

# Acoustic Attenuation in a Lossy Dielectric with Electromechanical Coupling

John Twomey , PhD, B.E.

Department of Computer Science, Cork Institute of Technology (CIT), Cork, Ireland.

Acoustic attenuation in solids has been investigated <sup>1,2</sup> in a number of different situations. In one of the earliest scenarios the media considered were insulating dielectrics having zero or negligible electromechanical coupling. In 1960 a new type of acoustic wave attenuation was shown to exist in Piezoelectric Semiconductors <sup>3</sup> in which an acoustic wave is coupled to mobile charges resulting in significant attenuation. Amplification <sup>4</sup> occurred when a dc drift field was applied to accelerate the charges faster than sound. In the current paper an attenuation mechanism will be examined in piezoelectric dielectrics with electrical loss. The dielectric loss mechanism will be characterised using the loss tangent <sup>7</sup> which results in a complex permittivity. There are two primary loss mechanisms <sup>7</sup> involved in the loss tangent namely electrical conduction losses and also dielectric damping caused by vibrating dipoles.. When a sound wave propagates in such a material it produces an electric field sometimes expressed as an acoustoelectric field. In simple terms this electrical field <sup>3</sup> interacts with the dielectric loss mechanisms causing acoustic absorption.. The theory is a full-mode <sup>5,6</sup> analysis and essentially small signal <sup>3</sup> and one dimensional with acoustic propagation only in the positive x-direction. It is based on the piezoelectric equations of state, Maxwell's Equations and Newton's Second Law. Although the analysis is directly applicable to piezoelectric media it is also valid for small signals in ferroelectric materials with electromechanical coupling. Such materials include Lead Zirconate Titanate (PZT) and Lithium Niobate. The piezoelectric equations of state are:-

$$T = c \frac{\partial u}{\partial x} - eE \quad (1)$$

$$D = \epsilon E + e \frac{\delta u}{\delta x} \quad (2)$$

Where the variables D,T,u and E are the electrical displacement, mechanical stress, mechanical displacement and electric field respectively. The mechanical strain is  $\frac{\delta u}{\delta x}$  and

the constants c,e and  $\epsilon$  are the appropriate elastic constant, piezoelectric constant and total permittivity. In a lossless dielectric medium the permittivity is wholly real but in this instance the medium will be assumed to be lossy. These losses will be represented using a complex expression for the permittivity <sup>7</sup> involving the loss tangent ( $\tan\delta$ ) which for small

losses results in the following expression<sup>7</sup> for the total permittivity:-

$$\epsilon = \epsilon_r \epsilon_0 (1 - j \tan \delta) \quad (3)$$

where  $\epsilon_r$  and  $\epsilon_0$  are the relative and free space permittivities respectively and it is assumed that the tangent term is small and much less than unity.

At this point, a term will be introduced known as the angular conductivity frequency <sup>3</sup> ( $\omega_c$ ). It is equal to the electrical conductivity divided by the real part of the dielectric permittivity. Its inverse is the dielectric relaxation time which is a measure of the response time of the dielectric to the build-up of space charge which arises from the acoustoelectric field. It will be assumed that the electrical conductivity frequency is very small compared to the angular frequency of the sound wave ( $\omega$ ). Hence there is insufficient time for the build-up of any appreciable space charge and consequently:-

$$\frac{\delta D}{\delta x} = 0 \quad (4)$$

Finally, the effects of diffusion currents associated with the spatially varying acoustoelectric field <sup>3</sup> will not be considered. These currents can be appreciable in media having large drift mobilities <sup>3</sup> of mobile charges but in ferroelectric<sup>12, 13</sup> materials such as Lithium Niobate and Potassium Lithium Tantalate Niobate the electrical drift mobility is extremely low.

Using Newton's Second Law yields:-

$$\frac{\delta T}{\delta x} = \rho \frac{\partial^2 u}{\partial t^2} \quad (\text{t=time variable}) \quad (5)$$

Plane wave propagation of the form  $\exp j(\omega t - kx)$  will be assumed where  $\omega$  is the angular frequency,  $\rho$  is the mass density and the propagation constant is  $k = \beta + j\alpha$  where  $\beta$  and  $\alpha$  are the phase and attenuation constants respectively. Substituting for  $\epsilon$  from equation (3) into (2) and expressing the variables T,E,u and D using the exponential notation outlined above results in four homogeneous equations. A non-trivial solution for these equations exists provided the determinant resulting from their coefficients is equal to zero. This condition results in the following dispersion equation for non-zero values of k:-

$$k^2 \left( 1 + \frac{K^2}{1 - j \tan \delta} \right) = \frac{\omega^2}{V_s^2} \quad (6)$$

where  $V_s = \text{sqrt}(c/\rho)$  is the velocity of sound in the absence of electromechanical coupling.

