International Journal of Applied Engineering Research ISSN 0973-4562 Volume 14, Number 9 (2019) pp. 2179-2181

Movable Restrained Independent Dominating Sets of a Graph

Jocecar Lomarda-Hinampas and Renario G. Hinampas, Jr.

College of Teacher Education, Bohol Island State University-Main Campus
CPG North Avenue, 6300 Tagbilaran City, Bohol, Philippines.

Abstract :
This paper shows the characterizations of 1-movable restrained independent dominating sets in the join and corona of two graphs and the bounds or exact values of the 1-movable restrained independent domination number were determined.

Mathematics Subject Classification: 05C69

Keywords: 1-movable restrained independent domination, restrained independent domination, 1-movable domination, restrained domination, independent domination,domination,

1. INTRODUCTION

One important concept in graph theory which made very popular in research is the concept of domination. This concept was formally defined by Berge in 1958 and Ore in 1962 and popularized by Cockayne and Hedetniemi(1975) and which was studied until now because of its significant and practical applications which contributed a lot in solving real life problems. The applications of these concepts motivate the graph theorists and the researchers to study extensively the other variants of domination.

Some variants of domination are independent domination, restrained domination, restrained independent domination and 1-movable domination. The independent domination was introduced by Cockayne and Hedetniemi as mentioned by Goddard and Henning in [6]. This was further studied by Allan and Liang in [1] and [9], respectively, where characterizations of the bounds of the independent domination numbers were established. Moreover, Canoy in [3] characterized the independent dominating sets under some binary operations of two graphs. The concept of restrained dominating sets and their properties were investigated in [5] and [7]. Another type of domination called 1-movable domination was introduced in 2011 by Blair, Gera and Horton [2]. They established the bounds of the 1-movable domination number of a graph and characterized those graphs which attained these bounds. This was further studied by Hinampas and Canoy [8] wherein the 1-movable dominating sets and the 1-movable domination numbers in some binary operations of two graphs were established.

This paper defined and investigated the properties of the 1-movable restrained independent dominating sets of a graph. In particular,the following properties were established: determined the bounds of the 1-movable restrained independent domination numbers of a graph; characterized those graphs which attained the given bounds; characterized 1-movable restrained independent dominating sets in the join and corona of two graphs and determined the exact values or bounds of the 1-movable restrained independent domination number in the join and corona of two graphs.

2. DEFINITIONS

Let $G = (V(G), E(G))$ be a graph and $v \in V(G)$. The *open neighborhood* of v is the set $N_G(v) = N(v) = \{u \in V(G) : uv \in E(G)\}$ and the *closed neighborhood* of v is the set $N_G[v] = N[v] = N(v) \cup \{v\}$.

A subset S of $V(G)$ is a *dominating set* of G if for every $v \in V(G)\backslash S$, there exists $u \in S$ such that $uv \in E(G)$. The *domination number* of G is denoted by $\gamma(G)$ which refers to the smallest cardinality of a dominating set of G. A dominating set of G with cardinality equal to $\gamma(G)$ is called a γ-*set* of G.

A subset S of $V(G)$ is an *independent set* of G if for every two elements $x, y \in S$, $xy \notin E(G)$. The *independence number* of G, denoted by $\beta(G)$, is the largest cardinality of an independent set in G. An independent set S of G is called a *maximum independent set* if $|S| = \beta(G)$. A nonempty subset S of $V(G)$ is an *independent dominating set* of G if S is both an independent set and a dominating set. The *independent domination number* of G, denoted by $\gamma_i(G)$, is the smallest cardinality of an independent dominating set of G. An independent dominating set of G with cardinality equal to $\gamma_i(G)$ is called a γ_i-*set* of G.

A dominating set S is called a *restrained dominating set* of G if $\langle V(G) \setminus S\rangle$ has no isolated vertex. The *restrained domination number* of G is the smallest cardinality of a restrained dominating set of G and is denoted by $\gamma_r(G)$. A restrained dominating set with cardinality equal to $\gamma_r(G)$ is called γ_r-*set* of G.

