International Journal of Applied Engineering Research ISSN 0973-4562 Volume 14, Number 9 (2019) pp. 2126-2133

Effect of non-inertial acceleration on heat transport by Rayleigh-Bénard magnetoconvection in Boussinesq-Stokes suspension with variable heat source

Ramachandramurthy[1], V., Uma, D.[2], Kavitha, N.[3]

[1,3]*Department of Mathematics, Ramaiah Institute of Technology, Bengaluru-560 056, India.*

[3]*Department of MCA, PES Institute of Technology, Bengaluru-560 085, India.*

Abstract
The effect of non-inertial acceleration and the temperature dependent volumetric heat source is discussed for the Rayleigh - Bénard convection of couple-stress fluid in the presence of applied magnetic field. Both linear and weakly non-linear analysis is performed in the present problem. The eigen boundary value problem is solved analytically for the external Rayleigh number. Linear stability analysis discusses the stationary mode of convection where as non-linear theory helps in analyzing the quantification of heat transport. The non-linear theory is based on the truncated representation of Fourier series.The Lorenz model for the system is derived and are solved using the numerical method. The thermal Nusselt number is calculated and the influence of Taylor number and the other parameters are discussed in the presence and absence of non-inertial acceleration.
Keywords: Rayleigh-Bénard convection; Boussinesq Stokes suspension; Boussinesq aproximation; Magnetoconvection;Taylor number; non-inertial acceleration.

1. INTRODUCTION

Thermal convection in a horizontal fluid layer in the presence of internal heat source subject to constant but of different temperatures at the boundaries has been extensively investigated by many researchers due to its practical importance in engineering and geophysical problems. The Rayleigh-Bénard convection in Newtonian fluids with heat sources has been analyzed by Sparrow et. al[1]. Many theoretical studies on thermal convection given by Roberts[2] ,McKenzie [3], Kulacki and Goldstein [4], Thirlby[5] et. al. Also the other researchers Palm[6], Tritton and Zarraga[7], Clever et. al. [8] gave the experimental results. In all the above works, the internal heat source Q^* is assumed as uniform but in many practical problems and applications, Q^* is nonuniform in nature, which is due to many internal factors such as heat release of chemical reaction that takes place in the fluids, heat source produced by radiation from external medium, radioactive decay and so on. Riahi[9] has studied the nonlinear convection in the harizantal layer with heat source. He showed that the effect of nonuniform internal heat source Q^* strongly affects the cell size, the stability of convective motion and the internal motion of hexagonal cells. Riahi[10] also studied the problem of nonlinear thermal convection in a low Prandtle number fluid with internal heating. Sparrow et al.[1] and Palm[6] studied the

<u>**Nomenclature**</u>

Latin symbols:

d	Depth of the fluid layer
g	acceleration due to gravity
H_0	Magnetic field
p	pressure
$\vec{q}$	Velocity (u,v,w)
Pr	Prandtl number
P_m	Magnetic Prandtl number
Q	Chandrasekhar number
C	Couple Stress Parameter
T_a	Taylor number
R_I	Internal Rayleigh number
R_E	External Rayleigh number
N_u	Nusselt Number
T	Temperature
t	Time
ΔT	Temperature difference between the plates

Greek symbols:

$\pi\alpha$	Wave number
μ	Dynamic Viscosity
μ'	Couple-stress Viscosity
μ_m	Magnetic permeability
β	Thermal expansion coefficient
χ	Constant Thermal diffusivity
ρ	Density
ψ	Stream function
Ψ	Perturbed Stream function
Θ	Perturbed Temperature
Ω	angular velocity of rotation
ζ	z - component of vorticity

Other symbols:

$\widehat{i}$	Unit vector normal in x - direction
$\widehat{k}$	Unit vector normal in z - direction
∇^2	Laplacian operator

thermal instability in a horizontal fluid layer with a uniformly distributed volumetric heat source. Bhattacharya and Jena[11] have analyzed the thermal instability of horizontal layer of micro polar fluid with heat source. They found that heat source and heat sink both have the same destabilizing effect in micro polar fluids. Siddheshwar and Stephan Titus[12] have made a detailed linear and nonlinear stability analysis for the Newtonian fluid with heat source/sink analytically using Lorenz and Ginzburg-Landau models. Siddheshwar and Meenakshi[13] studied the Rayleigh-Bénard convection for the twenty nanoliquids in the presence of volumetric heat source analytically.

In most of the above works researches have considered the effect of heat source for Newtonian fluids. But in many practical situations, most of the fluids are not so pure, which may contains suspended particles like polymeric suspensions, liquid crystals etc. Suspended particles plays a very important role in the analysis of fluid. Due to the presence of suspended particles there is a large stabilizing/destabilizing effect on the thermal convection of the fluids. Hence the study related to non-Newtonian fluids with heat source/sink in our modern science technology is desirable. The study on such fluids has been a field of sprightly research for the last few decades especially in many industrially important fluids like polymeric suspensions,chemicals, paints, solidifications of liquid crystals, pharmaceutical, food and beverage as well as refrigeration and so on.