K is the electromechanical coupling factor defined as

$$K^2 = \frac{e^2}{\epsilon c}$$

In the latter definition

it is assumed that the loss tangent is set equal to zero in the permittivity  $\epsilon$  term. Expand equation (6) using the binomial theorem noting that  $\tan \delta \ll 1$  so that higher order terms in the expansion may be neglected to give:-

$$k^2(1 + K^2 + jK^2 \tan \delta) = \frac{\omega^2}{V_s^2} \quad (7)$$

Next substitute for  $k = \beta + j\alpha$  in (7) noting that  $\alpha \ll \beta$  for small losses so that  $\alpha^2$  can be ignored in comparison with  $\beta^2$ . Then after equating real and imaginary parts the following expressions are obtained for the magnitudes of both  $\beta$  and  $\alpha$  :-

$$\alpha = \frac{\omega}{2V_s} \frac{1}{(1 + K^2)^{\frac{1}{2}}} \left( \frac{K^2}{1 + K^2} \right) \tan \delta \text{ Nepers/m}$$

and 
$$\beta = \frac{\omega}{V_s (1 + K^2)^{\frac{1}{2}}} \quad (8)$$

The expression for the phase constant shows that the sound velocity is increased by a factor of

$(1 + K^2)^{0.5}$ . This is due to the additional stiffness resulting from the piezoelectric coupling.

For materials with small electromechanical coupling such as crystalline quartz  $K^2 \ll 1$  and

the attenuation constant reduces to:-

$$\alpha = \frac{\omega K^2}{2V_s} \tan \delta \text{ Nepers/m} \quad (9)$$

Assuming that the dielectric damping loss component of the loss tangent is much less than that associated with the electrical conduction loss then using equation 7 of reference 7 it is shown that:-

$$\omega_c = \omega \tan \delta \quad (\text{for } \omega_c \ll \omega) \quad (10)$$

For the low coupling case as represented by equation (9) and after substituting from (10) the attenuation reduces to

$$\alpha = -0.5 \omega_c K^2 / V_s \text{ Nepers/m} \quad (11)$$

This expression is identical to that in Hutson and White's paper Ref 3 for the low conductivity condition... In this instance, the attenuation is essentially a function of  $\omega_c$ .

Finally, the magnitude of this attenuation will be evaluated for crystal quartz compared with the total measured value. The total measured value is the combined loss as described above plus other dielectric losses such as that proposed by Akhiezer. In X-Cut, L mode Quartz,<sup>2,8</sup> the total attenuation is 0.94dB/cm at 450Mhz. The other relevant physical parameters are  $V_s = 5750\text{m/s}$ ,  $K^2 = 0.0085$  and the loss tangent is  $9 \times 10^{-6}$  resulting in a loss at 450Mhz of 0.0016dB/cm using equation

(8). This corresponds to only 0.17% of the total loss and may be neglected

In the case of ferroelectrics the electromechanical coupling<sup>9</sup> can be significantly higher than in quartz with values of  $K^2$  of 0.25 for Lithium Niobate depending on the orientation and similarly for PZT it can vary up to 0.35. In the frequency range from 2Ghz to 4Ghz the loss tangent for Lithium Niobate is  $2 \times 10^{-3}$  whereas over a similar frequency range for quartz is only  $2.2 \times 10^{-5}$  nearly<sup>8</sup> two orders of magnitudes of a decrease. Higher values for the loss tangent also occur in other ferroelectrics<sup>11</sup> such as PZT and PMN-PT.. Consequently from the above comparative data this loss in the latter types of ferroelectrics represent a higher fraction of the total loss and should be factored into any analysis of sound absorption for these materials.

The author wish to acknowledge assistance with the text by Mr Pat Ahern, of CIT

## REFERENCES

- (1) Akhieser, A, Journal of Physics, 1, 278, (1939)
- (2) J. Lamb, J. Richter, Proceedings of the Royal Society London A, (08/1966)
- (3) A. R. Hutson, D. L. White, Journal of Applied Physics, 33, 1, (1962)
- (4) D. L. White, Journal of Applied Physics, 33, 2547, (1962)
- (5) C.A.A Greebe, Philips Res. Rep. 20, 1, (1965)
- (6) John Twomey, PhD Thesis, "Acoustoelectric Instability in Acoustically Amplifying CdS", University Of Glasgow, Scotland, U.K, (1970)
- (7) Shao Ying Huang, INET Search:= loss tangent shao ying huang
- (8) Jerzy Antoni Krupka, Warsaw University of Technology, August, (2002)
- (9) C.P. Wen, R.F. Mayo, Applied Physics, 9, 4, (1966)
- (10) Ru-Yuan Yang, Yan-Kuin Su et al., J. of Applied Physics, 101, 014101, (2007)
- (11) K C Cheng, H.L.W.Chan et al.. Proc.IEEE, International Symp, Ferroelectrics (2000)
- (12) Zhongxiang.Zhou, Yang Li, et al., Optics Communications, 7, 2624, (2009)
- (13) P.Nagels, Material Sci. Dept., B-2400, MOL, Belgium.