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Super (a, d) - Edge Antimagic Total Labeling of Union of Stars

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Abstract

An (a,d) -edge antimagic total labeling of a (p,q) -graph G is a bijection

$f: V \cup E \to \{1,2,3,\dots,p+q\}$ with the property that the edge-weight

$w(uv) = f(u) + f(v) + f(uv), uv \in E(G)$, form an arithmetic progression $a, a+d, \dots, a+(q-1)d$, where a > 0 and d ≥0 are two fixed integers. If G admits such a labeling , then G is called an (a, d) -edge antimagic total graph. Further, if the vertex labels are distinct integers from $\{1,2,\dots,p\}$, then f is called a super (a, d) -edge antimagic total labeling of G (in short (a, d) - SEAMT labeling) and a graph which admits such labeling is called super (a,d) -edge antimagic total graph (in short (a, d) -SEAMT graph). If d = 0, then the graph G is called super edge-magic total graph. In this paper, we investigate the existence of super (a, d) -edge antimagic total labeling of $nK_{1,r} \cup mK_{1,s}$ $for\ odd\ n \geq 3$, $even\ m > 3$, $r, s \geq 3$ $d \in \{0,2\}$ and $\delta(m,n) = 3$,where $\delta(m,n)$denotes the difference between m and n .

Keywords: Magic labelling and antimagic labelling.

Mathematics Subject: Classification (2000): 05C78.

1. INTRODUCTION

All graphs considered here are finite, undirected and simple. A (p, q) -graph is a graph G such that $|V(G)| = p$ and $|E(G)| = q$. Labeling of a graph G is a mapping that sends some set of graph elements to a set of non-negative integers. If the domain is the vertex / edge set of G, the labeling is called vertex / edge labeling of G . Moreover, if the domain is $V(G) \cup E(G)$ then the labeling is called total labeling.

If f is a vertex labeling of a graph G , then the edge-weight of uv∈ E(G) is defined as $w(uv) = f(u) + f(v)$. If f is a total labeling, then the edge-weight of uv ∈ E(G) defined as $w(uv) = f(u) + f(v) + f(uv)$.

By an (a, d) -edge antimagic vertex labeling of a (p, q) -graph G , we mean a bijective function f : V (G) → $\{1,2,3,\dots,p\}$ such that $\{w(uv): uv \in$ E(G) } form an arithmetic progression $a, a+d, a+2d, a+(q-1)d$, where a > 0 and d ≥0 are two fixed integers.

An (a,d) -edge antimagic total labeling of a (p,q) -graph G is a bijective function $f: V(G) \cup E(G) \to \{1,2,\dots,p+q\}$ with the property that $\{w(uv) = f(u) + f(v) + f(uv)$ }, uv ∈ E(G) , form an arithmetic progression $a, a+d, \dots, a+(q-1)d$, where a > 0 and d≥ 0 are two fixed integers.

If G admits such a labeling, then G is said to be an (a, d) -edge antimagic total graph. Further, f is a super (a, d) -edge antimagic total labeling of G if the vertex labels are the distinct integers $1,2,\dots,p$. Thus a super (a, d) -edge antimagic total graph is a graph that admits a super (a,d) -edge antimagic total labeling.

A star is a complete bipartite graph denoted by $K_{1,r}$. The study of magic labelings have been introduced by Simunjuntak et.al [7], a natural extension of the concept of magic valuation, studied by Kotzig and Rosa [5] and the concept of super edge magic labelings defined by Enomoto et.al [2]. Many authors discussed different forms of antimagic graphs in [4,6,8]. For good collection of results on labeling, the authors refered to the survey by J.A. Gallian [3]. For standard definitions and notations not defined here may be refered to D.B. West[9].

2 MAIN RESULTS

In this section, we investigate the existence of super (a,d) -edge antimagic total labeling of $nK_{1,r} \cup mK_{1,s}$ for odd $n \geq 3$, even $m > 3$, $r, s \geq 3$ $d \in \{0,2\}$ and $\delta(m,n) = 3$,where $\delta(m,n)$ denotes the difference between m and n.We prove the following results.