A nonempty subset S of $V(G)$ is a *1-movable dominating set* of G if S is a dominating set of G and for every $v \in S$, $S \setminus \{v\}$ is a dominating set of G or there exists a vertex $u \in (V(G)\backslash S) \cap N(v)$ such that $(S\backslash\{v\}) \cup \{u\}$ is a dominating set of G. The *1-movable domination number* of G denoted by $\gamma_m^1(G)$ is the smallest cardinality of a 1-movable dominating set of G. A 1-movable dominating set with cardinality equal to $\gamma_m^1(G)$ is called γ_m^1*-set* of G.

A nonempty subset S of $V(G)$ is a *1-movable restrained independent dominating set* of G if S is a restrained independent dominating set of G and for every $v \in S$, there exists $u \in (V(G) \setminus S) \cap N_G(v)$ such that $(S \setminus \{v\}) \cup \{u\}$ is a restrained independent dominating set of G. The *1-movable restrained independent domination number* of a graph G, denoted by $\gamma_{mri}^1(G)$, is the smallest cardinality of a 1-movable restrained independent dominating set of G. A 1-movable restrained independent dominating set of G with cardinality equal to $\gamma_{mri}^1(G)$ is called γ_{mri}^1*-set* of G.

3. RESULTS

Define a collection $\mathscr{R}_{mri}^1$ be the family of all graphs with 1-movable restrained independent dominating sets. This paper focuses only to those graphs which belong to $\mathscr{R}_{mri}^1$.

Remark 3.1. *For every connected graph G of order $n \geq 3$, $1 \leq \gamma_{mri}^1(G) \leq \beta(G)$.*

Theorem 3.2. *Let G be a connected graph G of order $n \geq 3$. Then $\gamma_{mri}^1(G) = 1$ if and only if $G \cong K_2 + H$ for some graph H.*

Proof. Suppose that $\gamma_{mri}^1(G) = 1$. Then G has a γ_{mri}^1-set say $S = \{x\}$ for some $x \in V(G)$. Since S is a 1-movable restrained independent dominating set of G, there exists $y \in (V(G) \setminus S) \cap N_G(x)$ such that $S \setminus \{x\} \cup \{y\} = \{y\}$ is a restrained independent dominating set of G. Hence, $xy \in E(G)$. Take $V(K_2) = \{x, y\}$ and $H = \langle V(G) \setminus V(K_2) \rangle$. Hence, $G \cong K_2 + H$.

For the converse, suppose $G = K_2 + H$ for some graph H. Let $V(K_2) = \{x, y\}$ and let $S = \{x\}$. Then S is an independent dominating set of G. Since $\langle V(G) \setminus S \rangle = \langle \{y\} \rangle + H$ has no isolated vertex, S is a restrained independent dominating set of G. Moreover, the set $S \setminus \{x\} \cup \{y\}$ is an independent dominating set of G and $\langle V(G) \setminus (S \setminus \{x\} \cup \{y\}) \rangle = \langle \{x\} \rangle + H$ has no isolated vertex. Hence, $S \setminus \{x\} \cup \{y\}$ is a restrained independent dominating set of G. Thus, S is a 1-movable restrained independent dominating set of G. Since $|S| = 1$, S is a γ_{mri}^1-set of G. Therefore, $\gamma_{mri}^1(G) = 1$. □

Corollary 3.3. *$\gamma_{mri}^1(K_n) = 1$ for all $n \geq 3$.*

Theorem 3.4. *Let G and H be connected nontrivial graphs. Then $S \subseteq V(G + H)$ is a 1-movable restrained independent dominating set of $G + H$ if and only if one of the following statements is true:*