The another most important aspect in the study of convection of fluids is due to the presence of rotating medium. The Rayleigh-Bénard convection in the presence of rotation has been studied by Sutton[14], Chandrasekhar [15], Acheson[16], Khiri[17], Chatterjee[18] et.al. and references therein. Rotating convection was first treated theoretically by Sutton[14], and studied the combined effect of non-uniform temperature gradient caused by rapid heating or cooling at the boundaries and Coriolis force rotation. Chandrasekhar[15] who devoted the third chapter of his book to the phenomena, in which he extended Rayleigh's analysis to the rotating case. But all these previous investigators have considered the effect of rotation/non-inertial acceleration in Newtonian fluids. The convective instability in a couple-stress fluid layer heated from below have been investigated in the recent past. Siddheshwar and Sakshath [19] studied both the linear and non-linear stability analyses of Rayleigh-Benard convection for the Newtonian nanoliquid in a rotating medium. They considered rigid-rigid isothermal boundaries for investigation and concluded that the onset of convection is delayed due to rotation and hence leads to decrease in heat transport. Balasubramanian and Robert[20] presented an experimental work on Rayleigh-Benard convection in the presence of rotation for Newtonian nano-liquids. They considered rigid-rigid isothermal boundaries for investigation. They observed that the onset of convection is delayed due to rotation and hence leads to decrease in the heat transport. Further they proved that the amount of heat transport is less in the case of rigid-rigid isothermal boundaries when compared to free-free isothermal boundaries. The same work have been done by Ramya Rajagopal, Shelin Elizabeth and Sangeetha George [21] for free-free isothermal boundaries theoriticaly. They discussed the non-linear stability analysis of Rayleigh-Benard convection in the presence of magnetic field and couple stress fluid an addition with rotating medium. The combined effect of rotational modulation and magnetic field for the Rayleigh-Benard convection of couple stress fluid using Ginzburg-Laundau model is discussed in detail. The Rayleigh–Benard convection in a square enclosure filled with viscoplastic fluid has been investigated by Hassan and Manabendra[22], numerically as well as experimentally. Sekhar and Jayalatha[23] have made a linear stability analysis of thermal convection in variable viscosity for Newtonian ferromagnetic liquid by considering all possible boundary combinations in the presence of heat source using Galerkin and shooting techniques.. Maruthamanikandan[24] et.al. presented a work on Marangoni convective instability in a ferromagnetic fluid layer in the presence of a spatial heat source and viscosity variation. A local nonlinear stability analysis using a eight-mode expansion is performed in arriving at the coupled amplitude equations for Rayleigh-Benard-Brinkman convection (RBBC) in the presence of LTNE effects by Siddheshwar and Kanchana[25]. Siddheshwar and Siddabasappa[26] studied the effect of local thermal non-equilibrium (LTNE) on onset of Brinkman–Bénard convection and on heat transport is investigated. The work carried out in present paper aims in analyzing the effects Coriolis force and Boussinesq-Stokes suspension for the fluid with Rayleigh-Bénard convection in presence with applied magnetic field and heat source/sink both linearly and non-linearly.

2. MATHEMATICAL FORMULATION

Consider the Boussinesq Stokes fluid between two parallel plates of infinite length and depth'd'. The lower and upper boundary are maintained at different temperatures so that there is temperature gradient $\Delta T > 0$. Here the boundaries are considered as stress free and isothermal (i.e maintained at constant temperature). The upper boundary is maintained at temperature T_0 where as lower boundary at $T_0 + \Delta T$. The transverse magnetic field H_0 is the applied vertically along the Z-direction. The schematic representation of the problem is shown in the Fig.1. For the sake of analysis all physical quantities of the fluid are assumed to be independent of y and considered x and z axis for the reference. Subjected to the Oberbeek-Boussinesq approximations, the governing equations describing the Rayleigh-Bénard instability situation for the electrically conducting couple stress fluid are

$$\nabla.\vec{q} = 0, \tag{1}$$

$$\nabla.\vec{H} = 0, \tag{2}$$

$$\left.\begin{aligned} \rho_0\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\,\vec{q}\right] = -\nabla p - \rho\,(T)\,g\hat{k} + \mu\nabla^2\vec{q} - \mu'\nabla^4\vec{q} \\ -\mu_m^2\left(\vec{H}.\nabla\right)\vec{H} - 2\rho_0(\vec{q}\times\vec{\Omega}), \end{aligned}\right\} \tag{3}$$

$$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \chi \nabla^2 T + Q^*(T - T_0), \quad (4)$$

$$\frac{\partial \vec{H}}{\partial t} + (\vec{q}.\nabla)\vec{H} = \left(\vec{H}.\nabla\right)\vec{q} + \nu_m \nabla^2 \vec{H}, \quad (5)$$

$$\rho(T) = \rho_0[1 - \beta(T - T_0)], \quad (6)$$

The term $Q^*(T - T_0)$ in the equation (4) represents the temperature dependent heat source/sink.

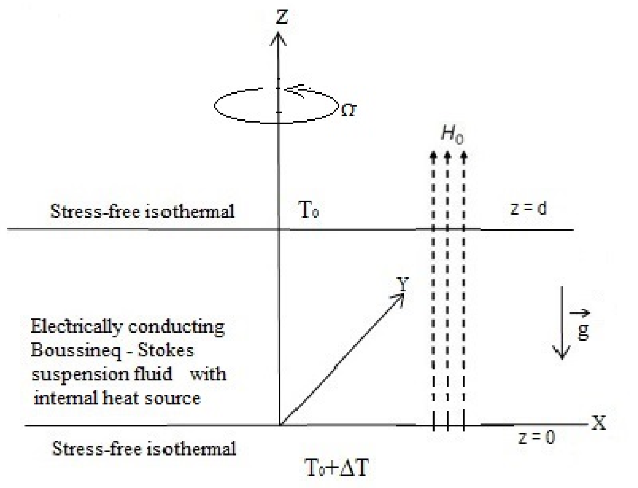

Figure 1: Schematic diagram of the flow configuration

Assuming the components of velocity $\vec{q}$, temperature T and density ρ in the basic state as $\vec{q_b}, T_b(z)$ and $\rho_b(z)$, the solutions obtained in quiescent state of the form :