***Theorem* 2.1** : $For\ odd\ n \geq 3$, $even\ m > 3$, $r \geq 3, s \geq 3$, $there\ exists\ a\ super$ $(a,0)$ edge antimagic total labeling $of\ nK_{1,r} \cup mK_{1,s} when\ \delta(m,n) = 3$, where $\delta(m,n)$ denotes the difference between m and n.

$Proof$: $Let\ G_1 = nK_{1,r}$. We denote the centre vertex of the $i^{th}\ star\ G_1\ by\ v_i$ and the pendent vertices connected to $v_i\ by\ v_j^i$ where i varies from 1,2,…,n and j varies from 1,2,…,r. $G_2 = mK_{1,s}$. We denote the centre vertex of the $i^{th}\ star\ G_2\ by\ u_i$ and the pendent vertices connected to $u_i\ by\ u_j^i$ where i varies from 1,2,…,m and j varies from 1,2,…,s. We define the vertex labeling $f: V(G) \cup E(G) \to \{1,2,\dots,p\}\ by$

$$f(v_1^i) = i, i = 1,2,\dots,n$$

$$f(v_2^i) = n + i, i = 1,2,\dots,n$$

$$f(v_3^i) = 2n + i, i = 1,2,\dots,n$$

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$$f(v_j^i) = n(j-1) + i, i = 1,2,\dots,n, j = 1,2,\dots,r$$

$$f(u_1^i) = f(v_r^n) + i, i = 1,2,\dots,m$$
$$f(u_2^i) = f(u_1^m) + i, i = 1,2,\dots,m$$

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$$f(u_j^i) = f(u_{j-1}^m) + i, i = 1,2,\dots,m, j = 2,3,\dots,s$$

Case 1. When $n > m$

$$f(v_{n-1}) = f(u_s^m) + 1$$
$$f(v_n) = f(v_{n-1}) + 1$$
$$f(v_{n-3}) = f(v_{n-1}) + 3$$
$$f(v_1) = f(u_s^m) + n + m$$
$$f(v_{n-2i}) = f(v_n) + 4i, i = 1,2,\dots,\left(\frac{m}{2}\right)$$
$$f(v_{n-(2i+3)}) = f(v_{n-3}) + 4i, i = 1,2,\dots,\left(\frac{m}{2}\right) - 1$$
$$f(u_1) = f(v_1) - 2$$
$$f(u_2) = f(u_1) - 2$$
$$f(u_3) = f(u_2) - 2$$

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$$f(u_i) = f(u_{i-1}) - 2, i = 2,3,\dots,m$$
$$f(v_iv_1^i) = [3(nr + ms) + 2n + m] - [f(v_i) + f(v_1^i)], i = 1,2,\dots,n$$
$$f(v_iv_2^i) = [3(nr + ms) + 2n + m] - [f(v_i) + f(v_2^i)], i = 1,2,\dots,n$$

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$$f(v_iv_j^i) = [3(nr + ms) + 2n + m] - [f(v_i) + f(v_j^i)], i = 1,2,\dots,n, j = 1,2,\dots,r$$
$$f(u_iu_1^i) = [3(nr + ms) + 2n + m] - [f(u_i) + f(u_1^i)], i = 1,2,\dots,m$$
$$f(u_iu_2^i) = [3(nr + ms) + 2n + m] - [f(u_i) + f(u_2^i)], i = 1,2,\dots,m$$

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$$f(u_iu_j^i) = [3(nr + ms) + 2n + m] - [f(u_i) + f(u_j^i)], i = 1,2,\dots,m, j = 1,2,\dots,s$$

