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Fuzzy Soft Generalized Closed Sets in Fuzzy Soft Topological Spaces

Zeba Tarrannum and Anil P Narappanavar*

Department of Mathematic, Global Academy of Technology,
Bengaluru - 560098, Karnataka, India.

**Corresponding author*

Abstract

In this paper we introduce fuzzy soft g-closed set, fuzzy soft g*- closed set and fuzzy soft strongly g*- closed set and study their relations. Also fuzzy soft g-continuous functions in fuzzy soft topological spaces have been introduced and few of their properties have been studied.

Keywords: Fuzzy soft Topology, Fuzzy soft g-closed set, Fuzzy soft g*-closed set, Fuzzy soft strongly g*-closed set and fuzzy soft g-continuous functions.

1. INTRODUCTION

The fuzzy set is a generalization of regular set. In a regular set an element is either a member of the set or not .But a fuzzy set has a graphical description. This graphical description is called a membership function valued in the unit interval [0, 1].L.A Zadeh [1] initiated the concept of Fuzzy soft set and provided a natural base to handle and improve mathematically the Fuzzy phenomenon found in different areas of knowledge. C.L Chang [2] studied and discussed Fuzzy topological spaces. B.Ahmad and A.Kharral [3] investigated fuzzy soft sets and established Fuzzy soft functions on Fuzzy soft classes. The notion of topological structure of Fuzzy soft sets was introduced by Tanay and Kandemir [4] in 2011 and studied further by Varol and Aygun [5].

The concept of generalized closed sets plays a significant role in topology. There are many research papers which deal with different types of generalized closed sets. In 1970, Levine [6] introduced the concept of generalized closed set in the topological spaces (briefly g -closed).g∗-closed sets were introduced and studied by Veerakumar [7] for general topology. Recently Parimelazhagan and Subramonia Pillai introduced strongly g ∗-closed sets in topological space [8]. In 2013 the strongly g*- closed sets in fuzzy topological spaces was introduced by T.Rajendra Kumar and Anandajothi [9]

Accordingly, the proposed study is to introduce a new class of fuzzy soft closed sets namely fuzzy soft generalized closed set, fuzzy soft g*-closed set, fuzzy soft strongly g*- closed sets in fuzzy soft topological space and study their properties. Further we define and investigate few properties of fuzzy soft g-continuous functions in fuzzy soft topological space.

2. PRELIMINARIES

Definition 2.1[10] : Let X be the initial universe and E be the set of parameters. I^x be the set of all fuzzy sets on X. Let $A \subseteq E$ and $f : A \to I^x$. A pair (f, A) is called fuzzy soft set over X. It is also denoted by f_A i.e. for every $a \in A$, $f(a) = f_a : X \to I$ is a fuzzy set on X.

Definition 2.2[11] :Let τ be a collection of all fuzzy soft sets over a universe X with a fixed set of parameter set E then a triplet (X, τ, E) is called fuzzy soft topological space [FSTS] if it satisfies the following axioms.

(i) $\tilde{0}_E, \tilde{1}_E \in \tau$

(ii)Arbitrary union of members of τ is a member of τ .

(iii) Finite intersection of members of τ is a member of τ .

Every member of τ is called fuzzy soft open set i.e. a fuzzy soft set $f_A \in \tau$ is called fuzzy soft open set in X and its complement $1 - f_A$ is called fuzzy soft closed set.

Definition 2.3[5] :The intersection of all fuzzy soft closed super sets of f_A is called fuzzy soft closure of f_A denoted by, $Fscl(f_A)$.

$$Fscl(f_A) = \bigcap \{ h_D,\ h_D \text{ is fuzzy soft closed set and } f_A \subseteq h_D \}$$

Definition 2.4[5] :The union of all fuzzy soft open subsets of g_B is called fuzzy soft interior of g_B denoted by $Fs\operatorname{int}(f_A)$.

$$Fs\operatorname{int}(g_B) = \bigcup \{ h_D,\ h_D \text{ is fuzzy soft open set and } h_D \subseteq g_B \}$$