$$\left.\begin{array}{l} \vec{q} = (0,0,0), \quad T_b = T_0 + \Delta T f\left(\frac{z}{d}\right) \\ \vec{\Omega} = \Omega_0 \hat{k}, \quad \vec{H_b} = H_0 \hat{k} \\ \rho_b\left(\frac{z}{d}\right) = \rho_0\left[1 - \beta \Delta T f\left(\frac{z}{d}\right)\right] \\ p_b\left(\frac{z}{d}\right) = -\int \rho_b\left(\frac{z}{d}\right) g d\left(\frac{z}{d}\right) + c_1 \end{array}\right\}, \quad (7)$$

Where, $f(\frac{z}{d}) = \frac{\left[sin\sqrt{R_I}\left(1-\frac{Z}{d}\right)\right]}{sin\sqrt{R_I}}$, $R_I = \frac{Q^* d^2}{\chi}$ (Internal Rayleigh number) and C_1 is the constant of integration

We now superimpose the finite amplitude perturbations for the basic state in the form,

$$\left.\begin{array}{c} \vec{q} = \vec{q_b} + \vec{q}', \quad T = T_b(Z) + T', \quad \rho = \rho_b(Z) + \rho' \\ p = p_b(Z) + \dot{p} \quad \vec{H} = \vec{H}_b(Z) + \vec{H}', \end{array}\right\} \quad (8)$$

where, the prime indicates a perturbed quantity. Substituting Eqn.(9) in the governing equations, we get the following component equations:

$$\nabla.\vec{q}^{'} = 0, \quad (9)$$

$$\nabla.\vec{H}^{'} = 0, \quad (10)$$

$$\left.\begin{array}{r} \rho_0\left[\frac{\partial \vec{q}'}{\partial t} + (\vec{q}'.\nabla)\vec{q}'\right] = -\nabla p' - \rho'(T) g\hat{k} + \mu\nabla^2\vec{q}' - \mu'\nabla^4\vec{q}' \\ -\mu_m^2\left(\vec{H}'.\nabla\right)\vec{H}' + \mu_m H_b \frac{\partial \vec{H}'}{\partial z} \end{array}\right\} \quad (11)$$

$$\frac{\partial T^{'}}{\partial t} + (\vec{q}^{'}.\nabla)T^{'} + w^{'}\frac{\partial T_b}{\partial z} = \chi\nabla^2 T^{'}, \quad (12)$$

$$\frac{\partial \vec{H}^{'}}{\partial t} + (\vec{q}^{'}.\nabla)\vec{H}^{'} - \left(\vec{H}^{'}.\nabla\right)\vec{q}^{'} - H_b\frac{\partial w\prime}{\partial z} = \nu_m \nabla^2 \vec{H}^{'}, \quad (13)$$

$$\rho^{'} = -\rho_0 \beta T^{'}. \quad (14)$$

Operating curl twice for the Eq.(11), to eliminate pressure p ,we get

$$\begin{aligned} \rho_0 \frac{\partial}{\partial t}(\nabla^2 w') =& \alpha\rho_0 g \nabla_1^2 \theta' + \mu\nabla^6 w' \\ & - \mu'\nabla^4 w' - \mu_m^2 \sigma H_0^2 w' - 2\rho_0\Omega\frac{\partial \zeta}{\partial z} \end{aligned} \quad (15)$$

where $\zeta = \left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\right)$ is the z-component of the vorticity, $\vec{w}' = \nabla \times \vec{q}'$. Now by differentiating x and y components of Eq.(11) partially with respect to y and x respectively, then subtracting the resulting equations from one another, which again gives the following equation

$$\rho_0 \frac{\partial \zeta}{\partial t} = 2\rho_0\Omega\frac{\partial w}{\partial z} + \mu\nabla^2\zeta - \mu'\nabla^4\zeta - \mu_m^2\sigma H_0^2 \zeta \quad (16)$$

In the view of analyzing the fluid, we consider only two dimensional disturbances, hence we introduce magnetic potential $\phi^{'}$ and stream function $\psi^{'}$ as

$$\left.\begin{array}{c} u' = -\frac{\partial \psi'}{\partial Z}, \ w' = \frac{\partial \psi'}{\partial X} \\ H'_x = -\frac{\partial \phi'}{\partial Z}, \ H'_z = \frac{\partial \phi'}{\partial X} \end{array}\right\} \quad (17)$$

The classical procedure of operating curl for the eq (12) helps in eliminating the pressure p and transforming the above system of equations to dimensionless equations using the scaling mentioned below,

$$\left.\begin{array}{l} (X,Z) = \left(\frac{x}{d}, \frac{z}{d}\right), \quad \Psi = \frac{\psi'}{\chi}, \quad \theta = \frac{T'}{\Delta T}, \quad \Phi = \frac{\phi}{dH_b} \\ \zeta^* = \frac{\zeta}{(\chi/d^2)}, \ t^* = \frac{t}{(d^2/\chi)}, \ w^* = \frac{w'}{(\chi/d)} \end{array}\right\} \quad (18)$$

