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Application of Exact Solution Approach in the Analysis of Thick Rectangular Plate

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Abstract

This work evaluates the exact relationships and static analysis of an isotropic thick rectangular plate with various kinds of support conditions. In this work, the use of higher order refined shear deformation theory for thick plate analysis was applied. Total potential energy equation of a thick plate was formulated from the first principle. These relationships enable the actual displacements, stresses, stress resultants and shear deformation rotations of the traditional third order plate theory to be determined by performing general variation of the total potential energy obtained. The coordinate functions, which must satisfy the geometric and natural boundary conditions, are carefully constructed and applied into refined shear deformation plate equation. The plate equation is thus integrated by direct integration approach and the integrand minimized to obtain the unknown coefficients which when substituted back in deformation equation of mid-surface of plate gives actual displacements, stresses, stress resultants and shear deformation of plate in analytical form. This work is helpful for evaluation of inplane displacements, deflections, bending moments, shear force and shear deformation rotations at any arbitrary point on the plates unlike the numerical methods which only give these results at nodal points. The results obtained from this study was compared with those of numerical obtained from refined theory with correction factor and those obtained using numerical and other methods at various aspect ratios of the plate for rectangular plates boundary conditions with clamped on two opposite short edges and simply supported on two opposite long edges (CSCS). The results obtained are in close agreement with the results obtained from the literature.

Keywords: Exact solution, shear deformation, displacement, stress, deflection, total potential energy.

I. INTRODUCTION:

There is no unanimity regarding this classification, some authors considering the ratio *a/t* > 10 (Wang *et al*., 2000; Qatu, 2004; Mazilu *et al*., 1986). Due to small thickness, for the thin plates can be used the most frequently 2-D theories, while the thick plate model requires the use of 3-D elasticity theories (Rehfield & Valisettz, 1984; Reissner, 1985; Wang *et al*., (2000). This leads to refined theory for thick plate analysis.

Refined shear deformation theories based on the power series expansion for displacements with respect to the thickness coordinate, and truncating the series at required order of thickness coordinate are called the higher-order shear deformation theories. This type of series expansion was initially proposed by Basset (1890). Refined plate theories have been characterized by the use of trigonometric displacement function. The refined plate theories; first, second and higher order shear deformation theory – HSDT can be obtained through the analogue means to solve the couples governing differential equations, consequently deduce deformation. Reddy's third-order, and Reissner's higher-order shear deformation plate theory which have two more unknowns' variables in comparison with the classical plate theory. One common observation is that most of these works are based mainly on trigonometric displacement functions. One can scarcely see work on thick plate based on polynomial displacement functions. This gap in literature is worth filling, hence the need for this present research work, which presents the higher order shear deformation theory – HSDT (third order shear deformation theory) using polynomial shear deformation function for thick plates analysis as:

$$F(z) = \frac{3}{2}\left(z - \frac{4z^3}{3t^2}\right) \qquad 1$$

In the course of this study, several methods of analyses especially the numerical methods such as finite element, finite difference, finite strip etc were extensively reviewed. The most widely accepted classical solution method, though acknowledged as satisfactory for most Engineering problems, is usually very tedious and rigorous. In view of these antecedent problems, direct variational method under the principle of total potential energy is formulated to circumvent the rigorous procedures inherent in the analysis of classical

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Solution

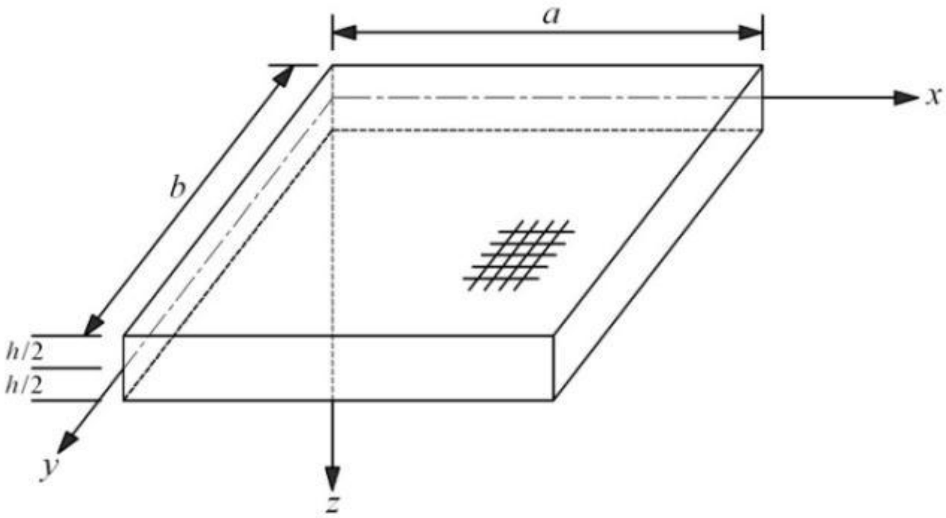

Figure **3.1** shows a bent elastic plate under lateral loading. Our intention is to obtain the displacement – strain relationships in terms of curvatures. From the assumptions, we have three displacement of thick place which includes the deflection, w(x,y) and the two inplane displacements, u(x,y,z), and v(x,y,z). Ibearugbulem (2015) in his lecture note defined deflection, w.

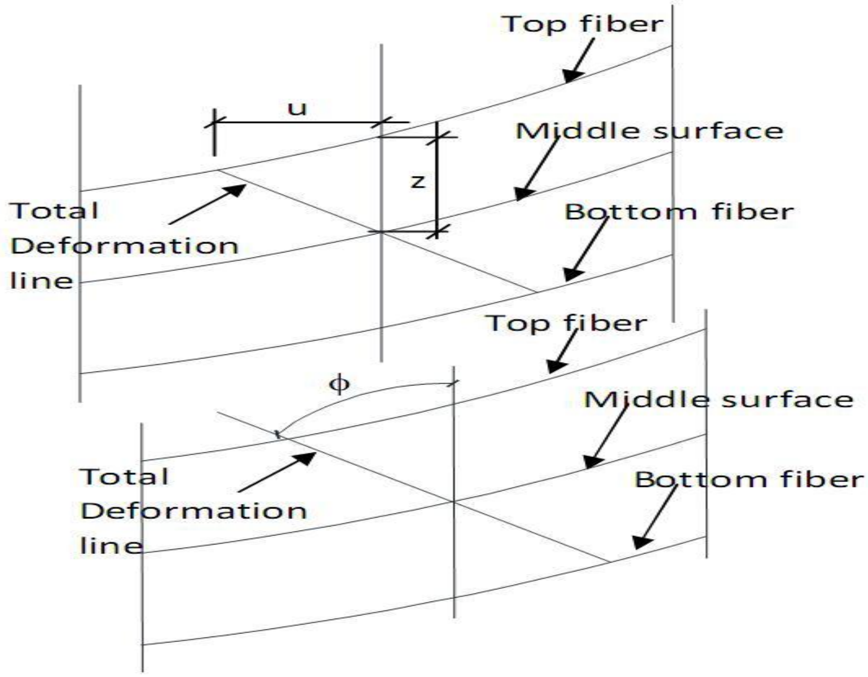

Figure 1: Deformation of a section of a thick plate

III. ENGINEERING STRAIN – DISPLACEMENT RELATIONS

The refined plate theory (RPT) displacements and strain as presented on figure1 are defined mathematically as:

