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On Semicircular Extreme-value distribution

R. Srinivas
Assistant Professor of Mathematics, Hindu College, Guntur, India

Y. Phani
Associate Professor of Mathematics, Shri Vishnu Engineering College for Women, Vishnupur, Bhimavaram, India.

S.V.S. Girija
Professor of Mathematics, Hindu College, Guntur, India

Abstract

This paper is aimed at construction of Semicircular Extreme-value distribution for modeling semicircular data by applying simple projection method on Extreme-value distribution. It is extended it to the l-axial Extreme-value distribution by a simple projection for modeling any arc of arbitrary length.

Keywords: characteristic function, semi circular models, l-axial data, projection, trigonometric moments.

AMS Subject Classification: 60E05, 62H11

1. INTRODUCTION

In some of the cases the directional / angular data does not require full circular models for modeling, this fact is noted in Guardiola (2004), Jones (1968) and Byoung et al(2008). For example, when a sea turtle emerges from the ocean in search of a nesting site on dry land, a random variable having values on a semicircle is well sufficient for modeling such data. Similarly, when an aircraft is lost but its departure and its initial headings are known, a semicircular random variable is sufficient for such angular data. And few more examples of semicircular data is available in Ugai et al (1977).

Guardiola (2004) obtained the semicircular normal distribution by using a simple projection and Byoung et at (2008) developed a family of the semicircular Laplace distributions for modeling semicircular data by simple projection, Phani et al (2013) constructed some semicircular distributions by applying inverse stereographic projection. In this paper the **Semicircular Extreme-value distribution** (**SCEVD**) is developed by projecting Extreme-value distribution over a semicircular segment. The graphs of the density and distribution functions for various values of parameters are plotted. The characteristic function is also derived. This model is extended to l-axial model.

2. SEMICIRCULAR EXTREME-VALUE DISTRIBUTION

A random variable X on the real line is said to have Extreme-value Distribution with location parameter γ and scale parameter $\lambda > 0$, if the probability density function and cumulative distribution function of X for $x, \gamma \in \mathbb{R}$ and $\lambda > 0$ are given respectively by

$$f(x) = \frac{1}{\lambda} \exp\left(\frac{-(x-\gamma)}{\lambda}\right) \exp\left(-\exp\left(\frac{-(x-\gamma)}{\lambda}\right)\right),$$

where $\lambda > 0$, and $\gamma,\ x \in \mathbb{R}$ (2.1)

$$F(x) = \exp\left(-\exp\left(\frac{-(x-\gamma)}{\lambda}\right)\right) . \tag{2.2}$$

Simple projection is defined by

mapping $x = v\tan(\theta)$ or $\theta = \tan^{-1}\left(\frac{x}{v}\right),\ v > 0 \in \mathbb{R}$.

Application of this simple projection on Extreme – value distribution results to a Semicircular Extreme-value distribution.

Definition:

A random variable X_{SC} on the Semicircle is said to have the Semicircular Extreme-value distribution with location parameter μ scale parameter $\sigma > 0$ denoted by **SCEVD**(σ, μ), if the probability density and the cumulative distribution functions are respectively given by

$$g(\theta) = \frac{1}{\sigma} \sec^2(\theta) \exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right) \exp\left\{-\exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right)\right\},$$

$$\text{where, } \mu = \frac{\gamma}{v},\ \sigma = \frac{\lambda}{v} > 0 \tag{2.3}$$

$$G(\theta)=\exp\left\{-\exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right)\right\}, \text{where } \sigma>0,\ -\frac{\pi}{2}<\theta<\frac{\pi}{2} \tag{2.4}$$

Hence the proposed new model **SCED** (σ,μ) is a Semicircular model.