- (i) *S is an independent dominating set of G such that*
  (a) *if $|S| \geq 2$, then S is a 1-movable independent dominating set of G and*
  (b) *if $|S| = 1$, then either S is a 1-movable independent dominating set of G or $\gamma(H) = 1$.*
- (ii) *S is an independent dominating set of H such that*
  (a) *if $|S| \geq 2$, then S is a 1-movable independent dominating set of H and*
  (b) *if $|S| = 1$, then either S is a 1-movable independent dominating set of H or $\gamma_i(G) = 1$.*

Proof. Suppose S is a 1-movable restrained independent dominating set of $G + H$. Then $S \subseteq V(G)$ or $S \subseteq V(H)$. Suppose $S \subseteq V(G)$. Then S is an independent dominating set of G. Suppose $|S| \geq 2$. Let $v \in S$. Since S is a 1-movable restrained independent dominating set of $G + H$, there exists $u \in (V(G + H) \setminus S) \cap N_{G+H}(v)$ such that $S \setminus \{v\} \cup \{u\}$ is a restrained independent dominating set of $G + H$. Since S is an independent set and $|S| \geq 2$, $u \in V(G) \setminus S \cap N_G(v)$. Thus, $S \setminus \{v\} \cup \{u\}$ is an independent dominating set of G. Hence, S is a 1-movable independent dominating set of G. Suppose $|S| = 1$. Let $S = \{x\}$ for some $x \in V(G)$. Since S is a 1-movable restrained independent dominating set of $G + H$, there exists $u \in (V(G + H) \setminus S) \cap N_{G+H}(x)$ such that $S \setminus \{x\} \cup \{u\}$ is a restrained independent dominating set of $G + H$. If $u \in V(G)$, then $S \setminus \{x\} \cup \{u\} = \{u\}$ is a independent dominating set of G. Thus, S is a 1-movable independent dominating set of G. If $u \notin V(G)$, then $u \in V(H)$. Hence, $S \setminus \{x\} \cup \{u\} = \{u\}$ is a dominating set of H. Thus, $\gamma(H) = |S| = 1$. Thus, (i) holds. Similarly, (ii) holds if $S \subseteq V(H)$.

For the converse, suppose (i) holds. Then S is an independent dominating set of $G + H$. Since $\langle V(G + H) \setminus S \rangle = \langle V(G) \setminus S \rangle + H$ has no isolated vertex, S is a restrained independent dominating set of $G + H$. Let $v \in S$ and suppose $|S| \geq 2$. By assumption, there exists $u \in (V(G) \setminus S) \cap N_G(v)$ such that $S \setminus \{v\} \cup \{u\}$ is an independent dominating set of G and hence of $G + H$. Moreover, $\langle V(G + H) \setminus [(S \setminus \{v\}) \cup \{u\}] \rangle = \langle V(G) \setminus [(S \setminus \{v\}) \cup \{u\}] \rangle + H$ has no isolated vertex. Hence, $S \setminus \{v\} \cup \{u\}$ is a restrained independent dominating set of $G + H$. Suppose $|S| = 1$. Let $S = \{v\}$ for some $v \in V(G)$. Suppose S is a 1-movable independent dominating set of G. By the previous argument, S is a 1-movable restrained independent dominating set of $G + H$. Suppose S is not a 1-movable independent dominating set of G. By assumption, $\gamma(H) = 1$. Let $D = \{y\}$ be a γ-set of H for some $y \in V(H)$. Then $S \setminus \{v\} \cup \{y\}$ is an independent dominating set of $G + H$. Since $\langle V(G + H) \setminus (S \setminus \{v\} \cup \{y\}) \rangle = G + \langle V(H) \setminus \{y\} \rangle$ has no isolated vertex, $S \setminus \{v\} \cup \{y\}$ is a restrained independent dominating set of $G + H$. Therefore, S is a 1-movable restrained independent dominating set of $G + H$. Similar arguments follow if (ii) holds. □

Corollary 3.5. *Let G and H be a connected nontrivial graphs, then*

$$\gamma_{mri}^1(G+H) = \begin{cases} 1, \text{ if } \gamma(G) = 1 = \gamma(H) \text{ or } \gamma_{mi}^1(G) = 1 \text{ or } \gamma_{mi}^1(H) = 1 \\ min\{\gamma_{mi}^1(G), \gamma_{mi}^1(H)\}, \text{ otherwise} \end{cases}$$

