

A New Ranking Method Using Dodecagonal Fuzzy Number To Solve Fuzzy Transportation Problem

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Abstract

In this paper, we introduce a new ranking method to solve fuzzy transportation problem where the transportation cost, demand and supply are in the form of dodecagonal fuzzy number. Compared to the conventional ranking methods we got a better result using the proposed new ranking method. Here we convert fuzzy transportation problem using proposed ranking technique to crisp problem and solve using a new method. To clarify our new method an example is explained.

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Keywords: Fuzzy transportation problem, Ranking function, Dodecagonal fuzzy number.

1 INTRODUCTION

Fuzzy set theory was introduced by L.A. Zadeh [5]. Dubis and H. Prade [1] introduced operations on fuzzy numbers. Fuzzy arithmetic was proposed by A. Kauffmann and M. Gupta [6]. The concept of decision making in fuzzy environment was introduced by Bellman and Zadeh [8]. In decision making ranking fuzzy number is very important. There are different ranking methods for fuzzy numbers. Tian-shy and Mao-jiun J Wang [7] and Shan Hu Chen [10] proposed many methods and theorems on ranking of fuzzy numbers.

The main aim of transportation problem is to transport different quantities which are stored at different sources to different destinations with minimum transportation cost. If some or all parameters of a transportation problem are fuzzy numbers then it is a fuzzy transportation problem. Several researchers proposed different methods to solve fuzzy transportation problem using triangular and trapezoidal fuzzy numbers. Krishna Prabha [14] use method of magnitude to solve the problem. Uma Maheswari and K Ganesan [11] proposed a method to solve fully fuzzy transportation problem using pentagonal fuzzy numbers. P. Elumalai [12] developed a ranking method using hexagonal fuzzy number. S.U. Malini [4] introduced a method to solve fuzzy transportation problem using octagonal fuzzy numbers. Dr. A. Sahaya Sudha, S. Karunambigai [13] solve transportation problem using a heptagonal fuzzy number. Kirtiwant P. Ghadle and Priyanka A. Pathade [14] solve the problem using generalized ranking

method. They use centroid ranking technique to convert fuzzy transportation problem into crisp problem. Dr. M.S. Annie Christi, Mrs. Malini. D [15] introduced octagonal fuzzy number. They use different ranking technique and find the optimum solution using best candidates method. K. Dhurai, A. Karpagam [16] proposed a new ranking function to solve octagonal fuzzy number. Nonagonal fuzzy number was introduced by A. Felix, S. Christopher and A. Victor Devadoss [17]. A. Felix, A. Victor Devadoss [2] proposed a new method on decagonal fuzzy number under uncertain linguistic environment. Jatinder Pal Singh and Neha Ishesh Thakur [9] proposed ranking of generalized dodecagonal fuzzy numbers using centroid of centroids.

In this paper the transportation cost, supply and demand are dodecagonal fuzzy number. Here we introduce a new ranking technique to convert the given fuzzy problem into crisp problem. Now we apply the proposed algorithm to find the solution of the given fuzzy problem.

2 PRELIMINARIES

2.1 Fuzzy Set

A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) / x \in X\}$, where $\mu_{\tilde{A}}(x)$ is called membership function of x in \tilde{A} which maps $X \rightarrow [0, 1]$.

2.2 Fuzzy Number

A fuzzy set \tilde{A} on R is said to be a fuzzy number if its membership function $\mu_{\tilde{A}} : R \rightarrow [0, 1]$ possess the following conditions.