We obtain the non-dimensional governing equations as

$$\left.\begin{array}{l} \frac{1}{Pr}\Big(\frac{\partial}{\partial \tau}(\nabla^2\Psi) + J(\Psi, \nabla^2\Psi)\Big) \\ \quad = R_E\frac{\partial \Theta}{\partial X} + QP_m\Big(\frac{\partial(\nabla^2\Phi)}{\partial Z} + J(\Phi, \nabla^2\Phi)\Big) \\ \quad - \sqrt{T_a}\frac{\partial}{\partial Z} - C\nabla^6\Psi + \mu\nabla^4\Psi \end{array}\right\} \quad (19)$$

$$\begin{aligned} \frac{\partial \Theta}{\partial \tau} =& \frac{\partial \Psi}{\partial X}\left(1 + 2\sum_{n=1}^{\infty}\frac{R_I}{R_I - n^2\pi^2}Cosn\pi Z\right) \\ & + \nabla^2\Theta + R_I\Theta - J(\Psi, \Theta) \end{aligned} \quad (20)$$

$$\left(\frac{\partial}{\partial\tau} - p_m\nabla^2\right)\Phi = \frac{\partial\Psi}{\partial Z} + J(\Psi,\Phi) \tag{21}$$

$$-\frac{1}{Pr}\left(\frac{\partial\zeta}{\partial\tau} + J(\Psi,\zeta)\right) = \nabla^2\zeta - C\nabla^4\zeta - \sqrt{T_a}\frac{\partial\Psi}{\partial Z} \tag{22}$$

where, The Jacobian $J(F_1,H_1) = \frac{\partial F_1}{\partial X}\frac{\partial H_1}{\partial Z} - \frac{\partial F_1}{\partial Z}\frac{\partial H_1}{\partial X}$ and

The boundary conditions for the present problem on velocity and temperature are

$$\left.\begin{array}{l} \Psi = \nabla^2\Psi = \Theta = D\Phi = 0 \quad at \quad Z = 0 \\ \Psi = \nabla^2\Psi = \Theta = D\Phi = 0 \quad at \quad Z = 1 \end{array}\right\} \tag{23}$$

Using (23), the solutions of dimensionless equations (19), (20), (21) and (22) are obtained.

2.1. Linear Theory

To perform the linear stability analysis, the linearized version of the Eqs.(19), (20), (21) and (22) are considered along with the stress free and isothermal boundary conditions (23). Which means that, it is essential to neglect the Jacobian's $\frac{\partial(\Psi,\nabla^2\Psi)}{\partial(X,Z)}$, $\frac{\partial(\Phi,\nabla^2\Phi)}{\partial(X,Z)}$ and $\frac{\partial(\Psi,\Theta)}{\partial(X,Z)}, \frac{\partial(\Psi,\Phi)}{\partial(X,Z)}$ in Eqs. (19), (20), (21) and (22). The neglect of the Jacobian's is essential to remove products of amplitudes which are very small in studying the linear theory.

The solution of linear versions of these equations are assumed to be periodic waves (see Chandrasekhar [17]) in the form

$$\left.\begin{array}{l} \Psi(X,Z,\tau) = \Psi_0 sin\pi\alpha X sin\pi Z \\ \Theta(X,Z,\tau) = \Theta_0 cos\pi\alpha X sin\pi Z \\ \Phi(X,Z,\tau) = \Phi_0 sin\pi\alpha X cos\pi Z \\ \zeta(X,Z,\tau) = \zeta_0 sin\pi\alpha X cos\pi Z \end{array}\right\} \tag{24}$$

then,

$$R_{E_c} = \frac{\pi^2 T_a + (1 + C\eta_1^2)(Q\pi^2\eta_1^2 + \eta_1^6(1 + C\eta_1^2))}{\pi^2\alpha^2(1 + C\eta_1^2)}. \tag{25}$$

Where, $\pi\alpha$ is the horizontal wave number. The quantities Ψ_0, Θ_0 , Φ_0 and ζ_0 are the amplitudes of the stream function, temperature, magnetic potential and vorticity function respectively.

Clearly, Ψ,Θ, Φ and ζ as assumed in Eqs.(24) satisfy the boundary conditions (23). Now Substituting the trial functions assumed in the equations (22) in the linearized version of the dimensionless equations (19), (20),(21) and (22) , using the standard Galerkin procedure (i.e integrating the equations with respect to X and Z between $[0, \frac{2\pi}{\pi\alpha}]$ and $[0, 1]$ respectively), we obtain the set of homogeneous equations in Ψ_0,Θ_0 ,Φ_0 and ζ_0. Hence the critical value of R_{E_c} as in equation (25) is obtained, which signifies the onset of convection.