Definition 2.5[7] :A Fuzzy soft set f_A in Fuzzy soft topological space (X, τ, E) is called Fuzzy soft semi-open if $f_A \le Fscl\, Fs\operatorname{int}(f_A)$, Fuzzy soft semi-closed if $Fs\operatorname{int} Fscl(f_A) \le f_A$

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Definition 2.6[12] : A Fuzzy soft set f_A in Fuzzy soft topological space (X,τ,E) is called Fuzzy soft pre-open if $f_A \le Fs\text{int}\, Fscl\,(f_A)$, Fuzzy soft pre-closed if $Fscl\,Fs\text{int}\,(f_A) \le f_A$

Definition 2.7[12] : A Fuzzy soft set f_A in Fuzzy soft topological space (X,τ,E) is called Fuzzy soft regular -open if $f_A = Fs\text{int}\, Fscl\,(f_A)$, Fuzzy soft regular –closed if $Fscl\,Fs\text{int}\,(f_A) = f_A$

3. FUZZY SOFT GENERALIZED CLOSED SETS

A Fuzzy soft set f_A in Fuzzy soft topological space (X,τ,E) is called Fuzzy soft generalized closed set (in short Fs g-closed) if $Fscl\,(f_A) \le H, \forall f_A \le H$ and H is Fuzzy soft open in X.

Theorem 3.1: Every fuzzy soft closed set is fuzzy soft generalized closed set in a fuzzy soft topological space (X,τ,E).

Proof: Let A be a fuzzy soft closed set in X. Let A $\le$ H where H is fuzzy soft open. Since A is fuzzy soft closed Fscl (A) = A $\le$ H. Thus we have Fscl (A) $\le$ H whenever A $\le$ H. Therefore, A is fuzzy soft generalized closed set.

The converse of the above theorem need not be true in general which can be seen from the following example.

Let $\tau = \{\tilde{0}, \tilde{1}, (F_1,E), (F_2,E)\}$, $(F_1,E) = \{0.5, 0.7\}$, $(F_2,E) = \{0.25, 0.2\}$

Let $H = \{0.5, 0.7\}$, $A = \{0.4, 0.2\}$

A is fuzzy soft generalized closed set but not fuzzy soft closed set in X... □

Theorem 3.2: For any two fuzzy soft generalized closed sets A and B $Fscl(A \vee B) = Fscl(A) \vee Fscl(B)$.

Proof: Let A and B be any two fuzzy soft generalized closed sets in FSTS (X,τ,E)

$$A \le A \vee B \Rightarrow Fscl(A) \le Fscl(A \vee B)$$

$$B \le A \vee B \Rightarrow Fscl(B) \le Fscl(A \vee B)$$

$$Fscl(A) \vee Fsbcl(B) \le Fscl(A \vee B)$$

Also $Fscl(A)$ *and* $Fsbcl(B)$ are the closed sets.

$\therefore Fscl(A) \vee Fsbcl(B)$ is also a closed set.

Again,

$$A \le Fscl(A) \text{ and } B \le Fscl(B) \Rightarrow A \vee B \le Fscl(A) \vee Fscl(B)$$

Thus $Fscl(A) \vee Fscl(B)$ is a closed set containing $A \vee B$

$$\therefore Fscl(A \vee B) \le Fscl(A) \vee Fscl(B)$$

Thus $Fscl(A \vee B) = Fscl(A) \vee Fscl(B)$

Theorem 3.3: Union of two fuzzy soft generalized closed sets in X is a fuzzy soft generalized closed set in X.

Proof: Let A and B be any two fuzzy soft generalized closed sets in FSTS (X,τ,E)

Let $A \vee B \le H$ where H is fuzzy soft open. Since A and B are fuzzy soft generalized closed sets

$$\Rightarrow Fscl(A) \vee Fscl(B) \le H$$

$$But \quad Fscl(A) \vee Fscl(B) = Fscl(A \vee B)$$

$$\therefore \quad Fscl(A \vee B) \le H$$

Hence $A \vee B$ is a fuzzy soft generalized closed set.

