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Farey Error Correcting Coding Text, Data and Images

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Abstract:

A new scheme for encryption and decryption of plain text, data and the coding of medical images through Farey Sequence is brought in the paper. It proposes the reduced arithmetic point representations and more of integer and whole number point implementations. The test case and outcomes are explained in the section 4 and 5.

Keywords: Error correcting codes, image coding, reduced arithmetic floating point, fixed point, digital signal processing and farey sequences

1. INTRODUCTION

The efficacy of Farey sequence applications for error correcting codes and image processing are brings out in the paper. John Farey came across and observed simple, irreducible and decimal division quantities or fractions which are proper in the interval [0 1] called as Farey sequences. Farey sequences studies and research works carried out are in [1-4].

Implementation of image processing algorithms consists of computer intensive arithmetic operations. Custom digital processing chips convert the floating point computations to fixed point computations and it take more time, space and memory.

This paper in Section 2 gives the properties [3-4] and Section 2 (properties), Section 3 gives *Gram-Schmidt orthogonalization, Section 4 gives. Farey Sequence and coding of medical images and also* Section 5 gives *Farey Sequence and Error Correcting Codes*

2. PRINCIPLES AND AXIOMS OF THE FAREY SEQUENCE

Farey Sequences for $Farey_1$...$Farey_8$ which are irreducible or simple decimal division quantities in [0, 1] for a given n is given below.

$Farey_1$ = [0/1, 1/1]

$Farey_2$ = [0/1, ½, 1/1]

$Farey_3$ = [0/1, 1/3, ½, 2/3, 1/1]

$Farey_4$ = [0/1, ¼, 1/3, ½, 2/3, ¾, 1/1]

$Farey_5$ = [0/1,1/5 ,1/4, 1/3, 2/5, 1/2 ,3/5, 2/3, 4/5, 1/1]

$Farey_6$ = [0/1, 1/6,/5, ¼, 1/3, 2/5, ½, 3/5, 2/3, ¾, 4/5, 5/6, 1/1]

$Farey_7$ = [0/1, 1/7, 1/6 ,1/5, ¼, 2/7,

1/3, 2/5, 3/7, ½, 4/7, 3/5, 2/3, 5/7, ¾, 4/5, 5/6, 6/7, 1/1]

$Farey_8$ = [0/1, 1/8 ,1/7, 1/6, 1/5, ¼,2/7,
1/3, 3/8, 2/5, 3/7, 1/2
,4/7, 3/5,5/8, 2/3, 5/7 ,3/4, 4/5 ,5/6, 6/7, 7/8, 1/1]

Fig. 1. Examples of Farey Sequences for F_1...F_8

The *Principles and axioms of the Farey sequence* , the number of decimal division quantities in Farey sequence and the aspects of rank of the division quantities or fractions in a *Farey sequence* are given in [3 4].

3. INVERSE OF A MATRIX:

A square **matrix** X, which is said to be non-singular (i.e. det (X) does not equal zero), if there exists an n × n **matrix** A^{-1} which is called the **inverse** of X such that: $XX^{-1} = X^{-1}X = I$, where I is the identity **matrix**. The calculation of X^{-1} involves the process of calculating the matrix of minors, then making them into the matrix consisting of Cofactors, and then finding the Adjugate, by multiplying and then dividing by the determinant. The equation solving involves matrix , inverse matrix and values

$$XY=Z$$

$$Y=X^{-1}Z$$

Where X is Matrix, Y is variables and Z is values.

3.1 *Gram-Schmidt orthogonalization:* It also called the **Gram-Schmidt** process, gives an orthogonal basis over a random interval with respect to a weighting function for a non-orthogonal set of linearly independent vectors.

A matrix A is set to be **orthogonal** if its inverse is equal to its transpose($A^{-1} = A^{T}$). The determinant value of any **orthogonal matrix** A needs to be +1 or −1.

Orthogonal matrices are very much required for proper transmission of binary elements 1 111..0000 or vectors.