$$w = A_1 h \qquad 2$$

From assumptions herein, the strain normal to z axis is zero. This left us with only five engineering strain components $\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}$ and γ_{yz}.

$$\varepsilon_x = \frac{du}{dy} \equiv \left(-\frac{zd^2w}{dx^2} + \frac{Fd\theta_{Sx}}{dx}\right) \quad 3$$

That is:

$$\varepsilon_x = [-A_1z + A_2F(z)]\frac{d^2h}{dx^2} \quad 4$$

$$\sigma_x = \frac{E}{1-\mu^2}\left[[-A_1z + A_2F(z)]\frac{d^2h}{dx^2} + \mu[-A_1z + A_3F(z)]\frac{d^2h}{dy^2}\right] \quad 5$$

Similarly reasoning in y direction, we shall obtain:

$$\varepsilon_y = \frac{dv}{dy} \equiv \left(-\frac{zd^2w}{dy^2} + \frac{Fd\theta_{Sy}}{dy}\right) \quad 6$$

That is:

$$\varepsilon_y = [-A_1z + A_3F(z)]\frac{d^2h}{dy^2} \quad 7$$

$$\gamma_{xy} = 2\frac{z\partial^2w}{\partial x\partial y} + F\left(\frac{d\theta_{Sx}}{dy} + \frac{Fd\theta_{Sy}}{dx}\right) \quad 8$$

That is:

$$\gamma_{xy} = \frac{du}{dy} + \frac{dv}{dx} \equiv [-2A_1z + A_2F(z) + A_3F(z)]\frac{d^2h}{\partial x\partial y} \quad 9$$

$$\gamma_{xz} = \frac{dF}{dz}.\theta_{Sx} \quad 10$$

That is:

$$\gamma_{xz} = A_2\frac{dF(z)}{dz}\frac{dh}{dx} \quad 11$$

$$\gamma_{yz} = \frac{dF}{dz}.\theta_{Sy} \quad 12$$

That is:

$$\gamma_{yz} = A_3\frac{dF(z)}{dz}\frac{dh}{dy} \quad 13$$

IV. CONSTITUTIVE RELATIONS

In the constitutive equations, there two in-plane and three transverse stresses:

$$\sigma_x = \frac{Ez}{1-\mu^2}\left(\left(\frac{-d^2w}{dx^2} + \frac{Fd\theta_{Sx}}{dx}\right) + \mu\left(\frac{d^2w}{dy^2} + \frac{Fd\theta_{Sx}}{dy}\right)\right) \quad 14$$

That is:

$$\sigma_x = \frac{E}{1-\mu^2}\left[[-A_1z + A_2F(z)]\frac{d^2h}{dx^2} + \mu[-A_1z + A_3F(z)]\frac{d^2h}{dy^2}\right] \quad 15$$

$$\sigma_y = \frac{Ez}{1-\mu^2}\left(\left(-\frac{d^2w}{dy^2} + \frac{Fd\theta_{Sx}}{dx}\right) + \mu\left(\frac{d^2w}{dx^2} + \frac{Fd\theta_{Sx}}{dy}\right)\right) \quad 16$$

That is:

$$\sigma_y = \frac{E}{1-\mu^2}\left[\mu[-A_1 z + A_2 F(z)]\frac{d^2h}{dx^2} + [-A_1 z + A_3 F(z)]\frac{d^2h}{dy^2}\right] \quad 17$$

Also, from known state Equation,

$$\tau_{xy} = \frac{E(1-\mu)}{(1-\mu^2)}\left(-\frac{z\partial^2 w}{\partial x\partial y} + F\left(\frac{d\theta_{Sx}}{dy} + \frac{d\theta_{Sy}}{dx}\right)\right) \quad 18$$

That is:

$$\tau_{xy} = \frac{Ez}{2(1-\mu^2)}[-2A_1 z + A_2 F(z) + A_3 F(z)]\frac{d^2h}{\partial x\partial y} \quad 19$$

$$\tau_{xz} = \frac{E}{2(1+\mu)}.\gamma_{xz} \quad 20$$

$$\tau_{xzm} = \frac{Ez(1-\mu)}{2(1-\mu^2)}A_2\frac{dF(z)}{dz}\frac{dh}{dx} \quad 21$$

Also;

$$\tau_{yz} = \frac{E}{2(1+\mu)}.\gamma_{yz} \quad 22$$

That is:

$$\tau_{yzm} = \frac{Ez(1-\mu)}{2(1-\mu^2)}A_3\frac{dF(z)}{dz}\frac{dh}{dy} \quad 23$$

Where τ_{xzm} and τ_{yzm} = Maximum vertical shear stresses.