Graphs of the probability density and cumulative distribution functions of the Semicircular Extreme-value distribution for various values of σ and μ

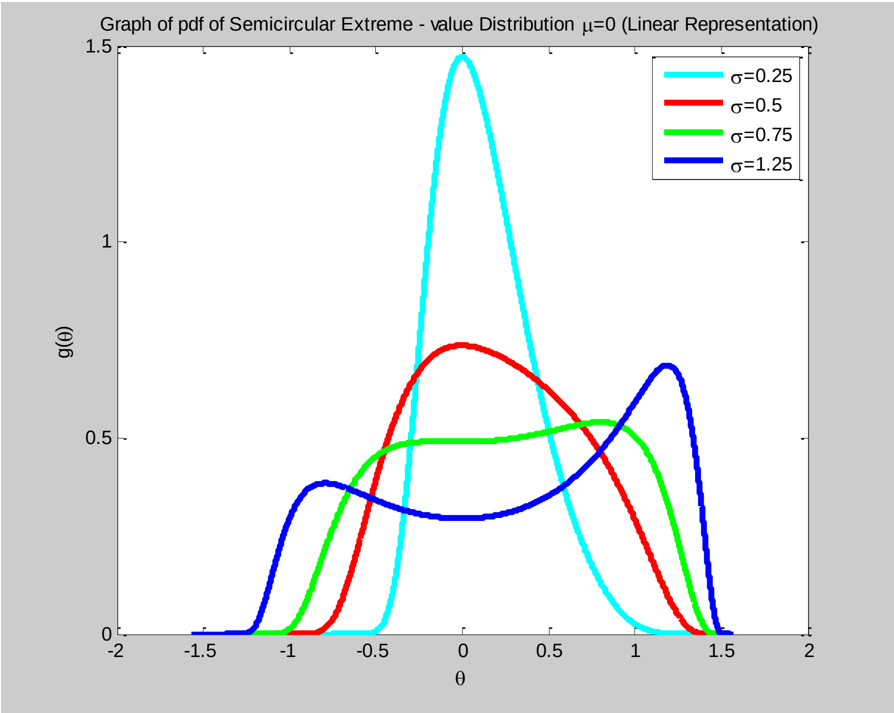

Fig1. Graph of pdf of Semicircular Extreme – value Distribution (Linear Representation)

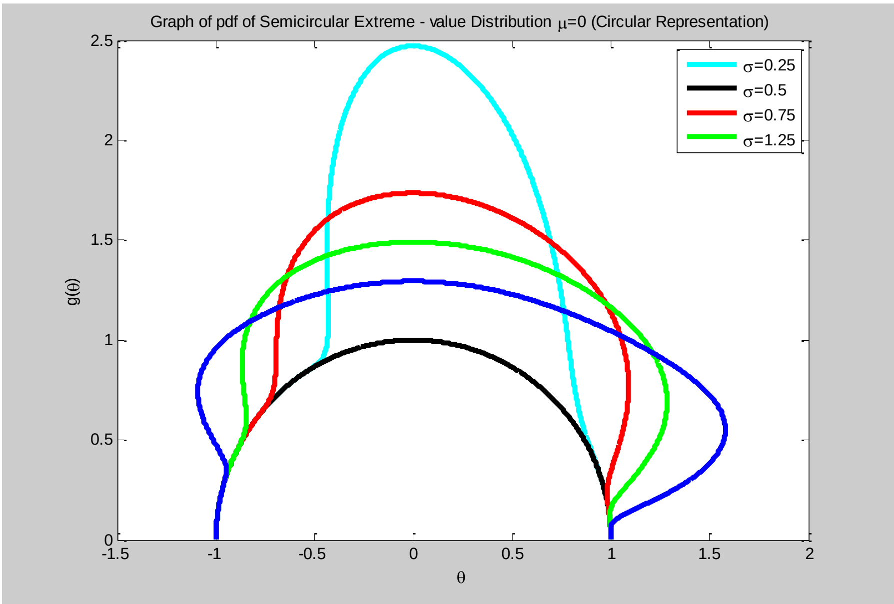

Fig2. Graph of pdf of Semicircular Extreme – value Distribution (Circular Representation)