Theorem 3.4: If a subset A of X is a fuzzy soft generalized closed set in X and A $\le B \le Fscl(A)$, then B is fuzzy soft generalized closed set in X.

Proof: Let A be a fuzzy soft generalized closed set such that A $\le B \le Fscl(A)$.

Let H be a fuzzy soft open set in X such that $B \le H$. Since A is fuzzy soft generalized closed set we have $Fscl(A) \le H$ *i.e* $Fscl(B) \le H$, H is fuzzy soft open in X.

Therefore B is fuzzy soft generalized closed set in X.

4. FUZZY SOFT g*-CLOSED SET

A Fuzzy soft set f_A in Fuzzy soft topological space (X,τ,E) is called Fuzzy soft g*-closed set (in short Fs g*-closed) if $Fscl\,(f_A) \le H, \forall f_A \le H$ and H is Fuzzy soft generalized open set in X.

Theorem 4.1: Every fuzzy soft closed set is fuzzy soft g*-closed set in a fuzzy soft topological space (X,τ,E).

Proof: Let A be a fuzzy soft closed set in X. Let A $\le$ H where H is fuzzy soft g-open. Since A is fuzzy soft closed Fscl (A) = A $\le$ H. Thus we have Fscl (A) $\le$ H whenever A $\le$ H, therefore A is fuzzy soft g* closed set.

Theorem 4.2: Every fuzzy soft g*- closed set is fuzzy soft g-closed set in a fuzzy soft topological space (X,τ,E).

Proof: Let A be a fuzzy soft g*- closed set in X. Let A $\le$ H where H is fuzzy soft g-open. Since A is fuzzy soft g*- closed Fscl (A) $\le$ H. Thus we have Fscl (A) $\le$ H whenever A $\le$ H.

Since every open set is g-open, therefore A is fuzzy soft g-closed set.

5. FUZZY SOFT STRONGLY g*-Closed Set

A Fuzzy soft set f_A in Fuzzy soft topological space (X,τ,E) is called Fuzzy soft strongly g*-closed set if $Fscl\ Fs\ \mathrm{int}(f_A) \le H, \forall f_A \le H$ and H is Fuzzy soft generalized open set in X.

Theorem 5.1: Every fuzzy soft closed set is a fuzzy soft strongly g*- closed set in a fuzzy soft topological space (X,τ,E).

Proof: Let A be a fuzzy soft closed set in X. Let H be fuzzy soft generalized open set in X such that A $\le$ H. Since A is fuzzy soft closed, Fscl (A) =A. Fscl (A) $\le$ H. Now, Fscl Fsint (A) $\le$ Fscl (A) $\le$ H. Hence A is fuzzy soft strongly g*-closed set in X.

The converse of the above theorem need not be true in general which can be seen from the following example.

Let $\tau = \{\tilde{0}, \tilde{1}, (F_1,E), (F_2,E)\}$

Where $(F_1, E) = \{0.5, 0.8\}$, $(F_2, E) = \{0.25, 0.2\}$

H= {0.5, 0.8}, A= {0.45, 0.3}. Then A is fuzzy soft strongly g*- closed but it is not fuzzy soft closed set in (X,τ,E).

Theorem 5.2: Every fuzzy soft generalized closed set is fuzzy soft strongly g*- closed set in a fuzzy soft topological space (X,τ,E).

Proof: Let A be a fuzzy soft generalized closed set in X. Let H be fuzzy soft generalized open set in X such that A $\le$ H. Since A is fuzzy soft generalized closed set, Fscl (A) $\le$ H.

Now, Fscl Fsint (A) $\le$ Fscl (A) $\le$ H. Hence A is fuzzy soft strongly g*- closed set in X.

The converse of the above theorem need not be true in general which can be seen from the following example.