4. FAREY SEQUENCE AND IMAGE CODING OF OF MEDICAL IMAGES

Implementation of image processing algorithms consists of computer intensive arithmetic operations like multiplications and divisions. It needs to be converted to integer or fixed point in order to save time, space and memory. The computation using farey Trees will reduce the floating point arithmetic and reduces to small number of lookups. The performance is much better than the more than known methods mentioned in [5-15]

An image frame of a medical image is considered. All the pixel values are divided by 255 or 65536 depending on the pixel size value for normalization or thresh holding. By normal division it involves more floating arithmetic and computation. It also consumes more power.

As per the procedure in the Fig 2 the farey approximation can be obtained. The approximation is reduced approximation. The numerator and denominator of the (irreducible) fraction like 159/255 is reduced to 15/17 or 8/9 which is a reduced fraction. Similarly any value like 255/65536 can be reduced to 1/257 reduced fraction. The numerator and denominator of the (irreducible) fraction like 146/255 is reduced to 47/82 or 19/37 which is a reduced fraction. Similarly any value like 246/255 can be reduced to 82/85 reduced fraction.

The Fig 3 give the reduced farey approximation of numerator and denominator of the reduced fraction when pixel value is divided by 255 value.

Further reduced numerator and denominator can be obtained by applying farey approximation as per the Fig 2 & 3 repeatedly till the gcd(Numerator, denominator) is 1. One can stop for particular approximation.

A farey approximation of reduced fraction cab be created for a image of suitable size can be created. By look up for particular fraction, the computation and calculations involving divisions can be reduced.

As per the Fig 3,4,5 When Pixel Values=[18 28 81 ....] are thresholded or divided by 255 will have big real value. But by the farey sequence method will have the reduced fraction consists of Numerator =[1 1 17.......] and Denomenator =[14 9 22......]. This helps in digital signal processing and fixed point implementation

5. FAREY ENCRYPTION AND DECRYPTION OF TEXT, DATA AND CODING OF MEDICAL IMAGES

All communications need Error correcting codes which are and are very much important to embedded systems. A fundamental application of coding theory is the detection and correction of errors. R W Hamming and Shannon did much of the early work into error detection and correction for coding theory.

Some of the layers in the ISO 7 layer system include provision to detect and even recover from transmission errors with the help of of parity bits, and cyclic redundancy codes. At the transmission side or the sender need to add extra bits to deduct correctly in the receiving end. Most of the error correcting codes need redundant data for parity data.

By using Farey sequency one can reduce transmission errors. The usefulness of farey sequence for error correction is shown below with typical examples.

fw is F_7 farey sequence matrix.

orfw is orthogonal matrix and orfw' is transpose of the orfw matrix.

Encoding Transmission matrix = orfw*t1' wheren t1 is test vector

Decoding = orfw'* Transmitted Matrix

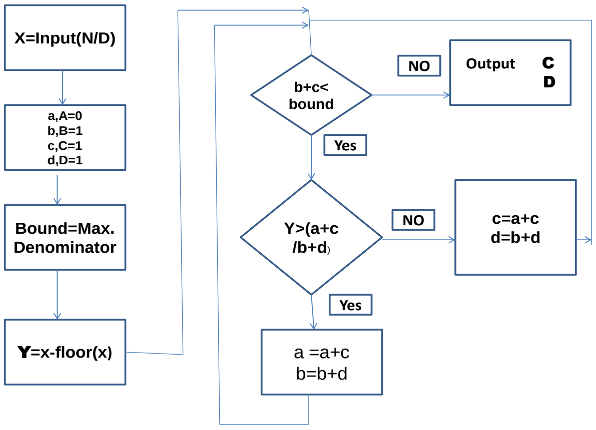

Fig 2. Procedure for Farey Approximation

F_7 =
{ 0/1, 1/7, 1/6, 1/5, 1/4, 2/7, 1/3, 2/5, 3/7, 1/2, 4/7, 3/5, 2/3, 5/7, 3/4, 4/5, 5/6, 6/7, 1/1
fw /*ferry Sequence*/

fw = $F_{7=}$

0.1429	0.2857	0.5000	0.7143
0.1667	0.3333	0.5714	0.7500
0.2000	0.4000	0.6000	0.8000
0.2500	0.4286	0.6667	0.8333

