-2Tt_1+t_1^2)+\frac{b}{6}(T^3-3Tt_1^2+2t_1^3)+\frac{c}{12}(T^4-4Tt_1^3+3t_1^4)]] \quad (4.8)$$

The equation (3.12) becomes

$$p_c[a+(b+a\theta)t_1+(c+b\theta)t_1^2+c\theta t_1^3-a\alpha t_1^{\beta}-b\alpha t_1^{\beta+1}-c\alpha t_1^{\beta+2}]$$

$$+(h_c+\theta d_c)[at_1+(b+\frac{a\theta}{2})t_1^2+(c+\frac{b\theta}{2})t_1^3+\frac{c\theta}{2}t_1^4-\frac{a\alpha\beta}{\beta+1}t_1^{\beta+1}-\frac{b\alpha\beta}{\beta+1}t_1^{\beta+2}-\frac{c\alpha\beta}{\beta+1}t_1^{\beta+3}]$$

$$+a_c[a\alpha t_1^{\beta}+b\alpha t^{\beta+1}+c\alpha t_1^{\beta+2}]+c_s[a(t_1-T)+bt_1(t_1-T)+ct_1^2(t_1-T)]=0 \quad (4.9)$$

which is the equation to find out the optimum value of t_1.

(d). If the demand rate is linear trended function of time then c = 0 and $d=0$

From (3.6), the total amount of inventory Q becomes

$$Q = at_1 + (\frac{b}{2} + \frac{a\theta}{2})t_1^2 + \frac{b\theta}{3}t_1^3 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{b\alpha}{\beta+2}t^{\beta+2} \tag{4.10}$$

From (3.11), the average total cost per unit time of the system during the cycle [0,T] becomes

$$TC(t_1) = \frac{1}{T}[A_0 + p_c[at_1 + (\frac{b}{2} + \frac{a\theta}{2})t_1^2 + \frac{b\theta}{3}t_1^3 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{b\alpha}{\beta+2}t^{\beta+2}]$$

$$+(h_c + \theta d_c)[\frac{a}{2}t_1^2 + (\frac{b}{3} + \frac{a\theta}{6})t_1^3 + \frac{b\theta}{8}t_1^4 - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} - \frac{b\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3}]$$

$$+ a_c[\frac{a\alpha}{\beta+1}t_1^{\beta+1} + \frac{b\alpha}{\beta+2}t^{\beta+2}] + c_s[\frac{a}{2}(T^2 - 2Tt_1 + t_1^2) + \frac{b}{6}(T^3 - 3Tt_1^2 + 2t_1^3)]] \tag{4.11}$$

The equation (3.12) becomes

$$p_c[a + (b + a\theta)t_1 + b\theta t_1^2 - a\alpha t_1^{\beta} - b\alpha t_1^{\beta+1}] + (h_c + \theta d_c)[at_1 + (b + \frac{a\theta}{2})t_1^2 + \frac{b\theta}{2}t_1^3$$

$$- \frac{a\alpha\beta}{\beta+1}t_1^{\beta+1} - \frac{b\alpha\beta}{\beta+1}t_1^{\beta+2}] + a_c\alpha(at_1^{\beta} + bt^{\beta+1}) + c_s\{a(t_1 - T) + bt_1(t_1 - T)\} = 0 \tag{4.12}$$

This gives the optimum value of t_1.

(e). If the demand rate is constant then b = 0, c = 0 and $d = 0$

From (3.6), the total amount of inventory Q becomes

$$Q = at_1 + \frac{a\theta}{2}t_1^2 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} \tag{4.13}$$

From (3.11), the average total cost per unit time of the system during the cycle [0,T] becomes

$$TC(t_1) = \frac{1}{T}[A_0 + p_c\{at_1 + \frac{a\theta}{2}t_1^2 - \frac{a\alpha}{\beta+1}t_1^{\beta+1}\} + (h_c + \theta d_c)\{\frac{a}{2}t_1^2 + \frac{a\theta}{6}t_1^3 - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2}\}$$

$$+ \frac{a_c a\alpha}{\beta+1}t_1^{\beta+1} + \frac{c_s a}{2}(T^2 - 2Tt_1 + t_1^2)] \tag{4.14}$$

The equation (3.12) becomes

$$p_c(a + a\theta t_1 - a\alpha t_1^{\beta}) + (h_c + \theta d_c)(at_1 + \frac{a\theta}{2}t_1^2 - \frac{a\alpha\beta}{\beta+1}t_1^{\beta+1}) + a_c a\alpha t_1^{\beta} + c_s a(t_1 - T) = 0 \tag{4.15}$$

which is the equation to find out the optimum value of t_1.

Numerical Example:

To illustrate the developed inventory model, let the values of parameters be as follows:

A_0 = \$500 per order; a=30; b=20; c=10; d=3; θ = 0.01; α = 0.001; β = 2; p_c = \$ 5 per unit, h_c = \$12 per unit; d_c = \$4 per unit; a_c = \$7 per unit; c_s = \$15 per unit; T = 1 year

Solving the equation (3.12) with the help of computer using the above values of parameters, we find the following optimum outputs

t_1^* = 0.37year, Q^* = 12.63 units, Purchasing Cost(PC*) = Rs.63.15, Holding Cost(HC*) = Rs.29.14, Cost of Deterioration(CD*)=Rs.0.0971, Amelioration Cost(AMC*)= Rs.0.0043, Shortage Cost(SC*) = Rs.137.03 and Total Inventory Cost TC^* = Rs.729.43

It is checked that this solution satisfies the sufficient condition for optimality.

Comparison study of different aspects of inventory models and concluding remarks:

The comparative study is furnished here to illustrate different aspects of the inventory models.

Inventory model	Optimum values of						
	Q	PC	HC	CD	AMC	CS	TC
No deterioration (θ=0)	12.67	63.33	29.37	00	0.0043	136.46	729.17
No amelioration (α =0)	12.63	63.15	29.14	0.0971	00	137.03	729.43
Quadratic Demand (d=0)	12.62	63.09	29.09	0.0970	0.0043	134.93	727.21
Linear Demand (c=d=0)	12.23	62.23	28.52	0.0951	0.0042	124.28	715.12
Constant demand (b=c=d=0)	11.09	55.43	24.52	0.0817	0.0035	89.63	669.66

Analyzing the results of table, the following observations may be made:

(i). The optimum value of on-hand inventory (Q) changes very less sensitively in the presence or absence of deteriorating and ameliorating items in the inventory model. Similarly the optimum total cost (TC) changes insignificantly for the presence or absence of deteriorating and ameliorating items in the proposed model. As a result, it can be concluded that the deterioration parameter (θ) and the shape parameter (α) have no significant effects on optimal values of the different aspects of the proposed model.

(ii). It is observed from the above table and illustrated example that the optimality of on-hand inventory level(Q), purchasing cost(PC), holding cost(HC), deteriorating cost(CD), ameliorating cost(AMC), shortage cost(CS) and total inventory cost(TC) are moderately changing for cubic, quadratic and linear trended demand rates, where as these are changing significantly towards the constant demand in nature. Also it is seen that the total inventory cost is minimum for the model where demand rate is constant. So the demand parameters b, c and d play an important role on the estimation of optimal cost of the present inventory model and we need adequate attention to estimate these parameters.

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