Let τ = {0, 1, (F_1, E), (F_2, E)}

Where (F_1,E)={ 0.5,0.8}, (F_2,E)={0.25,0.2}

H= {0.5, 0.8} A= {0.45, 0.3}. Then A is fuzzy strongly g*- closed but it is not fuzzy soft generalized set.

Theorem 5.3: Every fuzzy soft g*-closed set is a fuzzy soft strongly g*-closed set in X.

Proof: Suppose that A is fuzzy soft g*-closed set in X. Let H be a fuzzy soft generalized open set in X such that A $\le$ H. Then Fscl (A) $\le$ H, since A is fuzzy soft g* - closed set.

Now, Fscl Fsint (A) $\le$ Fscl (A) $\le$ H. Hence A is fuzzy soft strongly g*- closed set in X.

Theorem 5.4: If A is both open and strongly g*-closed in X then it is both regular open and regular closed in X.

Proof: Suppose A is a fuzzy soft open in X.

A= Fsint (A) = Fsint Fscl (A) since A is closed

Thus A is regular open.

Again A is open in X, Fscl Fsint (A)= Fscl (A)

As A is closed, Fscl Fsint (A) =A

Thus A is regular closed.

Theorem 5.5: In FSTS every fuzzy soft pre-closed set is fuzzy soft strongly g*-closed in X.

Proof: Let A be fuzzy soft pre-closed set in X $\Rightarrow$ Fscl Fsint (A) $\le$ A

Let A $\le$ H and H is g-open in X $\Rightarrow$ Fscl Fsint (A) $\le$ H where H is g-open in X.

$\Rightarrow$ A is fuzzy soft strongly g*-closed set in X.

6. FUZZY SOFT G-CONTINUOUS AND SEMI g-CONTINUOUS FUNCTIONS

Definition 6.1: A function $f:(X,\tau) \to (Y,\tau')$ is said to be fuzzy soft g - continuous (FSg- continuous) if for every fuzzy soft open set $(G,E) \in \tau'$, $f^{-1}(G,E)$ is fuzzy soft g-open set in τ.

Theorem 6.2: If A is a fuzzy soft g-closed set in X and if $f:(X,\tau) \to (Y,\tau')$ continuous and fuzzy soft closed, then f (A) is fuzzy soft g – closed in Y.

Proof: If f (A) $\le$ O' where O' is fuzzy soft open in Y, then A $\le f^{-1}(O')$ and hence

Fscl (A) $\le f^{-1}(O')$. Thus $Fscl(A) \le O'$ and $f[Fscl(A)]$ is a closed set. It follows that $Fscl(f[A]) \le Fscl(f[Fscl(A)]) = f[Fscl(A)] \le O'$. Then $Fscl(f[A]) \le O'$ and f (A) is fuzzy soft g-closed.

Definition 6.3: A function $f:(X,\tau) \to (Y,\tau')$ is called fuzzy soft semi - continuous if for every fuzzy soft closed set $(G,E) \in \tau'$, $f^{-1}(G,E)$ is fuzzy soft semi-closed set in τ.

Definition 6.4: A function $f:(X,\tau) \to (Y,\tau')$ is called fuzzy soft semi g - continuous if for every fuzzy soft g-closed set $(G,E) \in \tau'$, $f^{-1}(G,E)$ is fuzzy soft semi-closed set in τ.

Theorem 6.5: Every fuzzy soft semi-g-continuous function is fuzzy soft semi-continuous function.

Proof: Let f be a function $f:(X,\tau) \to (Y,\tau')$.Let V be a fuzzy soft open set in Y. Since every fuzzy soft open set is fuzzy soft g-open and f is fuzzy soft semi-g-continuous. Hence f is fuzzy soft semi-continuous function.

Converse of the above theorem is not true in general as shown by the following example.

Let $f:(X,\tau)\rightarrow(Y,\tau')$

$\tau=\{\widetilde{0},\widetilde{1},(F_1,E),(F_2,E),(F_3,E),(F_4,E)\}$

$\tau'=\{\widetilde{0},\widetilde{1},(G_1,E),(G_2,E)\}$ be fuzzy soft topological spaces.