Inv(fw) /*Inverse of fw*/
Invfw=

24.267	-33.6000	-19.333	28.0000
-16.333	0.0000	29.1667	-14.0000
-19.6000	33.6000	-14.000	0.0000
16.8000	-16.8000	2.0000	0.0000

Plian Text : A B C D /*Text Vector*/
ASCII value : [65 66 67 68]
sat1=[65 66 67 68]

invstest1=inv(fw)*sat1(1,1:4)'

invstest1=

-31.6000
-59.5000
5.000
117.2000

Invostest1=fw*invstest1

Invostest1= /*Decoded output*/

65.0000
66.0000
67.0000
68.0000

>> orfw /* Matrix Fw orthogonalization */

orfw =

-0.4385	-0.7006	0.0384	0.5616
-0.4795	-0.2727	-0.4807	-0.6817
-0.5181	0.1580	0.7987	-0.2620
-0.5562	0.6402	-0.3599	0.3889

>> orfw' /* transpose of orthogonalization matrix */

orfw'=

-0.4385	-0.4795	-0.5181	-0.5562
-0.7006	-0.2727	0.1580	0.6402
0.0384	-0.4807	0.7987	-0.3599
0.5616	-0.6817	-0.2620	0.3889

Text Data : A B C D /*Test Vector*/

ASCII Conversion : 65 66 67 68

sat1=[65. 00 66.00 67.00 68.00]

>>sat1 /*Test vector*/

sat1 =
65.00 66.00 67.00 68.00
stest1 = orfw*sat1(1,1:4)'

stest1 =
-33.9776
-127.7242
12.4480
8.8324
ostest1 = orfw'*stest1;
ostest1 = /*Decoded Output*/
65.00
66.00
67.00
68.00

Error proven ones
estest1=
-33.9775
-127.7241
12.4481
8.8323
oestest1=orfw'*estest1
oestest1 =
64.775
66.2559
66.8561
68.1555
round(oestest1) = /*Decoded Correctly*/
65
66
67
68

```
estest1=
   -33.8776
  -127.7241
    12.4481
     8.8323
oestest1=orfw'*estest1
oestest1 =
   64.7337
   66.1859
   66.8600
   68.2112
round(oestest1) =
   65
   66
   67
   68
```

```
>> s1  /* Test  vextor*/

s1 =

    1    2    1    1

>> s2   /* Test  vextor*/

s2 =


    1    0    1    0

>> test1

test1 =  orfw*test1' (Encoding)

   -1.2397
   -2.1872
    0.3346
    0.7532

>> trtest1 /*Transmission*/
```

Transmitted with error in element 1(-1.2387 instead of -1.2397, element 2 (-2.1873 instead of -2.1872, element 3(0.3342 instead of 0.3342) and element 4(0.7531 instead of 0.7532)

```
trtest1 =

   -1.2387
   -2.1873
    0.3342
    0.7531

>> dtest1 /*Decoded*/

dtest1 = orfw'*trtest1

    0.9999
    1.9992
    0.9998
    1.0007
Decodes as [ 1 2 1 1]
>> test2

test2 ==  orfw*test2' (Encoding)

   -0.4001
   -0.9602
    0.2806
   -0.9161


>> trtest2 /*Transmission*/
```

Transmitted with error in element 4(--0.9159 instead of -0.9161)

```
trtest2 =

   -0.4001
   -0.9602
    0.2806
   -0.9159

>> dtest2    /*Decoded*/

dtest2 = orfw'*trtest2

    0.0381
   -0.4822
    1.1586
   -0.6509
Decoded as 1 0 1 0
>>
```

Everey error prone digit in the transmitted digits like element 1(-1.2387 instead of -1.2397, element 2 (-2.1873 instead of -2.1872, element 3(0.3342 instead of 0.3342) and element 4(0.7531 instead of 0.7532) are decoded correctly, and finally round the value to get the correct value.

