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Miscellaneous Properties of Line Mycielskian Graph of a Graph.

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Abstract

The *line Mycielskian graph* [12] of a graph G, denoted by $L_\mu(G)$, is the graph obtained from $L(G)$ by adding $q+1$ new vertices $E' = \{e'_i : 1 \leq i \leq q\}$ and e, then for $1 \leq i \leq q$, joining e'_i to the neighbours of e_i and to e. The vertex e is called the *root* of $L_\mu(G)$. In this paper, we study the connectedness, connectivity, covering invariants, chromatic number, domination number of line Mycielskian graph of a graph.

Keywords: Line graph, Line Mycielskian graph, connectedness, connectivity, covering invariants.

2010 Mathematics Subject Classification: 05C76, 05C40.

1 Introduction

In this paper, we consider only simple finite and undirected graphs. For a graph
$G = (V, E)$, let $V(G)$, $E(G)$ denote the vertex set and edge set. If two distinct edges x and y are incident with a common vertex then they are called adjacent edges. The degree of a vertex v_i in G is the number of edges incident to v_i and is denoted by $d_i = deg(v_i)$. The minimum degree among the vertices of G is denoted by $\delta(G) = \delta$. The open–neighbourhood $N(e_i)$ of an edge e_i in $E(G)$ is the set of edges adjacent to e_i. $N(e_i) = \{e_j/e_i, e_j$ are adjacent in G$\}$. Let x be any number, then $\lceil x \rceil$ denote the least integer greater than or equal to x and $\lfloor x \rfloor$ denote the greatest interger less than or equal to x. For undefined terms and notations refer[6]. The line graph $L(G)$ of a graph G is the graph with vertex set as the edge set of G and two vertices of $L(G)$ are adjacent whenever the corresponding edges in G have a vertex in common [6].

Mycielski [13] used a fascinating construction in order to create a triangle free graphs with arbitrarily large chromatic numbers. Mycielski [2] introduced the graph–transformation as follows:

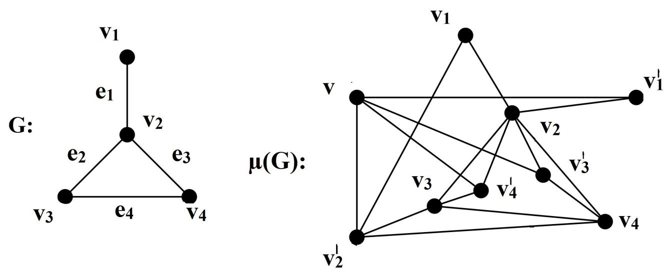

Figure 1: Graph G and Mycielskian graph of G

Let G be a graph with vertex set $V = \{v_i : 1 \leq i \leq p\}$. The *Mycielski graph* of G, denoted by $\mu(G)$, is the graph obtained from G by adding $p+1$ new vertices $V' = \{v'_i : 1 \leq i \leq p\}$ and u, then for $1 \leq i \leq p$, joining v'_i to the neighbours of v_i and to u. v_i and v'_i are known as *twin–vertices*, and V and V' are known as *twin–sets* in $\mu(G)$. The vertex u is called the *root* of $\mu(G)$. Clearly, $V[\mu(G)] = V \cup V' \cup \{u\}$. The beauty of Mycielski graph $\mu(G)$ is that it transforms the triangle–free graph G into a triangle–free graph $\mu(G)$, and it produces three new triangles for every triangle of G.

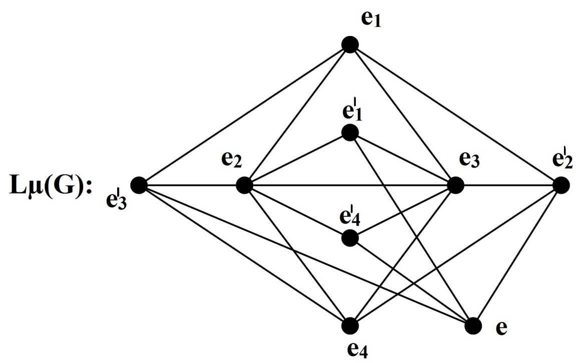

Figure 2: Line Mycielskian graph of G

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Motivated by this concept on the same lines the concept of line Mycielskian graph of a graph is introduced in [12] and is defined as follows:
Let G be a graph with edge set $E = \{e_i : 1 \leq i \leq q\}$. The *line Mycielskian graph* of a graph G, denoted by $L_\mu(G)$, is the graph obtained from $L(G)$ by adding $q + 1$ new vertices $E' = \{e'_i : 1 \leq i \leq q\}$ and e, then for $1 \leq i \leq q$, joining e'_i to the neighbours of e_i and to e. The vertex e is called the *root* of $L_\mu(G)$. Clearly, $V[L_\mu(G)] = E \cup E' \cup \{e\}$. The line graph $L(G)$ is an induced subgraph of $L_\mu(G)$.