Theorem 3.6. *Let H be a connected graph of order $n \geq 2$. Then $S \subseteq V(K_1 + H)$ is a 1-movable restrained independent dominating set of $K_1 + H$ if and only if one of the following statements holds:*

- (i) *$S = V(K_1)$ and $\gamma(H) = 1$.*
- (ii) *S is an independent dominating set of H such that if $|S| \geq 2$, then S is a 1-movable independent dominating set of H.*

Proof. Suppose S is a 1-movable restrained independent dominating set of $K_1 + H$. Since S is an independent set, $S = V(K_1)$ or $S \subseteq V(H)$. Suppose $S = V(K_1)$. Since S is a 1-movable restrained independent dominating set of $K_1 + H$, there exists $y \in V(H)$ such that $(S \setminus V(K_1)) \cup \{y\} = \{y\}$ is a restrained independent dominating set of $K_1 + H$. Hence, $\{y\}$ is a dominating set of H. Thus, $\gamma(H)1$. Suppose $S \subseteq V(H)$ and $|S| \geq 2$. Then S is an independent dominating set of H. Let $v \in S$. Since S is a 1-movable restrained independent dominating set of $K_1 + H$, there exists $z \in (V(H) \setminus S) \cap N(v)$ such that $S \setminus \{v\} \cup \{u\}$ is a restrained independent dominating setof $K_1 + H$. This implies that $S \setminus \{v\} \cup \{u\}$ is an independent dominating set of H. Thus, S is a 1-movable independent dominating set of H.

For the converse, suppose (i) holds. Then S is an independent dominating set of $K_1 + H$. Since $\langle V(K_1 + H) \setminus S\rangle = H$ has no isolated vertex, S is a restrained independent dominating set of $K_1 + H$. By assumption, there exists $y \in V(H)$ such that $\{y\}$ is an independent dominating set of H. Hence, $(S \setminus V(K_1)) \cup \{y\} = \{y\}$ is an indepedndent dominating set of $K_1 + H$. Since $\langle V(K_1 + H) \setminus (S \setminus V(K_1) \cup \{y\})\rangle = K_1 + \langle V(H) \setminus \{y\}\rangle$ has no isolated vertex, $S \setminus V(K_1) \cup \{y\} = \{y\}$ is a restrained indepedndent dominating set of $K_1 + H$. Thus, S is a 1-movable restrained independent dominating set of $K_1 + H$. Suppose (ii) holds. Then by definition of the join of graphs, S is an independent dominating set of $K_1 + H$. Since $\langle V(K_1 + H) \setminus S\rangle = K_1 + \langle V(H) \setminus S\rangle$ has no isolated vertex, S is a restrained independent dominating set of $K_1 + H$. Let $v \in S$. Suppose $|S| = 1$, say $S = \{y\}$ for some $y \in V(H)$. Then $S \setminus \{y\} \cup V(K_1) = V(K_1)$ is an independent dominating set of $K_1 + H$. Since $\langle V(K_1 + H) \setminus S \setminus \{y\} \cup V(K_1)\rangle = H$ has no isolated vertex, $S \setminus \{y\} \cup V(K_1) = V(K_1)$ is a restrained independent dominating set of $K_1 + H$. Suppose $|S| \geq 2$. Since S is a 1-movable independent dominating set of H, there exists $u \in (V(H) \setminus S) \cap N(v)$ such that $S \setminus \{v\} \cup \{u\}$ is an independent dominating of $K_1 + H$. Since $\langle V(K_1+H) \setminus (S \setminus \{v\} \cup \{u\})\rangle = K_1 + \langle V(H) \setminus (S \setminus \{v\} \cup \{u\})\rangle$ has no isolated vertex, $S \setminus \{v\} \cup \{u\}$ is a restrained independent dominating set of $K_1 + H$. Thus, S is a 1-movable restrained independent dominating set of $K_1 + H$. □