2.2. Non-linear Theory

The representation of a minimal double Fourier series for the stream function Ψ, magnetic potential Φ and the temperature Θ which describes the finite amplitude convection in the fluid is given by

$$\left.\begin{array}{l} \Psi(X,Z,\tau) = A(\tau) sin\pi\alpha X sin\pi Z \\ \Theta(X,Z,\tau) = B(\tau) cos\pi\alpha X sin\pi Z - D(\tau) sin2\pi Z \\ \Phi(X,Z,\tau) = E(\tau) sin\pi\alpha X cos\pi Z - F(\tau) sin2\pi\alpha X \\ \zeta(X,Z,\tau) = G(\tau) sin\pi\alpha X cos\pi Z - H(\tau) sin2\pi\alpha X \end{array}\right\}, \tag{26}$$

where A, B, D, E, F, G and H are the amplitudes to be determined from the dynamics of the system. Then substitute the Eq. (26) into the dimensionless equations.(19), (20),(21) and (22) and proceeding using the standard Galerkin procedure, we obtain the following non-linear autonomous system of differential equations:

$$\frac{dA}{d\tau_1} = \left[\begin{array}{r} (C\eta_1^2 + C_1)A + C_2 B - PmQ\left(\frac{4\pi^4\alpha^3 - 2\pi^2\alpha\eta_1^2}{\eta_1^4}\right)EF \\ -\frac{(PmQ\pi)}{\eta_1^2}E - -\frac{(Pm\,\pi\sqrt{Ta})}{\eta_1^2}F \end{array}\right]\Bigg\} \tag{27}$$

$$\frac{dB}{d\tau_1} = \frac{1}{\eta_1^2}\left[\frac{4\pi^3\alpha}{4\pi^2 - R_I}A - (R_I - \eta_1^2)B - \pi^2\alpha AD\right] \tag{28}$$

$$\frac{dD}{d\tau_1} = \frac{1}{\eta_1^2}\left[\frac{\pi^2\alpha}{2}AB - (4\pi^2 - R_I)D\right] \tag{29}$$

$$\frac{dE}{d\tau_1} = \frac{1}{\eta_1^2}[\pi A - p_m\eta_1^2 E + \pi^2\alpha AF] \tag{30}$$

$$\frac{dF}{d\tau_1} = \frac{1}{\eta_1^2}[-\pi^2\alpha AE - 4\pi^2\alpha^2 p_m F] \tag{31}$$

$$\frac{dG}{d\tau_1} = \frac{1}{\eta_1^2}\left[p_r\pi\sqrt{T_a}A + p_r(\eta_1^2 + C\eta_1^4)G - \frac{\pi^2\alpha}{4}AH\right] \tag{32}$$

$$\frac{dH}{d\tau_1} = \frac{1}{\eta_1^2}\left[-\frac{\pi^2\alpha}{2}AG - 4\pi^2\alpha^2 p_r H - 16\pi^4\alpha^4 p_r H\right] \tag{33}$$

where $C_1 = Pr\left[\left(\frac{\eta_1^2 - 2\pi^2}{2\eta_1^2}\right)a_2 - \frac{a_0}{2}\right]$,
$C_2 = \frac{Pr\pi^2}{\eta_1^2}\left[\frac{1+\alpha^2}{2}a_0 + \frac{1-\alpha^2}{2}a_2\right]$ and $\tau_1 = \eta_1^2\tau$.

The above system of non-linear differential equations are difficult to be solved using analytical procedure for the general time dependent variable. So these equations has been solved using numerical method like Runge-Kutta Fehlberg45 with the help of Mathematica 9.0 software .

3. HEAT TRANSPORT

The quantification of heat transport is very important in the study of convection in Boussinesq Stokes suspension fluids, as convection sets in with the increase of the temperature gradient across the layer of fluid, which can be detected by its effect on the heat transport.

The horizontally averaged Nusselt number Nu, for the stationary mode of magnetoconvection is given by

$$Nu(\tau) = \frac{\left[\frac{\alpha_c}{2}\left(\int\limits_{X=0}^{2/\alpha}(1-Z+\Theta)_Z dX\right)\right]_{Z=0}}{\left[\frac{\alpha_c}{2}\left(\int\limits_{X=0}^{2/\alpha}(1-Z)_Z dX\right)\right]_{Z=0}}, \quad (34)$$

Substituting the Eq. (24b) in Eq. (30), the Nusselt number $Nu(\tau)$ expression is obtained as:

$$Nu(\tau) = 1 + \frac{2}{\left(\frac{\pi^2\alpha^2 R_{E_c}}{\eta^6}\right)\left(1-\frac{R_I}{4\pi^2}\right)} \frac{tan\sqrt{R_I}}{R_I} D(\tau) \quad (35)$$

4. RESULTS AND DISCUSSIONS

In this paper the effect of rotation on the onset of magnetoconvection for the couple-stress fluid is discussed in the presence of heat source/sink. The following effects on the Rayleigh-Bénard magnetoconvection are considered.

1. Suspended particles
2. Heat source/sink
3. Applied magnetic field
4. Coriolis force

The effects of the above four are studied under the influence of the couple stress parameter C, Internal Rayleigh number R_I, Chandrasekhar number Q and the Taylor number T_a. The main emphasis of the present study is to consider the impact of these parameters on the onset of convention. A linear and non-linear stability analysis is performed in the problem with these constraints.

4.1. Linear theory

The magnetoconvection of the fluid is studied using Hartmann formulation. Before we move on to the discussion we note that in the case when the system is rotated it generates an additional component of velocity and hence some energy is used up in this. This results in the delay of onset of convection, which is reflected in the Table(1) and (2). Both the tables clearly explains the linear stability analysis under the influence of Taylor number T_a, Internal Rayleigh number R_I and Chandrasekhar number Q. The effect of increasing C corresponds to increase in the values of R_{E_c}. This is due to the fact that suspended particles stabilize the system. The similar effect can be observed even in the presence of Taylor number T_a and the applied vertical magnetic field, i.e increase in Q and T_a increases the external Rayleigh number R_{E_c}. They individually inhibits the onset of Rayleigh-Bénard convection. The effect of heat source/sink is opposite to these. From the Tables (1) and (2) it is evident that increase in R_I is to decrease in R_{E_c} exhibiting the destabilization of the system. The important result in the present study that the system is destabilized in the absence of rotation due to the presence of internal heat source even if the suspended particles exist in the system with an applied magnetic field. It is also clear from these tables that the system is destabilized in the absence of both suspended particles and the magnetic field and also for smaller values of these parameters.