V. TOTAL POTENTIAL ENERGY

Total potential energy is the summation of strain energy, U and external work, V. that's

$$\Pi = U + V \quad 24$$

Let's define external work as:

$$V = -q\ wyxdxdy \quad 25$$

Therefore, the total potential energy equation for a thick plate of traditional third order shear deformation theory is given as:

Let's define the span-depth aspect ratio as:

$$\rho = \frac{a}{t} \quad 26$$

$$\begin{aligned}\Pi = \frac{D}{2}\int_0^a\int_0^b \Bigg[& \left|g_1 A_1^2\left(\frac{\partial^2 h}{\partial x^2}\right)^2 - 2g_2 A_1 A_2\left(\frac{\partial^2 h}{\partial x^2}.\frac{\partial^2 h}{\partial x^2}\right) + g_3 A_2^2\left(\frac{\partial^2 h}{\partial x^2}\right)^2\right| \\ & + \left|2g_1 A_1^2\left(\frac{\partial^2 h}{\partial x\partial y}\right)^2 - 2g_2 A_1 A_2\left(\frac{\partial^2 h}{\partial x\partial y}.\frac{\partial^2 h}{\partial x\partial y}\right) - 2g_2 A_1 A_3\left(\frac{\partial^2 h}{\partial x\partial y}.\frac{\partial^2 h}{\partial x\partial y}\right)\right| \\ & + \left|(1+\mu)g_3 A_2 A_3\left(\frac{\partial^2 h}{\partial x\partial y}\right)\left(\frac{\partial^2 h}{\partial x\partial y}\right)\right| + \frac{(1-\mu)}{2}\left|g_3 A_2^2\left(\frac{\partial^2 h}{\partial x\partial y}\right)^2 + g_3 A_3^2\left(\frac{\partial^2 h}{\partial x\partial y}\right)^2\right| \\ & + \left|g_1 A_1^2\left(\frac{\partial^2 h}{\partial y^2}\right)^2 - 2g_2 A_1 A_3\left(\frac{\partial^2 h}{\partial y^2}.\frac{\partial^2 h}{\partial y^2}\right) + g_3 A_3^2\left(\frac{\partial^2 h}{\partial y^2}\right)^2\right| \\ & + \left|\frac{(1-\mu)}{2}g_4 A_2^2\left(\frac{\partial h}{\partial x}\right)^2 + \frac{(1-\mu)}{2}g_4 A_3^2\left(\frac{\partial h}{\partial y}\right)^2\right|\Bigg]\partial x\partial y - \int_0^a\int_0^b qA_1 h\,\partial x\partial y \quad 27\end{aligned}$$

VI. GENERAL VARIATION

The general variation on the total potential energy will be performed in order to obtain the solution of the resulting three simultaneous governing equations.

Here, to obtain the non-dimensional equations of equilibrium of forces, equation 27 must be differentiated with respect to w. That is;

$$\frac{d\Pi}{dw} = \frac{d\Pi}{d\emptyset_x} = \frac{d\Pi}{d\emptyset_y} = 0 \qquad 28$$

$$\int_0^1\int_0^1\left[g_1\left(\frac{d^4w}{dR^4} + \frac{2}{\propto^2}\frac{d^4w}{dR^2dQ^2} + \frac{1}{\propto^4}\frac{d^4w}{dQ^4}\right) - ag_2\frac{d^3\emptyset_{Sx}}{dR^3} - \frac{ag_2}{\propto^2}\frac{d^3\emptyset_{Sx}}{dRdQ^2} - \frac{ag_2}{\propto}\frac{d^3\emptyset_{Sy}}{dR^2dQ} - \frac{ag_2}{\propto^3}\frac{d^3\emptyset_{Sy}}{dQ^3}\right]dRdQ = \frac{qa^4}{D}\int_0^1\int_0^1 1.\,dRdQ \qquad 29$$

$$\int_0^1\int_0^1\left[-g_2\left(\frac{d^3w}{dR^3} + \frac{1}{\propto^2}\frac{d^3w}{dRdQ^2}\right) + ag_3\frac{d^2\emptyset_{Sx}}{dR^2} + \frac{(1-\mu)}{2\propto^2}ag_3\frac{d^2\emptyset_{Sx}}{dQ^2} + \frac{(1-\mu)}{2}a^3\beta^2g_4\emptyset_{Sx} + \frac{(1+\mu)}{2}ag_3\frac{d^2\emptyset_{Sy}}{dRdQ}\right]dRdQ \quad = 0 \qquad 30$$

$$\int_0^1\int_0^1\left[-g_2\left(\frac{1}{\propto}\frac{d^3w}{dR^2dQ} + \frac{1}{\propto^2}\frac{d^3w}{dQ^3}\right) + \frac{(1+\mu)}{2\propto}ag_3\frac{d^2\emptyset_{Sx}}{dRdQ} + \frac{(1-\mu)}{2}ag_3\frac{d^2\emptyset_{Sy}}{dR^2} + \frac{ag_3}{\propto^2}\frac{d^2\emptyset_{Sy}}{dQ^2} + \frac{(1-\mu)}{2}a^3\rho^2g_4\emptyset_{Sy}\right]dRdQ \quad = 0 \qquad 31$$

Obtaining values of deflection function $w, \emptyset_x$ *and* $\emptyset_y$ from equations 29, 32 and 31, let:

$$w = w_x.w_y \qquad 32$$

$$\emptyset_x = \emptyset_{xx}.\emptyset_{x_y} \qquad 33$$

$$\emptyset_y = \emptyset_{yx}.\emptyset_{yy} \qquad 34$$

The actual deflection is obtained by multiplying equations 36 and 37 and substitute into equation 32, we have:

$$w = \left(a_0 + a_1R + \frac{a_2R^2}{2} + \frac{a_3R^3}{6} + \frac{qa^4}{D}\left(\frac{n_1}{w_3}\right).\frac{R^4}{24}\right) \times \left(b_0 + b_1Q + \frac{b_2Q^2}{2} + \frac{b_3Q^3}{6} + \frac{qa^4}{D}\left(\frac{n_1}{w_3}\right).\frac{Q^4}{24}\right) 35$$

Where;

$$w_x = \left(a_0 + a_1R + \frac{a_2R^2}{2} + \frac{a_3R^3}{6} + \frac{qa^4}{D}\left(\frac{n_1}{w_3}\right).\frac{R^4}{24}\right) \qquad 36$$

$$w_y\left(b_0 + b_1Q + \frac{b_2Q^2}{2} + \frac{b_3Q^3}{6} + \frac{qa^4}{D}\left(\frac{n_1}{w_3}\right).\frac{Q^4}{24}\right) \qquad 37$$

$$F_{r4} = \frac{qa^4}{D}\left(\frac{n_1}{w_3}\right) \qquad 38$$

That is:

$$F_{a4} = \frac{qa^4}{D}\left(\frac{n_1}{w_3}\right) and\ F_{b4} = \frac{qa^4}{D}\left(\frac{n_1}{w_3}\right)$$

Similarly,

$$\emptyset_x = \left(a_4 + a_5R + \frac{a_6R^2}{2} + \frac{qa^3}{D}\left(\frac{n_4}{g_2\emptyset_3}\right).\frac{R^3}{6}\right) \times \left(b_7 + b_8Q + \frac{b_9Q^2}{2} + \frac{b_{10}Q^3}{6} + \frac{b_{11}Q^4}{24}\right) \quad 39$$