Case 2. When $n < m$

$$f(u_{m-1}) = f(u_s^m) + 1$$
$$f(u_m) = f(u_{m-1}) + 1$$
$$f(u_{m-3}) = f(u_{m-1}) + 3$$
$$f(u_1) = f(u_s^m) + n + m - 1$$
$$f(u_2) = f(u_1) + 1$$
$$f(u_{m-2i}) = f(u_m) + 4i, i = 1,2,\dots,\left(\frac{m}{2}\right) - 2$$
$$f(u_{m-(2i+3)}) = f(u_{m-3}) + 4i, i = 0,1,2,\dots,\left(\frac{m}{2}\right) - 3$$
$$f(v_1) = f(u_1) - 1$$
$$f(v_2) = f(v_1) - 2$$
$$f(v_3) = f(v_2) - 2$$

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$$f(v_i) = f(v_{i-1}) - 2, i = 2,3,\dots,n$$
$$f(v_iv_1^i) = [3(nr + ms) + 2m + n] - [f(v_i) + f(v_1^i)], i = 1,2,\dots,n$$
$$f(v_iv_2^i) = [3(nr + ms) + 2m + n] - [f(v_i) + f(v_2^i)], i = 1,2,\dots,n$$

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$$f(v_iv_j^i) = [3(nr + ms) + 2m + n] - [f(v_i) + f(v_j^i)], i = 1,2,\dots,n, j = 1,2,\dots,r$$
$$f(u_iu_1^i) = [3(nr + ms) + 2m + n] - [f(u_i) + f(u_1^i)], i = 1,2,\dots,m$$
$$f(u_iu_2^i) = [3(nr + ms) + 2m + n] - [f(u_i) + f(u_2^i)], i = 1,2,\dots,m$$

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$$f(u_iu_j^i) = [3(nr + ms) + 2m + n] - [f(u_i) + f(u_j^i)], i = 1,2,\dots,m, j = 1,2,\dots,s$$

The edge weight defined by $w(v_iu_j^i) = f(v_i) + f(u_j^i) + f(v_iu_j^i)$ from an arithmetic progression $14r + 8s + 18, 18r + 12s + 24, 27r + 16s + 30, \dots, a = 2(nr + ms) + 2n + m$, *when* $n > m$ *and* $6r + 12s + 15, 10r + 16s + 21, 14r + 20s + 27, \dots, a = 2(nr + ms) + 2m + n$, *when* $n < m$.

Theorem 2.2 : *For odd* $n \geq 3$, *even* $m > 3$, $r, s \geq 3$, there exists a super $(a, 2)$ edge antimagic total labeling of $nK_{1,r} \cup mK_{1,s}$ *when* $\delta(m, n) = 3$, *where* $\delta(m, n)$ denotes the difference between m and n.

Proof : *Let* $G_1 = nK_{1,r}$. We denote the centre vertex of the i^{th} *star* G_1 *by* v_i and the pendent vertices connected to v_i by v_j^i where i varies from 1,2,…,n and j varies from 1,2,…,r. $G_2 = mK_{1,s}$. We denote the centre vertex of the i^{th} $star$ G_2 by u_i and the pendent vertices connected to u_i by u_j^i where i varies from 1,2,…,m and j varies from 1,2,…,s. We define the vertex labeling $f: V(G) \cup E(G) \rightarrow \{1,2, \dots, p\}$ by

$$f(v_1^i) = i, i = 1,2, \dots, n$$

$$f(v_2^i) = n + i, i = 1,2, \dots, n$$

$$f(v_3^i) = 2n + i, i = 1,2, \dots, n$$

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$$f(v_j^i) = n(j-1) + i, i = 1,2, \dots, n, j = 2,3, \dots, r$$

$$f(u_1^i) = f(v_r^n) + i, i = 1,2,3, \dots, m$$

$$f(u_2^i) = f(u_1^m) + i, i = 1,2,3, \dots, m$$

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$$f(u_j^i) = f(u_{j-1}^m) + i, i = 1,2, \dots, m, j = 2,3, \dots, s$$