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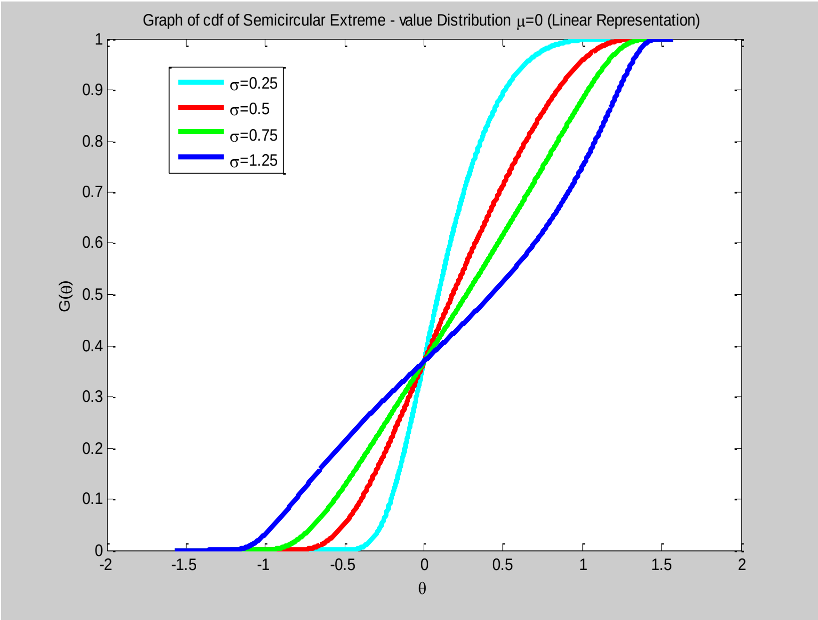

Fig3. Graph of cdf of Semicircular Extreme – value Distribution

3. The Characteristic function of Semicircular Extreme-value distribution

The characteristic function of a semicircular model with probability density function $g(\theta)$ is defined as

$$\Phi_{X_{SC}}(p)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{ip\theta}g(\theta)d\theta \quad ,p\in\mathbb{Z}.$$

$$\Phi_{X_{SC}}(p)=\frac{1}{\sigma}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{ip\theta}\sec^2(\theta)\exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right)\exp\left\{-\exp\left(-\frac{(\tan(\theta)-\mu)}{\sigma}\right)\right\}d\theta \quad ,p\in\mathbb{Z} \qquad (3.1)$$

As the integral cannot be obtained analytically, MATLAB techniques are applied for the evaluation of the values of the characteristic function. Population characteristics are also studied using first two trigonometric moments.

Graphs of the characteristic function of Semicircular Extreme -value distribution for various values of parameter are plotted.

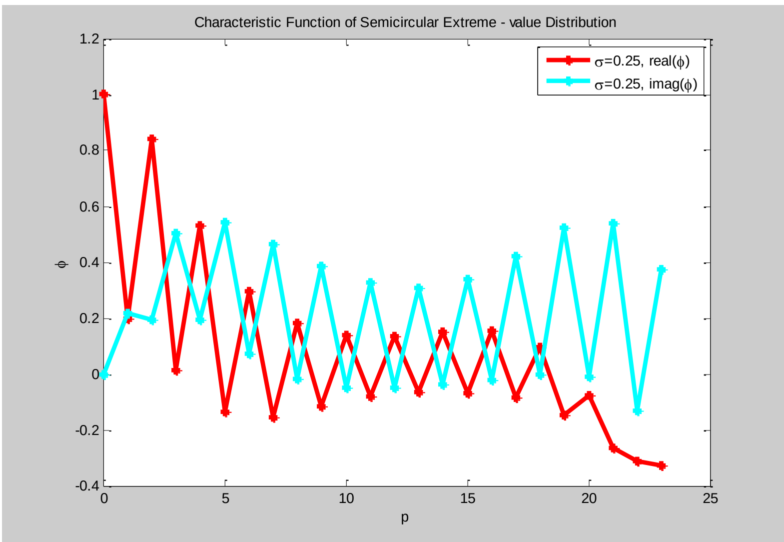

Fig4. Graph of Characteristic function of Semicircular Extreme – value Distribution

The expressions for mean direction, resultant length, circular variance, circular standard deviation, central trigonometric moments, skewness and kurtosis for circular distributions are available in Mardia and Jupp (2000). These characteristics for the Semicircular Extreme - value distribution are also based on their respective trigonometric moments and can be expressed in terms of trigonometric moments α_p and β_p which are presented here.