Let;

$$ka_7 = \frac{qa^3}{D}\left(\frac{n_4}{g_2\emptyset_3}\right) \quad 40$$

Therefore:

$$\emptyset_x = \left(a_4 + a_5R + \frac{a_6R^2}{2} + ka_7.\frac{R^3}{6}\right) \times \left(b_7 + b_8Q + \frac{b_9Q^2}{2} + \frac{b_{10}Q^3}{6} + \frac{b_{11}Q^4}{24}\right) \quad 41$$

Also;

$$\emptyset_y = \left(a_7 + a_8R + \frac{a_9R^2}{2} + \frac{a_{10}R^3}{6} + \frac{a_{11}R^4}{24}\right) \times \left(b_4 + b_5Q + \frac{b_6Q^2}{2} + \frac{qa^3}{D}\left(\frac{\propto^3 n_5}{g_2\emptyset_1}\right).\frac{Q^3}{6}\right) \quad 42$$

Where:

$$\emptyset_{yx} = \left(a_7 + a_8R + \frac{a_9R^2}{2} + \frac{a_{10}R^3}{6} + \frac{a_{11}R^4}{24}\right)$$

And;

$$\emptyset_{yy} = \left(b_4 + b_5Q + \frac{b_6Q^2}{2} + \frac{qa^3}{D}\left(\frac{\propto^3 n_5}{g_2\emptyset_1}\right).\frac{Q^3}{6}\right)$$

Let;

$$K_{b7} = \frac{qa^3}{6D}\left(\frac{\propto^3 n_5}{g_2\emptyset_1}\right) \quad 43$$

Therefore:

$$\emptyset_y = \left(a_7 + a_8R + \frac{a_9R^2}{2} + \frac{a_{10}R^3}{6} + \frac{a_{11}R^4}{24}\right) \times \left(b_4 + b_5Q + \frac{b_6Q^2}{2} + k_{b7}.\frac{Q^3}{6}\right) \quad 44$$

$$w = w_x.w_y = A_1h \quad 45$$

$$\emptyset_x = \emptyset_{xx}.\emptyset_{xy} = \left[\frac{dh}{dR}\right][A_2] \quad 46$$

Also, from equation 3.180;

$$\emptyset_y = \emptyset_{yx}.\emptyset_{yy} = \left[\frac{dh}{dQ}\right][A_3] \quad 47$$

$$Where\ G(z) = \frac{3}{2}\left(1 - 4\frac{z^2}{t^3}\right)\ and\ F(z) = \frac{3}{2}\left(z - 4\frac{z^3}{t^3}\right) \text{ (Okafor et al, 2018)}$$

$$g_1 = 1; \quad g_2 = 1.2; g_3 = 1.33; \ g_4 = 14.4$$

Also, the Stress Resultants are obtained as:

$$M_x = -D_1A_1\left[\frac{\partial^2 h}{\partial x^2} + \mu\frac{\partial^2 h}{\partial y^2}\right] + D_2\left[A_2\frac{d^2h}{\partial x^2} + \mu A_3\frac{d^2h}{\partial y^2}\right] \quad 48$$

Expressing equation in non-dimensional form, we have:

$$M_x = -D_1 \frac{A_1}{a^2}\left[\frac{d^2h}{dR^2} + \mu \frac{d^2h}{dQ^2}\right] + \frac{D_2}{a^2}\left[A_2 \frac{d^2h}{dR^2} + \mu A_3 \frac{d^2h}{dQ^2}\right] \quad 49$$

Similarly;

$$M_y = -D_1 \left[\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2}\right] + D_2 \left[\frac{d\theta_{Sy}}{dy} + \mu \frac{d\theta_{Sx}}{dx}\right] \quad 50$$

That is:

$$M_y = -D_1 \frac{A_1}{a^2}\left[\frac{d^2h}{dQ^2} + \mu \frac{d^2h}{dR^2}\right] + \frac{D_2}{a^2}\left[A_3 \frac{d^2h}{dQ^2} + \mu A_2 \frac{d^2h}{dR^2}\right] \quad 51$$

Also;

$$Q_x = -D_1 A_1 \left[\frac{\partial^3 h}{\partial x^3} + \mu \frac{\partial^3 h}{\partial y^3}\right] + D_2 \left[A_2 \frac{\partial^3 h}{\partial x^3} + \mu A_3 \frac{\partial^3 h}{\partial x^3}\right] \quad 52$$

Expressing equation in non-dimensional form, we have:

$$Q_x = -D_1 \frac{A_1}{a^3}\left[\frac{\partial^3 h}{\partial R^3} + \mu \frac{\partial^3 h}{\partial Q^3}\right] + \frac{D_1}{a^3}\left[A_2 \frac{\partial^3 h}{\partial R^3} + \mu A_3 \frac{\partial^3 h}{\partial Q^3}\right] \quad 53$$

$$Q_x = \overline{Q_x} q a \quad 54$$

Similarly;

$$Q_y = -D_1 \frac{A_1}{a^3}\left[\frac{\partial^3 h}{\partial Q^3} + \mu \frac{\partial^3 h}{\partial R^3}\right] + \frac{D_1}{a^3}\left[A_3 \frac{\partial^3 h}{\partial Q^3} + \mu A_2 \frac{\partial^3 h}{\partial R^3}\right] \quad 55$$

VII. DIRECT GOVERNING EQUATION

In minimization, when the differentiation is done with respect to the coefficient of the displacement, the result is called the direct governing equation. Here, the total potential energy shall be differentiated with respect to the coefficient of the deflection, shear deformation along x axis and shear deformation along y axis; A_1, A_2, and A_3, we have;

$$\frac{\partial \Pi}{\partial A_1} = \frac{\partial \Pi}{\partial A_2} = \frac{\partial \Pi}{\partial A_3} = 0 \quad 56$$

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \frac{qa^4}{D} \begin{bmatrix} k_6 \\ 0 \\ 0 \end{bmatrix} \quad 57$$

But from the matrix above, we got:

$$A_1 = \frac{qa^4}{D}\left(\frac{k_6}{r_{11}T_1 - r_{12}T_2 - r_{13}T_3}\right) \quad 58$$

Let:

$$\overline{A_1} = \left(\frac{k_6}{r_{11}T_1 - r_{12}T_2 - r_{13}T_3}\right) \quad 59$$