- $\mu_{\tilde{A}}$ is normal. It means that there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$
- $\mu_{\tilde{A}}$ is convex. It means that for every $x_1, x_2 \in R$, $\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}$, $\lambda \in [0, 1]$
- $\mu_{\tilde{A}}$ is upper semi-continuous.
- $\text{supp}(\tilde{A})$ is bounded in R

3 DODECAGONAL FUZZY NUMBER

The membership function of dodecagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12})$ where $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}$ are real numbers, is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq a_1 \\ k_1 \left(\frac{x-a_1}{a_2-a_1} \right), & a_1 \leq x \leq a_2 \\ k_1, & a_2 \leq x \leq a_3 \\ k_1 + (1-k_1) \frac{x-a_3}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ k_2, & a_4 \leq x \leq a_5 \\ k_2 + (1-k_2) \frac{x-a_5}{a_6-a_5}, & a_5 \leq x \leq a_6 \\ 1, & a_6 \leq x \leq a_7 \\ k_2 + (1-k_2) \frac{a_8-x}{a_8-a_7}, & a_7 \leq x \leq a_8 \\ k_2, & a_8 \leq x \leq a_9 \\ k_1 + (1-k_1) \frac{a_{10}-x}{a_{10}-a_9}, & a_9 \leq x \leq a_{10} \\ k_1, & a_{10} \leq x \leq a_{11} \\ k_1 \left(\frac{a_{12}-x}{a_{12}-a_{11}} \right), & a_{11} \leq x \leq a_{12} \\ 0, & a_{12} \leq x \end{cases} \quad (3.1)$$

where $0 < k_1 < k_2 < 1$

4 RANKING OF DODECAGONAL FUZZY NUMBER

We recall the definition of ranking of dodecagonal fuzzy number proposed by Basirzadeh [18].

Definition: Let \tilde{A} be a normal dodecagonal fuzzy number. The measure of \tilde{A} is

$$M_0^{DD}(\tilde{A}) = \frac{1}{4} \{ (a_1 + a_2 + a_{11} + a_{12})k_1 + (a_3 + a_4 + a_9 + a_{10})(k_2 - k_1) + (a_5 + a_6 + a_7 + a_8)(1 - k_2) \}. \text{ where } 0 < k_1 < k_2 < 1$$

4.1 Proposed Ranking Method

Definition: Let \tilde{A} be the normal dodecagonal fuzzy number. The measure of \tilde{A} is

$$M_0^{DDEC}(\tilde{A}) = \frac{1}{12} \{ (a_1 + a_{12}) + k_1(a_2 + a_3 + a_{10} + a_{11}) + k_2(a_4 + a_5 + a_8 + a_9) + (a_6 + a_7) \}. \text{ where } 0 < k_1 < k_2 < 1$$

8 NUMERICAL EXAMPLE

	D_1	D_2	D_3	D_4	Supply
S_1	(-3,-2,-1,0,1,2,3,4,5,6,7,8)	(-2,-1,0,1,2,3,4,5,6,7,8,9)	(6,7,8,9,10,11,12,13,14,15,16,17)	(2,3,4,5,6,7,8,9,10,11,12,13)	(-1,0,1,3,5,6,7,8,10,12,13,14)
S_2	(-4,-3,-2,-1,0,1,2,3,4,5,6,7)	(-5,-4,-3,-2,-1,0,1,2,3,4,5,6)	(0,1,2,4,5,6,7,8,9,11,12,13)	(-5,-4,-3,-1,0,1,2,4,5,6,7,9)	(-4,-3,-2,-1,0,1,2,3,4,5,6,7)
S_3	(0,1,2,3,4,5,6,7,8,9,10,11)	(1,2,3,6,7,8,9,10,12,13,15,16)	(8,9,11,12,14,15,16,17,18,21,22,23)	(2,3,5,6,8,9,10,11,12,15,16,17)	(2,4,5,6,8,10,12,13,15,17,18,19)
Demand	(2,3,4,5,6,7,8,9,10,11,12,13)	(-2,0,1,2,3,5,6,7,8,10,11,12)	(-2,-1,0,1,2,3,4,5,6,7,8,9)	(-3,-2,-1,0,1,2,3,4,5,6,7,8)	

5 FUZZY TRANSPORTATION PROBLEM

The fuzzy transportation problem is as follows.