$X=\{h_1,h_2,h_3\}\quad Y=\{x_1,x_2,x_3\}\quad A=\{e_1,e_2\}\quad B=\{e_1',e_2'\}$

$$(F_1,E)=\{\{0.5,0.3,0.2\},\{0.3,0.5,0.2\}\}$$
$$(F_2,E)=\{\{1,0,0.5\},\{0.5,0.3,1\}\}$$
$$(F_3,E)=\{\{0.5,0,0.2\},\{0.3,0.3,0.2\}\}$$
$$(F_4,E)=\{\{1,0.3,0.5\},\{0.5,0.5,1\}\}$$
$$(G_1,E)=\{\{0.2,0.3,0.5\},\{0.2,0.5,0.3\}\}$$
$$(G_2,E)=\{\{0.8,0.7,0.5\},\{0.8,0.5,0.7\}\}$$

Let us define the fuzzy soft mapping

$$f_{pu}:(X,\tau)\rightarrow(Y,\tau')$$
$$by\quad u(h_1)=x_3,u(h_2)=x_2,u(h_3)=x_1$$
$$and\ p(e_1)=e_1'\ p(e_2)=e_2'$$

Let A= $\{\{0.1,0.2,0.5\},\{0.1,0.2,0.1\}\}$

Here f is fuzzy soft semi continuous but it is not fuzzy soft semi-g-continuous

CONCLUSION

In this paper we have introduced fuzzy soft Generalized closed set, g*-closed set and strongly g*-closed set and then studied some properties of these closed sets in Fuzzy soft topological spaces. Also we have introduced fuzzy soft g-continuous function and fuzzy soft semi g-continuous function.

REFERENCES

[1] Zadeh L.A (1965) "Fuzzy sets", Information and Control, vol. 8, no. 3, pp. 338–353.
[2] Chang C.L(1968) "Fuzzy topological spaces", Journal of Mathematical Analysis and Applications, vol. 24, pp.182190.
[3] Kharal A and Ahmad B (2009) "Mappings on fuzzy soft classes", Advances in Fuzzy Systems, vol. 2009, Article ID 407890, 6 pages.
[4] Kandil A, Tanay O.A.E, El-Sheikh S.A and Abd El-latif A.M (2014)" Fuzzy semi open soft sets related properties in fuzzy soft topological spaces", Journal of mathematics and computer science, 13:94-114.
[5] Varol B.P and Aygün .H (2012)"Fuzzy soft topology", Hacettepe Journal of Mathematics and Statistics, vol. 41, no. 3, pp. 407–419.
[6] N. Levine (2004), Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, J.K. Park and J.H. Park, Mildly generalized closed sets, almost normal and mildly normal Spaces, Chaos, Solitions and Fractals 20, 1103–1111.
[7] M.K.R.S. Veera Kumar(2000), between closed sets and g-closed sets, Mem. Fac. Sci. Kochi Univ. (Math.) 21, 1–19.
[8] R. Parimelazhagan and V. Subramoniapillai, Strongly g* -Closed sets in topological spaces, Int. Jou. Of Math. Anal. 6(30) (2012) 1481-1489.
[9] T.Rajendrakumar and G.Anandajothi "On Fuzzy Strongly g-Closed Sets in Fuzzy Topological Spaces" Intern. J. Fuzzy Mathematical Archive vol. 3, 2013, 68-75
[10] Maji P.K, Biswas R and Roy A.R (2001) "Fuzzy soft sets," Journal of Fuzzy Mathematics, vol.9 no. 3, pp. 589–602.
[11] Tanay. B and Kandemir M.B (2011) "Topological structure of fuzzy soft sets", Computers & Mathematics with Applications, vol. 61, no. 10, pp. 2952–2957**.**
[12] Sabir hussain (2016) "On weak and strong forms of fuzzy soft open sets", Fuzzy Information and Engineering, vol 8, issue 4, pp 451-463.