That is:

$$A_1 = \overline{A_1}\left(\frac{qa^4}{D}\right) \quad 60$$

Similarly;

$$A_2 = \overline{A_1}\left(\frac{qa^4}{D}\right) \qquad 61$$

Similarly;

$$A_3 = \overline{A_3}\left(\frac{qa^4}{D}\right) \qquad 62$$

VIII. DEFINITION OF SOME QUANTITIES

By substituting equations 85, 86 and 87 into equations 45, 46 and 47 we got the non-dimensional displacements, stresses, stress resultant and shear deformation rotation equations of plate under pure bending as:

The displacement and shear deformation components as;

$$w = \overline{A_1}h\left(\frac{qa^4}{D}\right) \qquad 63$$

That is:

$$w = \overline{w}\left(\frac{qa^4}{D}\right) \qquad 64$$

Similarly;

$$u = [-\overline{A_1}s + \overline{A_2}F(s)]\frac{dh}{dR}\left(\frac{qa^4}{\rho D}\right) \qquad 65$$

Similarly;

$$v = \frac{1}{\propto}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{dh}{dQ}\left(\frac{qa^4}{\rho D}\right) \qquad 66$$

Also,

$$\emptyset_x = [\overline{A_2}]\left[\frac{dh}{dR}\right]\left(\frac{qa^4}{D}\right) \qquad 67$$

That is:

$$\emptyset_x = \overline{\emptyset_x}\left(\frac{qa^4}{D}\right) \qquad 68$$

And;

$$\emptyset_y = [\overline{A_3}]\left[\frac{dh}{dQ}\right]\left(\frac{qa^4}{D}\right) \qquad 69$$

That is:

$$\emptyset_y = \overline{\emptyset_y}\left(\frac{qa^4}{D}\right) \qquad 70$$

Similarly, let us define the stress components as;

$$\sigma_x = 12\left[[-\overline{A_1}s + \overline{A_2}F(s)]\frac{d^2h}{dR^2} + \frac{\mu}{\propto^2}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{d^2h}{dQ^2}\right](q\rho^2) \qquad 71$$

Similarly;

$$\sigma_y = q\rho^2\left[12\left[\mu[-\overline{A_1}s + \overline{A_2}F(s)]\frac{d^2h}{dR^2}\right] + \frac{\mu}{\propto^2}[-\overline{A_1}s + \overline{A_3}F(s)]\frac{d^2h}{dQ^2}\right] \quad 72$$

Similarly;

$$\tau_{xy} = 6\frac{(1-\mu)}{\propto}\left[-2\overline{A_1}s + \overline{A_2}F(s) + \overline{A_3}F(s).\frac{1}{\propto}\right]\frac{d^2h}{\partial R\partial Q}(q\rho^2) \quad 73$$

Similarly;

$$\tau_{xz} = 6(1-\mu)\,\overline{A_2}\frac{dF(z)}{dz}\frac{dh}{dR}(q\rho^2) \quad 74$$

Similarly;

$$\tau_{yz} = \frac{E}{2(1+\mu)}\left(\frac{qa^3}{D}\right)\overline{A_3}\frac{dF(z)}{dz}\frac{dh}{dQ} \quad 75$$

That is:

$$\tau_{yz} = \frac{6(1-\mu)}{\propto}\overline{A_3}\frac{dF(z)}{dz}\frac{dh}{dQ}(q\rho^2) \quad 76$$

Similarly, let us define the bending moment components as;

$$M_x = \left(-g_1\overline{A_1}\left[\frac{d^2h}{dR^2} + \mu\frac{d^2h}{dQ^2}\right] + g_2\left[\overline{A_2}\frac{d^2h}{dR^2} + \mu\overline{A_3}\frac{d^2h}{dQ^2}\right]\right)qa^2 \quad 77$$

That is:

$$M_x = \overline{M_x}qa^2 \quad 78$$

Let;

$$D = \frac{Et^3}{12(1-\mu^2)};\ D_1 = g_1D;\ and\ D_2 = g_2D$$

Where;

$$g_1 = 1\ and\ g_2 = 1.2$$

Similarly;

$$M_y = \left(-g_1\overline{A_1}\left[\frac{d^2h}{dQ^2} + \mu\frac{d^2h}{dR^2}\right] + g_2\left[\overline{A_3}\frac{d^2h}{dQ^2} + \mu\overline{A_2}\frac{d^2h}{dR^2}\right]\right)qa^2 \quad 79$$

That is:

$$M_y = \overline{M_y}qa^2 \quad 80$$

$$Q_x = qa\left(-\overline{A_1}\left[\frac{\partial^3h}{\partial R^3} + \mu\frac{\partial^3h}{\partial Q^3}\right] + \left[\overline{A_2}\frac{\partial^3h}{\partial R^3} + \mu\overline{A_3}\frac{\partial^3h}{\partial Q^3}\right]\right) \quad 81$$

That is:

$$Q_x = \overline{Q_x}qa \quad 82$$

Similarly;

$$Q_y = qa\left(-\overline{A_1}\left[\frac{\partial^3h}{\partial R^3} + \mu\frac{\partial^3h}{\partial Q^3}\right] + \left[\overline{A_2}\frac{\partial^3h}{\partial R^3} + \mu\overline{A_3}\frac{\partial^3h}{\partial Q^3}\right]\right) \quad 83$$

That is:

$$Q_y = \overline{Q_y}qa \quad 84$$

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IX. NUMERICAL PROBLEM

Determine the deflection at the center of CSCS thick plate. Determine also the shear deformation rotation along the direction of x coordinate and the shear deformation rotation along the direction of y coordinate at the edges of the plate. Polynomial displacement function shall be used. The actual deflection equation derived using polynomial displacement function is given as:

$w = \frac{F_{a4} \cdot F_{b4}}{576}(R - 2R^3 + R^4) \times (Q^2 - 2Q^3 + Q^4)$. The polynomial displacement shape function, h is given as: $h = (R - 2R^3 + R^4) \times (Q^2 - 2Q^3 + Q^4)$. The k values herein are given as:

$k_1 = 0.0076190476, k_2 = 0.0092517006, k_3 = 0.03936507931, k_4 = 0.0007709750,$