Corollary 3.7. *Let H be a connected graph of order $n \geq 2$. Then*

$$\gamma^1_{mri}(K_1 + H) = \begin{cases} 1, \textit{if } \gamma_i(H) = 1 \\ \gamma^1_{mi}(H), \textit{ otherwise} \end{cases}$$

Theorem 3.8. *Let G and H be connected nontrivial graphs with $\gamma_i(H) \neq 1$. A subset C of $V(G \circ H)$ is a 1-movable restrained independent dominating set of $G \circ H$ if and only if $C = \bigcup_{v \in V(G)} D_v$, where D_v is a 1-movable independent dominating set in H^v for each $v \in V(G)$.*

Proof. Suppose that C is a 1-movable restrained independent dominating set of $G \circ H$. Suppose that $C \cap V(G) \neq \varnothing$, say $v \in C \cap V(G)$. Since C is an independent set in $G \circ H$, $D_v = C \cap V(H^v) = \varnothing$. Since C is a 1-movable restrained independent dominating set of $G \circ H$, there exists $y \in V(H^v)$ such that $(C \setminus \{v\}) \cup \{y\}$ is a dominating set of $G \circ H$. Thus, $\{y\}$ is an independent dominating set in H^v. Hence,$\gamma_i(H) = 1$. This contradicts the assumption. Hence, $C \cap V(G) = \varnothing$ and $C = \bigcup_{v \in V(G)} D_v$, where $D_v = C \cap V(H^v)$ is an independent dominating set of H^v for each $v \in V(G)$. Since $\gamma_i(H) \neq 1$, $|D_v| \geq 2$ for each $v \in V(G)$. Since C is a 1-movable restrained independent dominating set, D_v is also a 1-movable restrained independent dominating set of H^v for each $v \in V(G)$.

For the converse, suppose that $C = \bigcup_{v \in V(G)} D_v$, where D_v is a 1-movable restrained independent dominating set in H^v. Clearly, C is a 1-movable restrained independent dominating set of $G \circ H$. □

Corollary 3.9. *Let G and H be connected nontrivial graphs. Then $\gamma^1_{mi}(G \circ H) = |V(G)|$ if $\gamma_i(H) \neq 1$.*

REFERENCES

[1] R. B. Allan, and R. Laskar, *On domination and independent domination of a graph*. Discrete Mathematics, 23 (1978),no.2,73-76.

[2] J. Blair, R. Gera and S. Horton. *Movable dominating sensor sets in networks*. Journal of Combinatorial Mathematics and Combinatorial Computing, 77 (2011) ,103-123.

[3] S.R. Canoy, Jr., *Another look at independent domination in graphs*. International Journal of Mathematical Analysis , 8 (2014), no. 42, 2075-2082.

[4] S.R. Canoy Jr., N. Tuan, and R.A. Namoco, *Restrained independent dominating sets and some realization problems*. International Journal of Mathematical Analysis, 8(2014), no.42, 2083-2092.

[5] G.S. Domke, J.H. Hattingh, S.T. Hedetniemi, R.C. Laskar, and L.R. Markus, *Restrained domination in graphs*. Discrete Mathematics, 203(1999), 61-69.

[6] W. Goddard, and M. Henning, *Independent domination in graphs: a survey and recent results*. Discrete Mathematics, 313 (2013), no. 7, 839-854.

[7] J.H. Hattingh, E. Jonck, E.J. Joubert, and A.R. Plummer, *Nordhaus-Gaddum results for restrained domination and total restrained domination in graphs*. (2006).

[8] R.G. Hinampas,Jr., and S.R. Canoy, Jr. *1-movable domination in graphs*. Applied Mathematical Sciences, 8 (2014), no. 172, 8565-8571.

[9] S. Liang, and W. Jianfang, *An upper bound for the independent domination number*. J. Combin. Theory, 76 (1999), 240-246.