4.2. Non-linear theory

The following are the important highlights of non-linear stability analysis.

1. the non-linear system of equations are obtained in the form of generalized Lorenz model,
2. quantification of heat transport is analyzed through Nusselt number for the stationary mode of magnetoconvection in the Boussinesq Stokes suspension fluid under the influence of rotation and variable heat source.
3. plotting of Nusselt number variation with respect to time in order to study the individual effects of the parameters involved.

Before we start on the discussion of the results, we note that the truncated Fourier series models are good enough to represent Rayleigh Bénard convection (Ref Siddeheshwar(12)). Also the Lorenz model of the problem is solved numerically using Runge-Kutta Fehlberg 45 method with the adoptive step size. In order to carry out the numerical integration of the coupled system, we have used the initial conditions $A(0) = B(0) = D(0) = E(0) = F(0) = G(0) = H(0) = 5$. Due to the suspended particles being present in the carrier liquid, the prandtl number of the couple-stress liquids is taken much higher than the Newtonian carrier liquid. Hence we have taken the value of the prandtl number as $P_r = 10$ and is maintained through out the discussion. There are many theoretical or experimental researches have been done on linear and non-linear analysis of the fluid, either by considering the individual effect or by considering the combined effect of the few of the parameters. Here an effort has been made in analyzing the combined effect of rotation, magnetic field, suspended particles in addition to heat source/sink on heat transfer. In order to study the amount of quantification of heat transfer, we computed the variation of space-averaged Nusselt number with time τ. The figures from (2) to (7) are the plots of Nusselt number N_u versus time τ for various values of $T_a, Q, C,$ and R_I. Here the figures from (2) to (4) are the plots of Nusselt number N_u for the couple stress parameter $C = 0.02$ and figures from (5) to (7) are the plots of Nusselt number N_u verses τ for $C = 0.04$. Due to the influence of suspended particles in the fluid heat transport decreases both in the presence or absence of rotation, which are reflected in the figures from (2) to (7). The similar effect can even be observed in the presence of magnetic field Q and Taylor number T_a. On comparing the Nusselt number N_u plots (2),(3),(4) and (5), (6) (7) we can notice the effect of internal heat source/sink.

5. CONCLUSION

1. The Effect of increases in couple stress parameter C is to decrease the Nusselt number, these indicates the stabilizing effect of the system.
2. The effect of increase in Chandrasekhar number Q and Taylor number T_a is to delay the heat transfer, i.e decrease in Nusselt number. Hence reducing heat transfer and stabilizing the system.
3. The effect of increase in internal Rayleigh number R_I increases the heat transfer and hence destabilize the system.

Table 1: Critical values of Rayleigh number R_{E_c} corresponds to the wave number $\pi\alpha_c$ for Q = 2

R_I	C	T_a = 0		T_a = 100		T_a = 1000	
		$\pi\alpha_c$	R_{E_c}	$\pi\alpha_c$	R_{E_c}	$\pi\alpha_c$	R_{E_c}
-1	0.02	2.212	997.59	2.449	1149.44	3.267	1977.77
	0.04	2.133	1207.8	2.306	1345.03	2.994	2133.74
0	0.02	2.180	910.93	2.419	1054.13	3.239	1839.02
	0.04	2.104	1101.28	2.278	1230.44	2.966	1975.64
1	0.02	2.143	826.8	2.388	961.54	3.2044	1703.93
	0.04	2.072	997.97	2.243	1119.22	2.934	1821.91

Table 2: Critical values of Rayleigh number R_{E_c} corresponds to the wave number $\pi\alpha_c$ for Q = 4

R_I	C	T_a = 0		T_a = 100		T_a = 1000	
		$\pi\alpha_c$	R_{E_c}	$\pi\alpha_c$	R_{E_c}	$\pi\alpha_c$	R_{E_c}
-1	0.02	2.275	1061.52	2.485	1205.89	3.283	2018.5
	0.04	2.190	1275.11	2.347	1405.9	3.006	2178.42
0	0.02	2.243	970.49	2.454	1106.8	3.252	1877.26
	0.04	2.158	1163.87	2.318	1287.09	2.978	2017.44
1	0.02	2.205	882.08	2.419	1010.5	3.217	1739.72
	0.04	2.127	1055.93	2.284	1171.74	2.947	1860.89