$k_5 = 0.0092517006, k_6 = 0.0066666667.$

X. RESULTS AND DISCUSSIONS

A close look at tables 1 to 6 reveals that the values of in-plane displacement in x axis, bending moment along y axis and shear deformation at both x and y axis increases as the span-depth ratio increases, while values of in-plane displacement in y axis, shear force, stresses and bending moment along x axis decrease as the span-depth ratio increases. The variation of the out-of-plane quantities decreases as the span-depth ratio increases and becomes insignificant from span-depth ratio of 20. The design factors $\overline{w}, \overline{u}, \overline{v}, \overline{\sigma_x}, \overline{\sigma_y}, \overline{\tau_{xy}}, \overline{\tau_{xz}}, \overline{\tau_{yz}}, \overline{M_x}, \overline{M_y}, \overline{Q_x}, \overline{Q_y}, \emptyset_x, \emptyset_y$ for displacements, stresses, stress resultants and shear deformation rotation along x coordinate and y coordinate respectively of rectangular plates at varying aspect ratio, $\propto$ and ρ, are obtained using equations as given in the previous sections at the center of the plate. The result shows slight changes for these values when compared with CPT results, this disprove the theory that thick plate is the plate with aspect ratio *a/t* > 10 (Wang *et al*., 2000; Qatu, 2004; Mazilu *et al*., 1986). But confirming from this work that span-depth aspect ratio *a/t* of 20 up to 100, a plate can be taking as moderately thick. Therefore, it can be concluded that from this research that thick plate is the one whose span-depth ratios value is 4 up to 20. The values of non-dimensional form of deflection for span-depth ratios of 100 and above are equal to the value from CPT. Tables 1, 2, 3, 4, 5 and 6 showed that at span-depth ratio above 100, the values obtained from the models used herein coincide exactly with values from CPT.

Rectangular plate's boundary conditions with clamped on two opposite short edges and simply supported on two opposite long edges (CSCS) are considered and results compared with solution presented by Reddy. J. N, ABAQUS and Crooke and Levinson. From table 6, the studies in comparison present good interpretations of the four results except for the aspect ratio ρ of 5.0 which absolute differences are 27.15232%, 20.28986% and 20.28986% respectively. However, the slight higher value of the absolute difference is attributed to the slight higher value from this work, the reason is because the model is of a higher order than those in comparison. From the percentage different obtained from table, it is clear that the results of exact solution as obtained herein agree very well with that of approximating solution by J.N Reddy, *ABAQUS* and Crooke and Levinson. The average percentage difference for the out of plane displacement of 15.70074% is negligible.

Looking closely on table 7 and 8, it can be observed that the deflection values are in agreement with those obtained in the literature, thus confirming the correctness of the derived relationship. Though the disparity between the values in bending is obvious. It does not invalidate the results, since the values from the present solution are upper bound results, which will not put the structure being design in danger. The missing result in the table indicates the paucity of literature for such a plate.

It is observed that values of out plane displacement using this exact approach are slightly higher, showing some level of accuracy and safety of the analysis. Hence, the exact function according to the characteristic orthogonal polynomial can be used reliably for analysis of deflection and shear deformation of a rectangular plate with all edges simply supported.

However, a critical observation of the values reveals that the values from different models are very good approximation of one another. Hence, the model derived herein is quite sufficient for thick plate (Traditional third order refined plate theory) analyses.

Table 1: Non-dimensional centroidal deflection, shear deformations and its coefficients of CSCS plate for a/t = 4.0

$\alpha = \frac{b}{a}$	$A_1 = \overline{A_1}\left(\frac{qb^4}{D}\right)$	$A_2 = \overline{A_2}\left(\frac{qb^4}{D}\right)$	$A_3 = \overline{A_3}\left(\frac{qb^4}{D}\right)$	$w = \overline{w}\left(\frac{qb^4}{D}\right)$	$\overline{\emptyset_x}\left(\frac{qb^4}{D}\right)$	$\overline{\emptyset_y}\left(\frac{qb^4}{D}\right)$
	$\overline{A_1}$	$\overline{A_2}$	$\overline{A_3}$	$\overline{w}$	$\overline{\emptyset_x}$	$\overline{\emptyset_y}$
1	0.118634	0.030481	0.008318	0.0023171	0.001508816	0.000499104
1.1	0.15521	0.036655	0.009437	0.0030315	0.001814415	0.000566209
1.2	0.195425	0.042968	0.01047	0.0038169	0.002126895	0.00062818
1.3	0.238146	0.049268	0.011399	0.0046513	0.002438748	0.000683931
1.4	0.282229	0.055424	0.012215	0.0055123	0.002743467	0.000732906
1.5	0.326619	0.061331	0.012917	0.0063793	0.0030359	0.00077501
1.6	0.370422	0.066916	0.013508	0.0072348	0.003312352	0.000810498
1.7	0.412931	0.072131	0.013998	0.0080651	0.003570499	0.000839864
1.8	0.453635	0.076953	0.014396	0.0088601	0.003809196	0.000863742
1.9	0.492192	0.081378	0.014714	0.0096131	0.004028221	0.000882819
2	0.52841	0.085415	0.014963	0.0103205	0.004228034	0.00089778

Table 2: Non-dimensional centroidal deflection shear deformations and its coefficients of CSCS plate for a/t = 5.0

$\alpha = \frac{b}{a}$	$A_1 = \overline{A_1}\left(\frac{qb^4}{D}\right)$	$A_2 = \overline{A_2}\left(\frac{qb^4}{D}\right)$	$A_3 = \overline{A_3}\left(\frac{qb^4}{D}\right)$	$w = \overline{w}\left(\frac{qb^4}{D}\right)$	$\overline{\emptyset_x}\left(\frac{qb^4}{D}\right)$	$\overline{\emptyset_y}\left(\frac{qb^4}{D}\right)$
	$\overline{A_1}$	$\overline{A_2}$	$\overline{A_3}$	$\overline{w}$	$\overline{\emptyset_x}$	$\overline{\emptyset_y}$
1	0.112678	0.020068	0.005246	0.0022007	0.000993344	0.000314788
1.1	0.147668	0.024051	0.005944	0.0028841	0.001190524	0.000356648
1.2	0.186075	0.028107	0.006586	0.0036343	0.001391298	0.000395153
1.3	0.2268	0.032139	0.007161	0.0044297	0.001590864	0.000429656
1.4	0.268738	0.036063	0.007664	0.0052488	0.00178513	0.00045985
1.5	0.310884	0.039816	0.008095	0.006072	0.001970915	0.000485712
1.6	0.352395	0.043353	0.008457	0.0068827	0.00214599	0.000507433
1.7	0.392612	0.046647	0.008756	0.0076682	0.002309012	0.000525348
1.8	0.431062	0.049684	0.008998	0.0084192	0.002459374	0.000539869
1.9	0.467438	0.052466	0.009191	0.0091296	0.002597043	0.000551435
2	0.501568	0.054998	0.009341	0.0097962	0.0027224	0.00056048