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^m \sum_{j=1}^n \tilde{C}_{ij} \tilde{x}_{ij}$$

subject to $\sum_{j=1}^n \tilde{x}_{ij} = \tilde{a}_i, i = 1, 2, \dots, m$

$$\sum_{i=1}^m \tilde{x}_{ij} = \tilde{b}_j, j = 1, 2, \dots, n$$

$$\sum_{i=1}^m \tilde{a}_i = \sum_{j=1}^n \tilde{b}_j, \tilde{x}_{ij} \geq 0, \text{ for all } i \text{ and } j$$

where, \tilde{a}_i = fuzzy accessibility at i^{th} source

\tilde{b}_j = fuzzy demand at j^{th} destination

\tilde{C}_{ij} = fuzzy transportation cost from source to destination

\tilde{x}_{ij} = number of fuzzy units carry over from source to destination.

6 PROCEDURE TO SOLVE FUZZY TRANSPORTATION PROBLEM

We developed a new algorithm to find the optimal solution for the defuzzified transportation problem.

Step 1: Convert the fuzzy transportation problem into crisp values by using the proposed ranking method. Here we are using dodecagonal fuzzy number.

Step 2: Continue the crisp problem using the proposed method.

7. NEW ALGORITHM FOR SOLVING TRANSPORTATION PROBLEM

Step 1: Verify whether the problem is balanced or not. If it is not balanced make it balanced.

Step 2: Determine the difference between the largest and the next to largest costs for each row of the transportation table. Also, find the difference for each column.

Step 3: Select the row or column with the smallest difference. If a tie occurs, choose an arbitrary choice. Let the smallest difference correspond to i^{th} row and let C_{ij} be the largest cost in the i^{th} row. Allocate the maximum feasible amount $x_{ij} = \min(a_i, b_j)$ in the $(i, j)^{th}$ cell. Now strike out either the i^{th} row or j^{th} column.

Step 4: Once more determine the row and column difference. Then goto step 2. Continue the process up to the demand and supply are exhausted.

Using the proposed ranking method we convert the fuzzy problem of dodecagonal numbers into the crisp value. This is done by taking $k_1 = 0.2$ and $k_2 = 0.6$.

$$R(-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8) = \frac{1}{2}\{(-3+8) + (0.2)(-2-1+6+7) + (0.6)(0+1+4+5) + (2+3)\} = \frac{1}{2}\{5+2+6+5\} = \frac{1}{2} \times 18 = 1.5$$

$$R(-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) = 2.1$$

$$R(6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17) = 6.9$$

$$R(2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13) = 4.5$$

$$R(-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7) = 0.9$$

$$R(-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6) = 0.3$$

$$R(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13) = 3.9$$

$$R(-5, -4, -3, -1, 0, 1, 2, 4, 5, 6, 7, 9) = 1.0$$

$$R(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11) = 3.3$$

$$R(1, 2, 3, 6, 7, 8, 9, 10, 12, 13, 15, 16) = 5.1$$

$$R(8, 9, 11, 12, 14, 15, 16, 17, 18, 21, 22, 23) = 9.2$$

$$R(2, 3, 5, 6, 8, 9, 10, 11, 12, 15, 16, 17) = 5.6$$

$$R(-1, 0, 1, 3, 5, 6, 7, 8, 10, 12, 13, 14) = 3.9$$

$$R(-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7) = 0.9$$

$$R(2, 4, 5, 6, 8, 10, 12, 13, 15, 17, 18, 19) = 6.4$$

$$R(2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13) = 4.5$$

$$R(-2, 0, 1, 2, 3, 5, 6, 7, 8, 10, 11, 12) = 3.1$$

$$R(-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) = 2.1$$

$$R(-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8) = 1.5$$

The crisp transportation table

	D_1	D_2	D_3	D_4	Supply
S_1	1.5	2.1	6.9	4.5	3.9
S_2	0.9	0.3	3.9	1	0.9
S_3	3.3	5.1	9.2	5.6	6.5
Demand	4.5	3.1	2.1	1.5	

Using new algorithm we obtain the initial solution as.