Recently, there has been an increasing interest in the study of Mycielskians graph, especially, in the study of their circular chromatic numbers [2, 7, 8, 9, 10] and which are also studied in [1, 3, 4, 11, 13, 14]. In this paper, we study the connectedness, connectivity, covering invariants, chromatic number, domination number of line Mycielskian graph of a graph.

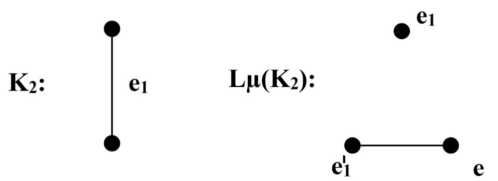
Remark 1.1. *[12] $L(G)$ is an induced subgraph of $L_\mu(G)$.*

2 Connectedness of Line Mycielskian graph of a graph

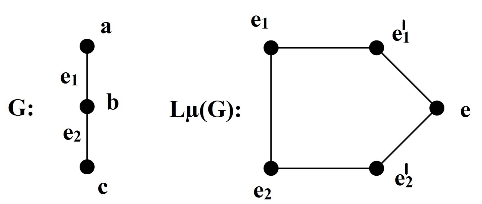
In the following theorem, we study when line Mycielskian graph $L_\mu(G)$ is connected and disconnected.

Theorem 2.1. *The line Mycielskian graph $L_\mu(G)$ of a graph G is disconnected iff $G = K_2$.*

Proof. Suppose $G = K_2$, then $L(G)$ has exactly one vertex. By definition, in $L_\mu(K_2)$



for each edge e_1 of G, a new vertex e'_1 is taken and a new vertex e is introduced which is adjacent to new point e'_1. Further e_1 is not adjacent to either e'_1 nor e and results in a disconnected graph of order 3. By Remark 1.1, it is clear that, $L_\mu(G) = L(G) \cup K_2$. Hence $L_\mu(G)$ is disconnected. Conversely, assume $L_\mu(G)$ is disconnected $L_\mu(G) = L(G) \cup K_2$.



Suppose G is not K_2 and assume that G has atleast two edges. Then $L(G) \cup K_2$ has $q + 2$ vertices where as $L_\mu(G)$ has $2q + 1$ vertices, $q > 1$. Hence $L(G) \cup K_2$ has less number of vertices than $L_\mu(G)$. Clearly $L_\mu(G) \neq L(G) \cup K_2$, a contradiction. Hence G must be K_2. □

Theorem 2.2. *For any connected (p, q) graph G with $p \geq 3$, $L_\mu(G)$ is connected.*

Proof. Let G be a connected graph with $p \geq 3$ vertices. Let $V[L_\mu(G)] = E_1 \cup E_2 \cup \{e\}$ where $\langle E_1 \rangle = L(G)$ and E_2 is the set of newly introduced vertices such that e_i implies e'_i is a bijective map from E_1 onto E_2 satisfying $N(e'_i) = N(e_i) \cap E_1$, for all $e_i \in E_1$ and the vertex e is called the root of $L_\mu(G)$. Let $a, b \in V(L_\mu(G))$.