Table 3: Non-dimensional centroidal deflection, shear deformations and its coefficients of CSCS plate for a/t = 10

$\alpha = \frac{b}{a}$	$A_1 = \overline{A_1}\left(\frac{qb^4}{D}\right)$	$A_2 = \overline{A_2}\left(\frac{qb^4}{D}\right)$	$A_3 = \overline{A_3}\left(\frac{qb^4}{D}\right)$	$w = \overline{w}\left(\frac{qb^4}{D}\right)$	$\overline{\emptyset_x}\left(\frac{qb^4}{D}\right)$	$\overline{\emptyset_y}\left(\frac{qb^4}{D}\right)$
	$\overline{A_1}$	$\overline{A_2}$	$\overline{A_3}$	$\overline{w}$	$\overline{\emptyset_x}$	$\overline{\emptyset_y}$
1	0.10456	0.005227	0.001284	0.002042187	0.000258736	7.70193E-05
1.1	0.137375	0.006232	0.001452	0.002683099	0.000308476	8.7105E-05
1.2	0.173317	0.007249	0.001606	0.003385091	0.000358829	9.63326E-05
1.3	0.211329	0.008255	0.001743	0.004127526	0.000408603	0.000104558
1.4	0.25037	0.009228	0.001862	0.004890038	0.000456804	0.00011172
1.5	0.289502	0.010155	0.001964	0.005654339	0.000502683	0.000117825
1.6	0.32795	0.011025	0.002049	0.006405275	0.000545733	0.000122931
1.7	0.365118	0.011832	0.002119	0.007131213	0.000585668	0.000127125
1.8	0.400585	0.012573	0.002175	0.007823927	0.000622379	0.000130512
1.9	0.434083	0.01325	0.00222	0.008478186	0.000655895	0.000133201
2	0.46547	0.013865	0.002255	0.00909121	0.000686338	0.000135297

Table 4: Non-dimensional centroidal deflection, shear deformations and its coefficients of CSCS plate for a/t = 20

$\propto = \frac{b}{a}$	$A_1 = \overline{A_1}\left(\frac{qb^4}{D}\right)$	$A_2 = \overline{A_2}\left(\frac{qb^4}{D}\right)$	$A_3 = \overline{A_3}\left(\frac{qb^4}{D}\right)$	$w = \overline{w}\left(\frac{qb^4}{D}\right)$	$\overline{\emptyset_x}\left(\frac{qb^4}{D}\right)$	$\overline{\emptyset_y}\left(\frac{qb^4}{D}\right)$
	$\overline{A_1}$	$\overline{A_2}$	$\overline{A_3}$	$\overline{w}$	$\overline{\emptyset_x}$	$\overline{\emptyset_y}$
1	0.102493186	0.00132108	0.000319054	0.00200182	6.53935E-05	1.91433E-05
1.1	0.134752865	0.001572746	0.00036067	0.002631892	7.78509E-05	2.16402E-05
1.2	0.170067913	0.001827138	0.000398694	0.003321639	9.04433E-05	2.39217E-05
1.3	0.207393769	0.002078248	0.000432545	0.00405066	0.000102873	2.59527E-05
1.4	0.245703617	0.00232111	0.000461983	0.004798899	0.000114895	2.7719E-05
1.5	0.284078617	0.002551993	0.000487049	0.00554841	0.000126324	2.92229E-05
1.6	0.32176012	0.002768409	0.000507989	0.006284377	0.000137036	3.04793E-05
1.7	0.358167861	0.002968971	0.000525175	0.006995466	0.000146964	3.15105E-05
1.8	0.392893314	0.003153194	0.000539043	0.007673698	0.000156083	3.23426E-05
1.9	0.425678264	0.003321264	0.000550046	0.008314029	0.000164403	3.30027E-05
2	0.456386973	0.003473833	0.000558619	0.008913808	0.000171955	3.35171E-05

Table 5: Non-dimensional deflection and Stress resultants; Bending Moment and Shear Force for a CSCS thick square plate

$\rho = \frac{a}{t}$	$w = \overline{w}\left(\frac{qb^4}{D}\right)$	$M_x = \overline{M_x}qa^2$	$M_y = \overline{M_y}qa^2$	$M_y = \overline{M_y}qa^2$	$Q_x = \overline{Q_x}qa$	$Q_y = \overline{Q_y}qa$
	$\overline{w}$ (0.5,0.5)	$\overline{M_x}$(0.5,0.5)	$\overline{M_y}$(0.5,0.5)	$\overline{M_y}$(0.5,0.0)	$\overline{Q_x}$(1.0,0.5)	$\overline{Q_y}$(0.5,0.0)
4	0.002317071	0.025572	0.038569	-0.06791	0.116409	-0.48231
5	0.002200745	0.026585	0.038228	-0.06649	0.102569	-0.44615
6	0.002136671	0.027175	0.03803	-0.06566	0.094796	-0.4265
7	0.002097699	0.027545	0.037905	-0.06515	0.090012	-0.41464
8	0.002072259	0.027791	0.037822	-0.0648	0.086867	-0.40694
9	0.002054748	0.027963	0.037764	-0.06456	0.084691	-0.40166
10	0.002042187	0.028087	0.037722	-0.06439	0.083124	-0.39788
15	0.002012316	0.028387	0.037621	-0.06397	0.079381	-0.38892
20	0.00200182	0.028493	0.037586	-0.06382	0.078059	-0.38579
25	0.001996954	0.028543	0.037569	-0.06375	0.077445	-0.38433
30	0.001994309	0.02857	0.03756	-0.06371	0.077111	-0.38354
35	0.001992713	0.028586	0.037554	-0.06369	0.076909	-0.38307
40	0.001991678	0.028597	0.037551	-0.06367	0.076778	-0.38276
45	0.001990967	0.028604	0.037548	-0.06366	0.076689	-0.38255
50	0.001990459	0.028609	0.037546	-0.06366	0.076624	-0.3824
55	0.001990083	0.028613	0.037545	-0.06365	0.076577	-0.38229
60	0.001989797	0.028616	0.037544	-0.06365	0.076541	-0.3822
65	0.001989574	0.028618	0.037543	-0.06364	0.076513	-0.38213
70	0.001989398	0.02862	0.037543	-0.06364	0.07649	-0.38208
75	0.001989139	0.028623	0.037542	-0.06364	0.076457	-0.382
80	0.001989139	0.028627	0.037557	-0.06367	0.076457	-0.38183
85	0.001989042	0.028624	0.037542	-0.06364	0.076445	-0.38198
90	0.001988961	0.028625	0.037541	-0.06363	0.076435	-0.38195
95	0.001988892	0.028625	0.037541	-0.06363	0.076426	-0.38193
100	0.001988834	0.028626	0.037541	-0.06363	0.076419	-0.38191
1000	0.001988297	0.028631	0.037539	-0.06363	0.076351	-0.38175