Case 1. When $n > m$

$$f(v_{n-1}) = f(u_s^m) + 1$$

$$f(v_n) = f(v_{n-1}) + 1$$

$$f(v_{n-3}) = f(v_{n-1}) + 3$$

$$f(v_1) = f(u_s^m) + n + m$$

$$f(v_{n-2i}) = f(v_n) + 4i, i = 1,2, \dots, \left(\frac{m}{2}\right)$$

$$f(v_{n-(2i+3)}) = f(v_{n-3}) + 4i, i = 1,2, \dots, \left(\frac{m}{2}\right) - 1$$

$$f(u_1) = f(v_1) - 2$$

$$f(u_2) = f(u_1) - 2$$

$$f(u_3) = f(u_2) - 2$$

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$$f(u_i) = f(u_{i-1}) - 2, i = 2,3, \dots, m$$

$$f(v_{n-1}v_1^{n-1}) = f(v_1) + 1$$

$$f(v_n v_1^n) = f(v_{n-1}v_1^{n-1}) + 2$$

$$f(v_{n-2i}v_1^{n-2i}) = f(v_n v_1^n) + 2i, i = 1,2, \dots, \left(\frac{m}{2}\right)$$

$$f\left(v_{n-(2i+1)}v_1^{n-(2i+1)}\right) = f(v_1) + 2i, i = 1,2, \dots, \left(\frac{m}{2}\right)$$

$$f(v_1 v_1^1) = f(v_1) + n - 1$$

$$f(v_{n-1}v_2^{n-1}) = f(v_{n-1}v_1^{n-1}) + n$$

$$f(v_n v_2^n) = f(v_1 v_1^n) + n$$

$$f(v_{n-2i}v_2^{n-2i}) = f(v_n v_1^n) + 2i + n, i = 1,2, \dots, \left(\frac{m}{2}\right)$$

$$f\left(v_{n-(2i+1)}v_2^{n-(2i+1)}\right) = f(v_1) + 2i + n, i = 1,2, \dots, \left(\frac{m}{2}\right)$$

$$f(v_1 v_2^1) = f(v_1) + n - 1 + n$$

$$f(v_{n-1}v_3^{n-1}) = f(v_{n-1}v_1^{n-1}) + 2n$$

$$f(v_n v_3^n) = f(v_n v_1^n) + 2n$$

$$f(v_{n-2i}v_3^{n-2i}) = f(v_n v_1^n) + 2i + 2n, i = 1,2, \dots, \left(\frac{m}{2}\right)$$

$$f\left(v_{n-(2i+1)}v_3^{n-(2i+1)}\right) = f(v_1) + 2i + 2n, i = 1,2, \dots, \left(\frac{m}{2}\right)$$

$$f(v_1 v_3^1) = f(v_1) + n - 1 + 2n$$

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$$f(v_{n-1}v_j^{n-1}) = f(v_{n-1}v_1^{n-1}) + (j-1)n, j = 2,3, \dots, r$$

$$f(v_n v_j^n) = f(v_n v_1^n) + (j-1)n, j = 2,3, \dots, r$$

$$f(v_{n-2i}v_j^{n-2i}) = f(v_n v_1^n) + 2i + (j-1)n, i = 1,2, \dots, \left(\frac{m}{2}\right), j = 2,3, \dots, r$$

$$f\left(v_{n-(2i+1)}v_j^{n-(2i+1)}\right) = f(v_1) + 2i + (j-1)n, i = 1,2, \dots, \left(\frac{m}{2}\right), j = 2,3, \dots, r$$

$$f(v_1 v_j^1) = f(v_1) + n - 1 + (j-1)n, j = 2,3, \dots, r$$

$$f(u_1 u_1^1) = f(v_{n-1}v_1^{n-1}) + 3n + m - i, i = 1,2, \dots, m$$

$$f(u_1 u_2^1) = f(v_{n-1}v_1^{n-1}) + 3n + m - i + m, i = 1,2, \dots, m$$

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$$f(u_i u_j^i) = f(v_{n-1}v_1^{n-1}) + 3n + m - i + (j-1)m, i = 1,2, \dots, m, j = 2,3, \dots, s$$