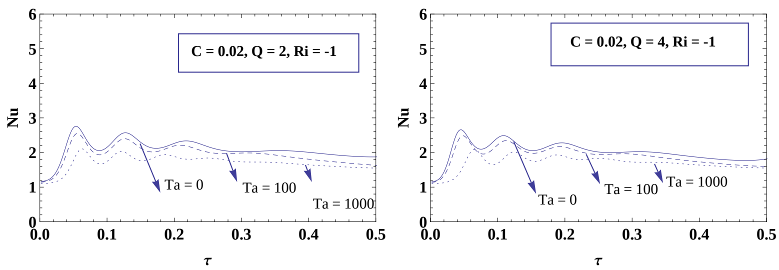

Figure 2: Nusselt number N_u variation plot with τ

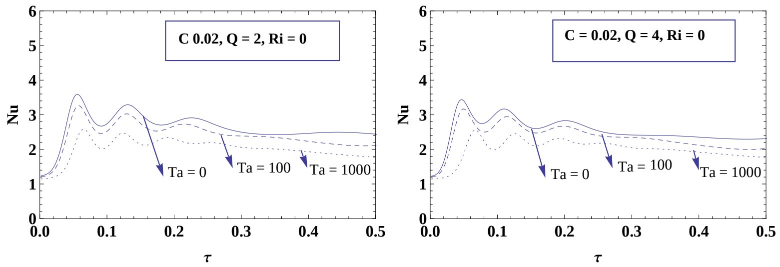

Figure 3: Nusselt number N_u variation plot with τ

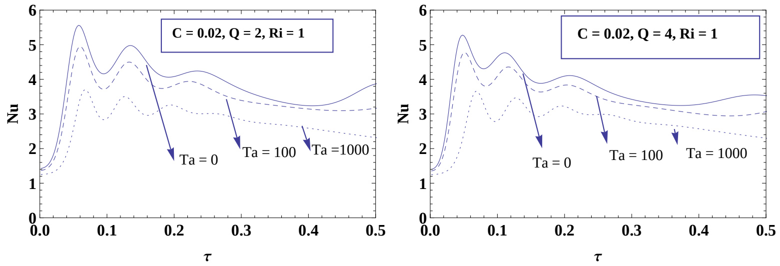

Figure 4: Nusselt number N_u variation plot with τ

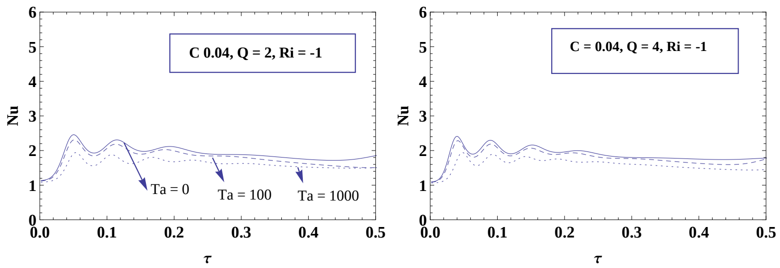

Figure 5: Nusselt number N_u variation plot with τ

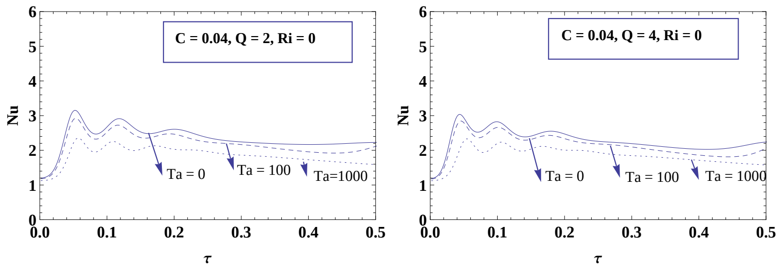

Figure 6: Nusselt number N_u variation plot with τ

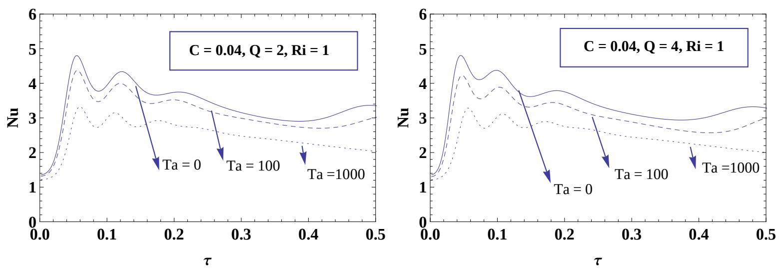

Figure 7: Nusselt number N_u variation plot with τ

ACKNOWLEDGEMENTS

The first and second authors are grateful to the principal and management of Ramaiah Institute of Technology, Bengaluru. Where as the second author is grateful to the management of PES university for the encouragement of research activities.

REFERENCES

[1] Sparrow,E.M., Goldstein,R.J., 1964, "Thermal instability in harizantal fluid layer", J. Fluid Mech. 18(4), pp 513-528.

[2] Roberts, P. H., 1967, "Convection in Horizontal Layers With Internal Heat Generation", J. Fluid Mech., 30, pp. 33-9.

[3] McKenzie, D. P., Roberts, J. M., and Weiss, N. O., 1974, "Convection in the Earth's Mantle: Towards a Numerical Simulation", J. Fluid Mech.,62, 465-538.

[4] Kulacki, F.A., Goldstein,R.J., 1972, "Thermal convection in harizantal fluid layer with volumetric heat source", J. Fluid Mech. 55(2), 271-287.

[5] Thirlby, R., 1970, "Convection in an Internally Heated Layer", J. Fluid Mech.,44, pp. 673-693.

[6] Palm,E., 1975, "Nonlinear Thermal Convection", J. Fluid Mech.,7,39,.

[7] Tritton,D.J, Zarraga,M.N.,1967, "Convection in Horizontal Layers With Internal Heat Generation", Experiments, J. Fluid Mech. pp.30-21.

[8] Clever, R.M., 1977, "Heat Transfer and Stability Properties of Convection Rolls in an Internally Heated Fluid Layer with an Internal Heat Source", J. Phys. Soc. Jpn, 53, pp.4169-4178.