Table 6: Non-dimensional displacement and stresses of CSCS plate for b/a = 1. 0

$\rho = \frac{a}{t}$	$u = \bar{u}\left(\frac{qa^4}{\rho D}\right)$	$v = \bar{v}\left(\frac{qa^4}{\rho D}\right)$	$\sigma_x = \overline{\sigma_x}(q\rho^2)$	$\sigma_y = \overline{\sigma_y}(q\rho^2)$	$\tau_{xy} = \overline{\tau_{xy}}(q\rho^2)$	$\tau_{xz} = \overline{\tau_{xz}}(q\rho^3)$	$\tau_{yz} = \overline{\tau_{yz}}(q\rho^3)$
	$\bar{u}$	$\bar{v}$	$\overline{\sigma_x}$	$\overline{\sigma_y}$	$\overline{\tau_{xy}}$	$\overline{\tau_{xz}}$	$\overline{\tau_{yz}}$
4	-0.002433	-0.003393	0.174215	0.245222	-0.085237	0.006337	0.0020962
5	-0.002458	-0.003275	0.173126	0.238227	-0.084059	0.004172	0.0013221
6	-0.002474	-0.00321	0.172631	0.234339	-0.08344	0.0029447	0.0009104
7	-0.002485	-0.003169	0.172367	0.231962	-0.083074	0.0021854	0.0006653
8	-0.002492	-0.003143	0.172211	0.230405	-0.082839	0.0016844	0.0005076
9	-0.002498	-0.003124	0.172111	0.22933	-0.08268	0.0013371	0.0004001
10	-0.002502	-0.003111	0.172043	0.228558	-0.082567	0.0010867	0.0003235
15	-0.002511	-0.00308	0.171895	0.226718	-0.082302	0.0004869	0.0001432
20	-0.002515	-0.003068	0.171847	0.22607	-0.08221	0.0002747	8.04E-05
25	-0.002517	-0.003063	0.171825	0.225769	-0.082167	0.000176	5.142E-05
30	-0.002517	-0.00306	0.171814	0.225606	-0.082144	0.0001223	3.569E-05
35	-0.002518	-0.003059	0.171807	0.225507	-0.08213	8.991E-05	2.622E-05
40	-0.002518	-0.003058	0.171803	0.225443	-0.082121	6.885E-05	2.007E-05
45	-0.002519	-0.003057	0.1718	0.225399	-0.082115	5.441E-05	1.586E-05
50	-0.002519	-0.003056	0.171798	0.225368	-0.082111	4.408E-05	1.284E-05
55	-0.002519	-0.003056	0.171796	0.225345	-0.082107	3.643E-05	1.061E-05
60	-0.002519	-0.003056	0.171795	0.225327	-0.082105	3.062E-05	8.918E-06
65	-0.002519	-0.003055	0.171794	0.225313	-0.082103	2.609E-05	7.598E-06
70	-0.002519	-0.003055	0.171793	0.225302	-0.082102	2.25E-05	6.551E-06
75	-0.002519	-0.003055	0.171792	0.225286	-0.082099	1.723E-05	5.016E-06
80	-0.002519	-0.003056	0.171807	0.225336	-0.08211	1.723E-05	-5.02E-06
85	-0.002519	-0.003055	0.171792	0.22528	-0.082098	1.526E-05	4.443E-06
90	-0.002519	-0.003055	0.171791	0.225275	-0.082098	1.361E-05	3.963E-06
95	-0.002519	-0.003055	0.171791	0.225271	-0.082097	1.222E-05	3.557E-06
100	-0.002519	-0.003055	0.171791	0.225267	-0.082097	1.103E-05	3.21E-06
1000	-0.00252	-0.003054	0.171788	0.225234	-0.082092	1.103E-07	3.21E-08

Table 7: Results comparation for the non-dimensionalized maximum deflection, $w = Wo^{m}(a/2,0)D/q_{o}^{4})$ of square plates (b/a = 1) with a clamped boundary on two sides (SCSC) for two different thickness-to-side ratios **ρ** of uniformly loaded square.

$\rho = \frac{a}{t}$	ABAQUS $\bar{w}$	Present study $\bar{w}$	Percent difference	Crooke and Levinson	Present study $\bar{w}$	Percent difference	Reddy, J. $\bar{w}$	Present study $\bar{w}$	Percent difference	CPT
4.0	-	0.00231	0.00231	-	0.00231	0.00231	-	0.00231	0.00231	0.0020
5.0	0.00302	0.00220	27.15232	0.00276	0.00220	20.28986	0.00302	0.00220	27.15232	-
10.0	0.00221	0.00204	7.692308	0.00213	0.00204	4.225352	0.00221	0.00204	7.692308	-
Average difference			17.42231			12.2576			17.42231	-

Table 8: Non-dimensional Stress resultants; Bending Moment and Shear Force for a CSCS thick square plate

$\rho = \frac{a}{t}$	Point (a/x, b/y)	Stress resultant	Present Study	Xiao et al (MQ MLPG1)	Kant and Hinton	Lee et al.
5.0	(0.5,0.5)	M_x	0.0281	0.0299	0.0292	0.0292
	(0.5,0.5)	M_y	0.0377	0.0338	0.0330	0.0331
	(0.5,0.5)	M_y	0.0644	0.0651	0.0626	0.0627
	(0.5,0.5)	Q_x	0.0831	0.2520	0.2510	0.2510
	(0.5,0.5)	Q_y	0.3979	0.5040	0.4750	0.4750
10.0	(0.5,0.5)	Mx	0. 0266	0.0255	0.0258	0.0258
	(0.5,0.5)	My	0. 0382	0.0330	0.0332	0.0333
	(0.5,0.5)	My	0.0665	0.0697	0.0697	0.0680
	(0.5,0.5)	Qx	0. 1026	0.2485	0.2430	0.2430
	(0.5,0.5)	Qy	0.4462	0.5160	0.5000	0.5000

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