	D_1	D_2	D_3	D_4	Supply
S_1	1.5	2.1 1.8	6.9 2.1	4.5	3.9
S_2	0.9	0.3 0.9	3.9	1	0.9
S_3	3.3 4.5	5.1 0.4	9.2	5.6 1.5	6.5
Demand	4.5	3.1	2.1	1.5	

Hence we got the total cost of the transportation as:

$$(2.1)(1.8) + (6.9)(2.1) + (0.3)(0.9) + (3.3)(4.5) + (5.1)(0.4) + (5.6)(1.5) = 43.83$$

8.1 Solving the example using ranking method [5]

Consider the following fuzzy transportation problem

	D_1	D_2	D_3	D_4	Supply
S_1	(-3,-2,-1,0,1,2,3,4,5,6,7,8)	(-2,-1,0,1,2,3,4,5,6,7,8,9)	(6,7,8,9,10,11,12,13,14,15,16,17)	(2,3,4,5,6,7,8,9,10,11,12,13)	(-1,0,1,3,5,6,7,8,10,12,13,14)
S_2	(-4,-3,-2,-1,0,1,2,3,4,5,6,7)	(-5,-4,-3,-2,-1,0,1,2,3,4,5,6)	(0,1,2,4,5,6,7,8,9,11,12,13)	(-5,-4,-3,-1,0,1,2,4,5,6,7,9)	(-4,-3,-2,-1,0,1,2,3,4,5,6,7)
S_3	(0,1,2,3,4,5,6,7,8,9,10,11)	(1,2,3,6,7,8,9,10,12,13,15,16)	(8,9,11,12,14,15,16,17,18,21,22,23)	(2,3,5,6,8,9,10,11,12,15,16,17)	(2,4,5,6,8,10,12,13,15,17,18,19)
Demand	(2,3,4,5,6,7,8,9,10,11,12,13)	(-2,0,1,2,3,5,6,7,8,10,11,12)	(-2,-1,0,1,2,3,4,5,6,7,8,9)	(-3,-2,-1,0,1,2,3,4,5,6,7,8)	

We calculated the measure by taking the values of $k_1 = 0.2$ and $k_2 = 0.6$.

$$R(-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8) = 2.50$$

$$R(-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) = 3.50$$

$$R(6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17) = 11.5$$

$$R(2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13) = 7.5$$

$$R(-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7) = 1.5$$

$$R(-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6) = 0.5$$

$$R(0, 1, 2, 4, 5, 6, 7, 8, 9, 11, 12, 13) = 6.5$$

$$R(-5, -4, -3, -1, 0, 1, 2, 4, 5, 6, 7, 9) = 1.75$$

$$R(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11) = 5.5$$

$$R(1, 2, 3, 6, 7, 8, 9, 10, 12, 13, 15, 16) = 8.5$$

$$R(8, 9, 11, 12, 14, 15, 16, 17, 18, 21, 22, 23) = 15.5$$

$$R(2, 3, 5, 6, 8, 9, 10, 11, 12, 15, 16, 17) = 9.5$$

$$R(-1, 0, 1, 3, 5, 6, 7, 8, 10, 12, 13, 14) = 6.5$$

$$R(-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7) = 1.5$$

$$R(2, 4, 5, 6, 8, 10, 12, 13, 15, 17, 18, 19) = 10.75$$

$$R(2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13) = 7.5$$

$$R(-2, 0, 1, 2, 3, 5, 6, 7, 8, 10, 11, 12) = 5.25$$

$$R(-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) = 3.5$$

$$R(-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8) = 2.5$$

The crisp transportation problem is as following.

	D_1	D_2	D_3	D_4	Supply
S_1	2.5	3.5	11.5	7.5	6.5
S_2	1.5	0.5	6.5	1.75	1.5
S_3	5.5	8.5	15.5	9.5	10.75
Demand	7.5	5.25	3.5	2.5	

Using new algorithm we obtain the solution as.