Farey tables for transmission and decoding can be used for fast processing. Farey sequence can be used for real and integer data for error correcting data.

s1=[1.2000 1.2000 -1.1000 -1.3000]
test1=orfw*s1(1,1:4)'
test1=
-2.1392
0.5123
-0.9701
-0.0090
Decoded Dtest1=orfw'*test1(Encoding)
[1.2000 1.2000 -1.1000 -1.3000]

By corrupting test1 in the element1 and transmitting as trtest1=[-2.1292 0.5123 -0.9701 -0.0090]

Decoding
Dtest1= orfw'*trtest1
Dtest1=[1.1956 1.1930 -1.0996 -1.2944]
Rounding yields
[1.2000 1.2000 -1.1000 -1.3000]

Which is same as s1

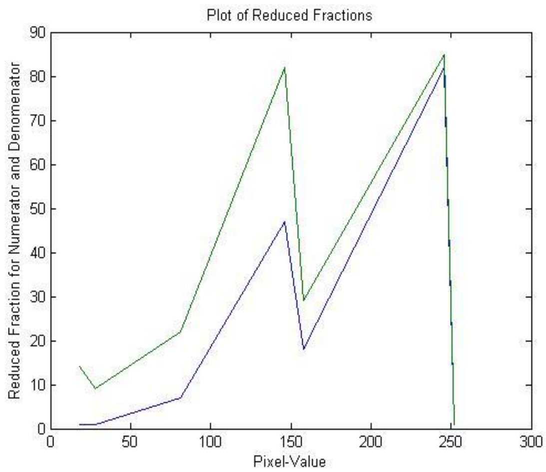

Fig 3. Pixel Value and Further Reduced Fractions

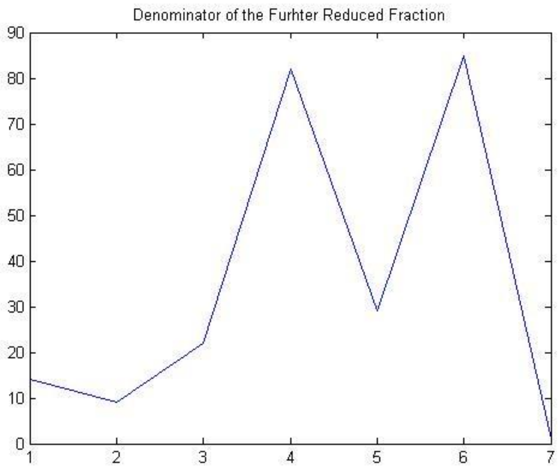

Fig 4. Denominator the Further Reduced Fractions

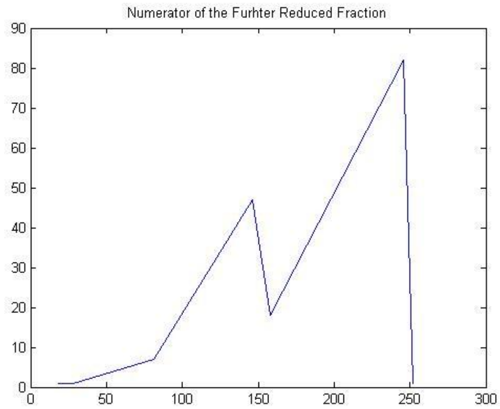

Fig 5 Numerator of the Further Reduced Fractions

CONCLUSION

This paper shows adaptation of farey sequences for error correcting codes and obtaining a decimal fraction in the Farey sequence. Handling of the fractions can been improved by using the Farey approximation approach and proposed variable processing architecture. New scheme to obtain a decimal division quantities closest to any random decimal division quantities in a given Farey sequence are also explained in the paper. Most of the operations are reduced real point representations and also are of reduced time, space and memory.

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