We consider the following cases:

Case 1. $a, b \in E_1$. Since G is a connected graph with $p \geq 3$, $L(G)$ is a nontrivial connected graph. Since by Remark 1.1, there exists an $a - b$ path in $L_\mu(G)$.
Case 2. $a \in E_1$ and $b \in E_2$. Let $e \in E_1$ be such that $N(b) = N(e) \cap E_1$. Choose $w \in N(b)$. Since a and $w \in E_1$, as in Case 1, a and w are joined by a path in $L_\mu(G)$. Hence a and b are connected by a path in $L_\mu(G)$.
Case 3. $a, b \in E_2$. As in Case 2, there exists c and d in E_1 such that $c \in N(a)$ and $d \in N(b)$. Consequently, $ca, db \in E[L_\mu(G)]$. Also c and d are joined by a path in $L_\mu(G)$. Hence a and b are connected by a path in $L_\mu(G)$.
Case 4. $a \in E_2$ and a root vertex $\{e\}$. By definition of $L_\mu(G)$, all points of E_2 are joined by a root vertex e. Hence there exists a path from the vertices of E_2 to the root vertex of $L_\mu(G)$.
In all the cases, a and b are connected by a path in $L_\mu(G)$. Thus $L_\mu(G)$ is connected. □

3 Connectivity and Edge Connectivity of line Mycielskian graph of a graph

Connectivity or vertex–connectivity is a minimum number of vertices whose removal from G results into a disconnected or trivial graph and is denoted by $\kappa(G)$. *Edge–connectivity* of a graph is the minimum number of edges whose removal from G results into a disconnected or trivial graph and is denoted by $\lambda(G)$.
In the following theorem, we determine vertex–connectivity of line Mycielskian graph $\kappa(L_\mu(G))$ and the edge–connectivity of line Mycielskian graph $\lambda(L_\mu(G))$ of a graph.

Theorem 3.1. *For a connected (p, q) graph G, the vertex–connectivity of line Mycielskian graph of a graph is*

$$\kappa(L_\mu(G)) \quad = \quad min\{2\kappa(L(G)) + 1, \delta(L(G)) + 1\}.$$

Proof. From Whitney's result, we have

$$\kappa(L_\mu(G)) \leq \lambda(L_\mu(G)) \leq \delta(L_\mu(G)) \text{ and}$$

$$\kappa(L(G)) \leq \lambda(L(G)) \leq \delta(L(G)).$$

By Remark 1.1 we have, $\kappa(L_\mu(G)) \geq \kappa(L(G))$.

Case 1. If $\kappa(L(G)) = 0$, then obviously $\kappa(L_\mu(G)) = 0$.

Case 2. If $\kappa(L(G)) = 1$, then $L(G) = K_2$ or it is a connected graph with a cutvertex e_i.

We have the following subcases:

Subase 2.1. If $L(G) = K_2$. Then $L_\mu(G) = C_5$.

Consequently, $\kappa(L_\mu(G)) = \delta(L(G)) + 1 = 2$.

Subcase 2.2. $L(G)$ is connected with a cutvertex e_i. If $\delta(L(G)) = 1$, then let e_j be a pendant vertex of $L(G)$ which is adjacent to e_i. Then e'_i is a vertex of $L_\mu(G)$ such that $deg_{L_\mu(G)}(e'_i) = 2$ and so, removal of e'_i results in a connected graph with root vertex e as a cutvertex. So removal of root vertex e results in a disconnected graph. Hence $\kappa(L_\mu(G)) = \delta(L(G)) + 1$.

If $\delta(L(G)) \geq 2$, then the removal of a cutvertex e_i of $L(G)$ and its corresponding vertex e'_i and the root vertex e from $L_\mu(G)$ results in a disconnected graph. Hence $\kappa(L_\mu(G)) = 2\kappa(L(G)) + 1$. Now, suppose $\kappa(L(G)) = n$, where n is an integer. Then $L(G)$ has a minimum vertex–cut $\{e_l : 1 \leq l \leq n\}$ whose removal from $L_\mu(G)$ results in a disconnected graph.

There are two types of vertex–cuts in $L_\mu(G)$ depending on the structure of $L(G)$.

[1] vertex–cut containing exactly $2n + 1$ vertices, that is, $\{e_l, e'_l, e : 1 \leq l \leq n\}$ whose removal increases the number of components of $L_\mu(G)$.

[2] vertex–cut containing $\delta(L(G)) + 1$ vertices.