From the population characteristics, it can be observed that with increasing value of scale parameter σ , the circular variance gradually decrease, the distribution is negatively skewed and remained platykurtic

Table 3.1 Population Characteristics of Semicircular Extreme – value model

	$\sigma=0.25$	$\sigma=0.5$	$\sigma=0.75$	$\sigma=1.25$
Trigonometric Moments				
α_1	0.1995	0.1074	-0.0183	-0.1525
α_2	0.8426	0.6299	0.4812	0.1586
β_1	0.2176	0.3461	0.3727	0.2671
β_2	0.1926	0.2203	0.2488	0.3660
Resultant Length				
ρ_1	0.2952	0.3624	0.3732	0.3076
ρ_2	0.8643	0.6673	0.5417	0.3989
Mean Direction μ_0	0.8288	1.2700	-1.5218	-1.0518
Circular Variance v_0	0.7048	0.6376	0.6268	0.6124
Circular Standard Deviation				
σ_0	1.5622	1.4249	1.4041	1.5356
	0.5400	0.8994	1.1074	1.3558
Central Trigonometric Moments				
α_1^*	0.2952	0.3624	0.3732	0.3076
α_2^*	0.1188	-0.3947	-0.5032	-0.3958
β_1^*	0	0	0	0
β_2^*	-0.8561	-0.5381	-0.2005	-0.0493
Skewness γ_1^0	-1.4467	-1.0568	-0.4040	-0.0856
Kurtosis γ_2^0	0.2239	-1.0131	-1.3299	-0.8442

4 Extension to l -axial distribution

The newly proposed model is extended to the l -axial distribution, which is applicable to any arc of arbitrary length say $2\pi/l$ for $l=1,2,...,$. So it is desirable to extend the Semicircular Extreme-value distribution to its l – axial model. To construct the l -axial Extreme-value distribution, the density function of Semicircular Extreme-value distribution is considered and the transformation $\phi = 2\theta/l$, $l=1,2,...,$ is used. The probability density function of ϕ is given by

$$g(\phi) = \frac{l}{2\sigma}\sec^2\left(\frac{l\phi}{2}\right) e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}} e^{-e^{-\frac{\tan\left(\frac{l\phi}{2}\right)}{\sigma}}}$$

$$\text{where } -\frac{\pi}{l} < \phi < \frac{\pi}{l} \qquad (4.1)$$

Case (1) When $l=2$, the probability density function (4.1) is the same as the probability density function of Semicircular **Extreme-value distribution .**

Case (2) When $l=1$, the probability density function (4.1) is the same as that of **Stereographic Extreme-value Distribution** [Phani et al., 2012] which is circular distribution.

If X has Extreme-value with scale parameter λ and location parameter $\gamma=0$, then $Y=e^X$ has a **log-Extreme-value distribution** with the same parameters. So, using $y=v\tan(\theta)$, a probability density function is

$$g(\theta)=\frac{2}{\lambda\sin(2\theta)}e^{-\frac{1}{\lambda}(\log(v\tan(\theta)))}e^{-e^{-\frac{1}{\lambda}(\log(v\tan(\theta)))}} \qquad 0<\theta<\frac{\pi}{2}, \qquad (4.2)$$

Suppose $\phi=\frac{4\theta}{l}$, $l=1,2,...,$ then ϕ has a probability density function ,

$$g(\phi)=\frac{1}{2\lambda\sin\left(\frac{l\phi}{2}\right)}e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{l\phi}{4}\right)\right)\right)}e^{-e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{l\phi}{4}\right)\right)\right)}}, \ 0<\theta<\frac{2\pi}{l} \qquad (4.3)$$

It is said that ϕ follows the l **- axial log-Extreme-value distribution**.

Case (1): When $l=1$,

$$g(\phi)=\frac{1}{2\lambda\sin\left(\frac{\phi}{2}\right)}e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{\phi}{4}\right)\right)\right)}e^{-e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{\phi}{4}\right)\right)\right)}}, \ 0<\theta<2\pi \qquad (4.4)$$

It is called as a **Circular log-Extreme-value distribution**.

Case (2): When $l=2$,

$$g(\phi)=\frac{1}{2\lambda\sin(\phi)}e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{\phi}{2}\right)\right)\right)}e^{-e^{-\frac{1}{\lambda}\left(\log\left(v\tan\left(\frac{\phi}{2}\right)\right)\right)}}, \ 0<\theta<\pi \qquad (4.5)$$

It is called as a **Semicircular log-Extreme-value distribution**.

Case (3): When $l=4$,

$$g(\phi)=\frac{1}{2\lambda\sin(2\phi)}e^{-\frac{1}{\lambda}(\log(v\tan(\theta)))}e^{-e^{-\frac{1}{\lambda}(\log(v\tan(\theta)))}}, \ 0<\theta<\frac{\pi}{2} \qquad (4.6)$$

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