Case 2. When $n < m$

$$f(u_{m-1}) = f(u_s^m) + 1$$

$$f(u_m) = f(v_{m-1}) + 1$$

$$f(u_{m-3}) = f(u_{m-1}) + 3$$

$$f(u_1) = f(u_s^m) + n + m - 1$$

$$f(u_2) = f(u_1) + 1$$

$$f(u_{m-2i}) = f(u_m) + 4i, i = 1,2, \dots, \left(\frac{m}{2}\right) - 2$$

$$f(u_{m-(2i+3)}) = f(u_{m-3}) + 4i, i = 1,2, \dots, \left(\frac{m}{2}\right) - 3$$

$$f(v_1) = f(u_1) - 1$$

$$f(v_2) = f(v_1) - 2$$

$$f(v_3) = f(v_2) - 2$$

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$$f(v_i) = f(v_{i-1}) - 2, i = 2,3, \dots, n$$

$$f(v_i v_1^i) = f(u_2) + n + 1 - i, i = 1,2, \dots, n$$

$$f(v_i v_2^i) = f(u_2) + n + 1 - i + n, i = 1,2, \dots, n$$

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$$f(v_i v_j^i) = f(u_2) + n + 1 - i + (j-1)n, i = 1,2, \dots, n, j = 2,3, \dots, r$$

$$f(u_{m-1} u_1^{m-1}) = f(v_1) + 2n + m$$

$$f(u_m u_1^m) = f(u_{m-1} u_1^{m-1}) + 2$$

$$f(u_{m-2i} u_1^{m-2i}) = f(u_m u_1^m) + 2i, i = 1,2, \dots, \left(\frac{m}{2}\right) - 2$$

$$f\left(u_{m-(2i+3)} u_1^{m-(2i+3)}\right) = f(u_m u_1^m) + 1 + 2i, i = 1,2, \dots, \left(\frac{m}{2}\right) - 3$$

$$f(u_1 u_1^1) = f(u_2) + nr + m - 2$$

$$f(u_2 u_1^2) = f(u_1 u_1^1) + 1$$

$$f(u_{m-1} u_2^{m-1}) = f(v_1) + 2n + 2m$$

$$f(u_m u_2^m) = f(u_{m-1} u_1^{m-1}) + 2 + m$$

$$f(u_{m-2i} u_2^{m-2i}) = f(u_m u_1^m) + 2i + m, i = 1,2, \dots, \left(\frac{m}{2}\right) - 1$$

$$f\left(u_{m-(2i+3)} u_2^{m-(2i+3)}\right) = f(u_m u_1^m) + 1 + 2i + m, i = 0,1,2, \dots, \left(\frac{m}{2}\right) - 3$$

$$f(u_1 u_2^1) = f(u_2) + nr + m - 2 + m$$

$$f(u_2 u_2^2) = f(u_1 u_1^1) + 1 + m$$

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$$f(u_{m-1} u_j^{m-1}) = f(v_1) + 2n + m + (j-1)m, j = 2,3, \dots, s$$

$$f(u_m u_j^m) = f(u_{m-1} u_1^{m-1}) + 2 + (j-1)m, j = 2,3, \dots, s$$

$$f(u_{m-2i} u_j^{m-2i}) = f(u_m u_1^m) + 2i + (j-1)m, i = 1,2, \dots, \left(\frac{m}{2}\right), j = 2,3, \dots, s$$

$$f\left(u_{m-(2i+3)} u_j^{m-(2i+3)}\right) = f(u_m u_1^m) + 1 + 2i + (j-1)m, i = 0,1,2, \dots, \left(\frac{m}{2}\right), j = 2,3, \dots, s$$

$$f(u_1 u_j^1) = f(u_2) + nr + m - 2 + (j-1)m, j = 2,3, \dots, s$$

$$f(u_2 u_j^2) = f(u_1 u_1^1) + 1 + (j-1)m, j = 2,3, \dots, s$$

From an arithmetic progression $14r + 8s + 19, 18r + 12s + 25, 22r + 16s + 31, \dots, a = 2(nr + ms) + 2n + m + 1, when\ n > m\ and\ 6r + 12s + 16, 10r + 16s + 22, 14r + 20s + 28, \dots, a = 2(nr + ms) + 2m + n + 1, when\ n < m$.

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