[9] Riahi, N, 1984, "Nonlinear Convection in a Horizontal Layer With an Internal Heat Source", J. Phys. Soc. Jpn, 53, pp.4169-4178.

[10] Riahi, N., 1986, "Convection in a low Prantle number fluid with internal heating", Int J.Non-lin.Mech. Jpn, 21, pp.97.

[11] Bhattacharyya,S.P., and Jena, S.K., 1984, "Thermal instability of harizantal layer of micro polar fluid with heat source", Proceedings of Indian academy of mathematical sciences 93(1), pp. 13-26 .

[12] Siddheshwar, P.G., and Stephen Titus, P., 2013,"Nonlinear Rayleigh-Benard convection with variable heat source", J. heat transfer vol. 135, 122502.

[13] Siddheshwar, P.G, Meenakshi, N., 2018," A theoretical study of enhanced heat transfer in nanoliquids with volumetric heat source", J. applied mathematics and computing, 57,703–728.

[14] Sutton,O.G., 1950, "On the stability of fluid heated from below", Proceedings of Roy.Soc.A, pp 204-257.

[15] Chandrasekhar,S., 1961, "Hydrodynamic and Hydromagnetic Stability", Oxford University Press, Oxford.

[16] Acheson,D.J., 1979, "On magneto convection in a rotating fluid layer of high thermal conductivity ", J. Astrophysics and Space Science, 65, pp 453-458.

[17] Khiri,R., 2004, " Coriolis effect on convection for a low Prandtl number fluid", International J. Non-Linear Mechanics, 39(4), pp .593-604.

[18] Chatterjee,S., Basak,T.,and Das.S., 2008, "Onset of natural convection in a rotating fluid layer with non-uniform volumetric heat sources", International Journal of Thermal Sciences, 47(6), June , pp. 730-741.

[19] Siddheshwar,P. G., and Sakshath., 2017, "Rayleigh-Benard-Taylor convection of Newtonian nanoliquid, Int. J. of Mechanical and Mechatronics Engineering Vol 11(6), pp. 1225-1229.

[20] Balasubramanian,S., and Robert,E.E., 2013, "Experimental study of Rayleigh-Benard convection in the presence of rotation", International J. of Materials, Mechanics and manufacturing, Vol 1(2), pp. 142-152.

[21] Ramya Rajagopal, Shelin Elizabeth, and Sangeetha George,K., 2018, "A non-linear stability analysis of Rayleigh-Benard magneto-convection of a couple stress fluid in the presence of rotational modulation", Int. J. of mathematics trends and technology, 54, pp. 477-484.

[22] Hassan,M.A., Manabendra Pathak, Mohd. KaleemKhan, 2015, "Rayleigh–Benard convection in Herschel–Bulkley fluid, J. of non-newtonian fluid mechanics", 226, pp 32-45.

[23] Sekhar, G. N., Jayalatha, G., Prakash, R., 2017, "Thermal convection in variable viscosity ferromagnetic liquids with heat source", Int. J. Appl. Comput. Maths, 3, pp 3539–3559.

[24] Maruthamanikandan, S., Nisha Mary Thomas, and Soya Mathew, 2018 "Thermorheological and magnetorheological effects on Marangoni-ferroconvection with internal heat generation", J. Physics Conf. Series, 1139 , 012024.

[25] Siddheshwar, P. G. , Vanishree, R. K., and Kanchana, C., 2017, "Study of Rayleigh-Benard-Brinkman convection using LTNE model and coupled, real Ginzburg-Landau equations", International scholarly and scientific research and innovation 11(6).

[26] Siddheshwar, P.G., Siddabasappa,C., 2017, "linear and weakly nonlinear stability analysis of two-dimensional, steady Brinkman–Bénard convection using local thermal non-equilibrium model", J. Transport on porous media, 120, pp 605–631.

[27] Saravanan,S., and Senthil Nayaki.V.P.M., 2014," Thermorheological effect on thermal non equilibrium porous convection with heat generation", Int. J. Engg. Sciences, 74, pp 55-64.

[28] Straughan,B., 2015, "Convection with local thermal non-equilibrium and microfluidic effects", Springer, New York.

[29] Malashetty, M.S., and Mahantesh Swamy, 2010," Effect of rotation on the onset of thermal convection in a sparsely packed porous layer using a thermal non-equilibrium model", Int. J. Heat and Mass Tran., 53(15), pp. 3088-3011.

[30] Sekhar, G. N., and Jayalatha, G., 2010, "Elastic effects on Rayleigh-Benard convection in liquids with temperature-dependent viscosity", Int. J. of Thermal Sci., 49, pp. 67-79.

[31] Sekhar,G.N., Jayalatha,G. and Prakash,R., 2013, "Thermorheological and magnetorheological effects on Rayleigh-Benard-Marangoni convection in ferromagnetic liquids with non-uniform basic temperature gradient", Proc. ASME, Vol 7A, pp.1-10.

[32] Bhadauria,B.S., Siddheshwar,P.G., and Suthar, Om.P., 2012, "Nonlinear thermal instability in a rotating viscous fluid layer under temperature/gravity modulation", J. of heat transfer, ASME, 134, 102502.

[33] Hirdesh Varshney, Mirza Faisal Baig., 2008, " Rotating Rayleigh-Benard convection under the influence of transverse magnetic field", Int. J. of heat and mass transfer, 51, pp. 4095-4108.