	D_1	D_2	D_3	D_4	Supply
S_1	2.5	3.5 $\boxed{3}$	11.5 $\boxed{3.5}$	7.5	6.5
$S_{1.5}$	1.5	0.5 $\boxed{1.5}$	6.5	1.75	1.5
S_3	5.5 $\boxed{7.5}$	8.5 $\boxed{0.75}$	15.5	9.5 $\boxed{2.5}$	10.75
Demand	7.5	5.25	3.5	2.5	

we got the solution as:

$$(3.5)(3) + (11.5)(3.5) + (0.5)(1.5) + (5.5)(7.5) + (8.5)(0.75) + (9.5)(2.5) = 122.875$$

9. CONCLUSION

Here we convert the fuzzy transportation problem into crisp transportation problem using new ranking technique and solve the crisp problem using new algorithm. We illustrate the new ranking technique and the existing ranking method with an example. Compared to the conventional ranking method used by Basirzadeh we got a better result using the proposed ranking method.

REFERENCES

- [1] D. Dubois and H. Prade, *Operations on Fuzzy numbers*, International Journal of System Science 9(6)(1978), 613-626
- [2] A.Felix and A.Victor Devadoss, *A new decagonal fuzzy number under uncertain linguistic environment*, International journal of mathematics and its application, Volume 3, Issue 1(2015), 89-97.
- [14] S.Krishna Prabha, S.Vimala, *ATM for solving fuzzy transportation problem using method of magnitude*, Jaetsd journal for advanced research in applied science, Volume 5, Issue 3, March 2018.
- [4] S.U.Malini, F.C.Kennedy *An approach for solving fuzzy transportation using octagonal fuzzy number*, Applied mathematical science, 54(2013) 2661-2673.
- [5] L.A.Zadeh, *Fuzzy sets*, Information and control, 8(3) 1965, 338- 353.
- [6] Kauffman, A., *Introduction to fuzzy arithmetic: Theory and application*, Van Nostrand reinhold, New york, (1980).
- [7] Tian-Shy Liou, Mao-Jiun.J.Wang, *Ranking fuzzy numbers with integral value*, Fuzzy Sets and Systems, Volume 50, Issue 3, 1992, 247-255.
- [8] Bellmann.R, Zadeh L.A, *Decision making in a fuzzy environment*, Management Science, 17(B) (1970), 141-164.
- [9] Jatinder Pal Singh, Neha ishesh Thakur *An approach for solving fuzzy transportation problem using dodecagonal fuzzy number*, International journal of mathematical archive, 6(4), (2015), 105 - 112.
- [10] Shan-Huo-Chen, *Ranking fuzzy numbers with maximizing set and minimizing set*, Fuzzy sets and systems, Volume 17, Issue 2(1985), 113 - 129.
- [11] Uma Maheswari, K.Ganesan *Solving fully fuzzy transportation problem using pentagonal fuzzy numbers*; IOP conf.series:Journal of physics:conf. series 1000(2018) 012014
- [12] P.Elumalai *Fuzzy transportation problem using hexagonal fuzzy numbers*; Emperor International Journal of Finance and Management Research, July, 2017
- [13] Dr. A.Sahaya Sudha, S.Karunambigai *Solving a Transportation Problem using a Heptagonal Fuzzy Number*, International Journal of Advanced Research in Science, Engineering and Technology, Vol. 4, Issue 1, January 2017.
- [14] Kirtiwant P. Ghadle, Priyanka A. Pathade, *Solving Transportation Problem with Generalized Hexagonal and Generalized Octagonal Fuzzy Numbers by Ranking Method*, Global Journal of Pure and Applied Mathematics, Vol 13, Issue 9, (2017), 6367-6376.
- [15] Dr. M. S. Annie Christi, Mrs. Malini. D *An approach to solve transportation problem with octagonal fuzzy numbers using best candidate method and different ranking techniques*; International Journal of Computer Application, Volume 66 No.1, January- February 2016.
- [16] K. Dhurai, A. Karpagam *New ranking function on octagonal fuzzy number for solving fuzzy transportation problem*, International Journal of Pure and Applied Mathematics, Volume 119 No. 9 2018, 125-131
- [17] A.Felix1, S.Christopher, A.Victor Devadoss, *A Nonagonal Fuzzy Number and Its Arithmetic Operation*, International Journal of Mathematics And its Applications Volume 3, Issue 2 (2015), 185-195.
- [18] Basirzadeh.H, *An approach for solving fuzzy transportation problem*, Applied mathematical science, 5(2011), 1549-1566.