Thus,

$$\kappa(L_\mu(G)) = \begin{cases} 2n+1, & \text{if } n \leq \frac{\delta(L(G))+1}{2}; \\ \delta(L(G)) + 1, & \text{otherwise.} \end{cases}$$

Hence,

$$\kappa(L_\mu(G)) = min\{2\kappa(L(G)) + 1, \delta(L(G)) + 1\}.$$

□

Theorem 3.2. *If G is a (p, q) connected graph, then the edge–connectivity of line Mycielskian of a graph G is given as*

$$\lambda(L_\mu(G)) = \begin{cases} (\delta(L(G)) + 1)\ \lambda(L(G)), & \text{if } n \leq \delta(L(G)) + 1; \\ \delta(L(G)) + 1, & \text{otherwise.} \end{cases}$$

Proof. From, Whitney's result we have

$\kappa(L_\mu(G)) \leq \lambda(L_\mu(G)) \leq \delta(L_\mu(G))$ and by Remark 1.1 we have $\lambda(L_\mu(G)) \geq \lambda(L(G))$.

We now consider the following cases.

Case 1. If $\lambda(L(G)) = 0$, then obviously $\lambda(L_\mu(G)) = 0$.

Case 2. If $\lambda(L(G)) = 1$, then $L(G) = K_2$ or it is a connected graph with a bridge $x = e_ie_j$, say.

We have the following subcases of this case.

Subcase 2.1. If $L(G) = K_2$. Then $L_\mu(G) = C_5$. Consequently,

$\lambda(L_\mu(G)) = \delta(L(G)) + 1 = 2$.

Subcase 2.2. $L(G)$ is a connected with a bridge e_ie_j. If e_i is a pendant vertex in $L(G)$ then $L_\mu(G)$ is a connected graph having a vertex e'_i with a degree $deg_{L_\mu(G)}(e'_i) = 2$. The removal of edges incident with e'_i disconnects $L_\mu(G)$. Thus, $\lambda(L_\mu(G)) = \delta(L(G)) + 1 = 2$. If neither e_i nor e_j is a pendant vertex in $L(G)$, and $\delta(L(G)) = 2$, then $\delta(L_\mu(G)) = 3$ and let e'_k be a vertex of $L_\mu(G)$ with a $deg_{L_\mu(G)}(e'_k) = 3$. In $L_\mu(G)$, there are only three edges incident with e'_k and the removal of these edges disconnects $L_\mu(G)$. So, $\lambda(L_\mu(G)) = \delta(L(G)) + 1$. If $\delta(L(G)) = m \geq 3$, where m is an integer. Then let e_k be a vertex in $L(G)$ with $deg_{L(G)}(e_k) = \delta(L(G)) = m$. Then, $deg_{L_\mu(G)}(e'_k) = m + 1$ and the removal of the set $\{e_le'_k, ee'_k : 1 \leq l \leq m\}$ results in a disconnected graph, so that

$$\begin{aligned} \lambda(L_\mu(G)) &= (m+1)\lambda(L(G)) \\ &= (\delta(L(G)) + 1)\ \lambda(L(G)). \end{aligned}$$

Case 3. If $\lambda(L(G)) = n$, where n is an integer. Then $L(G)$ has minimum edge–cut $\{e_l : e_l = u_lv_l, 1 \leq l \leq n\}$ whose removal makes $L(G)$ disconnect. As above, there are two types of edge–cuts in $L_\mu(G)$ depending on the structure of $L(G)$. Among these, one edge–cut contains exactly $(\delta(L(G)) + 1)\ \ \lambda(L(G))$ edges whose removal increases the number of components of $L_\mu(G)$ and the other is $\delta(L(G)) + 1$ edge cuts.

Thus, we have

$$\lambda(L_\mu(G)) = \begin{cases} (\delta(L(G)) + 1)\ \lambda(L(G)), & \text{if } n \leq \delta(L(G)) + 1; \\ \delta(L(G)) + 1, & \text{otherwise.} \end{cases}$$

□

4 Covering invariants of line Mycielskian graph of a graph

A vertex and an edge are said to *cover* each other if they are incident. A set of vertices in a graph G is a *vertex covering set*, which covers all the edges of G. The *vertex covering number* $\alpha_0(G)$ of G is the minimum number of vertices in a vertex covering set of G. A set of edges in a graph G is an *edge covering set*, which covers all vertices of G. The *edge covering number* $\alpha_1(G)$ of G is the minimum number of edges in an edge covering set of G. A set of vertices in a graph G is independent if no two of them are adjacent. The maximum number of vertices in such a set is called the *vertex independence number* of G and is denoted by $\beta_0(G)$. The set of edges in a graph G is independent if no two of them are adjacent. The maximum number of edges in such a set is called the *edge independence number* of G and is denoted by $\beta_1(G)$. The following theorem is useful in the proof of the result:

Theorem 4.1. *[5] If $L(G)$ is the line graph of a nontrivial connected graph G, then*

$$\text{(i) } \alpha_1(L(G)) = \left\lceil \frac{q}{2} \right\rceil$$

$$\text{(ii) } \beta_1(L(G)) = \left\lfloor \frac{q}{2} \right\rfloor$$

$$\text{(iii) } \alpha_0(L(G)) = q - \beta_1(G)$$

$$\text{(iv) } \beta_0(L(G)) = \beta_1(G).$$

In the following theorem, we obtain vertex covering number, edge covering number of line Mycielskian graph of a graph. Also, we establish the upper and lower bounds for the edge covering number and edge independence number for line Mycielskian graph of a graph.

Theorem 4.2. *For any connected (p, q) graph G of order $p \geq 3$, we have*

$$\text{(i) } \alpha_0(L_\mu(G)) = min\{2q+1, 2\alpha_0(L(G))+1\}$$

$$\text{(ii) } \beta_0(L_\mu(G)) = max\{2q+1, 2\beta_0(L(G))+1\}.$$

Proof. (i) By Remark 1.1, we have $\alpha_0(L(G)) < \alpha_0(L_\mu(G))$. There are two kinds of covering sets of $L_\mu(G)$, depending upon the structure of G. The first kind is the set $E(G) \cup E^{'}(G) \cup \{e\}$, where $E(G)$ is a trivial edge covering set of G and also, a trivial vertex covering set of $L(G)$. So, $\alpha_0(L_\mu(G)) \leq 2q+1$.

The second kind of vertex covering set of $L_\mu(G)$ is as follows: Let $N_0 = \{e_i : 1 \leq i \leq p\}$ be a minimum vertex covering set in $L(G)$. Then $N_0' = \{e_i' : 1 \leq i \leq p\}$ is an independent subset of $V(L_\mu(G))$ such that $N_0 \cup N_0' \cup \{e\}$ is a trivial vertex covering set of $L_\mu(G)$. Then by definition, we have

$$\alpha_0(L_\mu(G)) = min\{2q+1, 2\alpha_0(L(G))+1\}.$$

(ii) $L_\mu(G)$ has $2q+1$ vertices and $\alpha_0(L_\mu(G)) + \beta_0(L_\mu(G)) = 2q+1$. By substitution the value of $\alpha_0(L_\mu(G))$ from (i), we have

$$\beta_0(L_\mu(G)) = max\{2q+1, 2\beta_0(L(G))+1\}.$$

□

Theorem 4.3. *For any connected (p, q) graph G with $p \geq 3$, we have*

$$q+1 \leq \alpha_1(L_\mu(G)) \leq 2\left\lceil \frac{q}{2} \right\rceil + 1 \text{ and}$$

$$2\left\lfloor \frac{q}{2} \right\rfloor + 1 \leq \beta_1(L_\mu(G)) \leq q+1.$$

Proof. Let G be a connected (p, q) graph G with $p \geq 3$ vertices and $e_1, e_2, e_3, ..., e_q$ edges. Let $E_1 = e_1', e_2', e_3', ..., e_q'$ be the set of newly introduced vertices in the construction of $L_\mu(G)$. For each pair $\{e_i, e_j\}$ of adjacent edges of G, we have an edge e_ie_j in $L(G)$. Corresponding to this edge e_ie_j, there are edges $e_ie_j, e_ie_j', e_i'e_j$ in $L_\mu(G)$. Among these, e_ie_j' and $e_i'e_j$ are independent in $L_\mu(G)$. Thus, each pair of adjacent edges of G gives rise to two independent edges in $L_\mu(G)$. That is, each edge of $L(G)$ gives rise to two indepenent edges in $L_\mu(G)$. So, $\beta_1(L(G))$ independent edges of $L(G)$ give rise $2\beta_1(G)$ independent edges in $L_\mu(G)$. Hence $\beta_1(L_\mu(G)) \geq 2\beta_1(L(G))$. By Theorem 4.1, it follows that $2\left\lfloor \frac{q}{2} \right\rfloor + 1 \leq \beta_1(L_\mu(G))$. Since $L_\mu(G)$ has $2q+1$ vertices and $\alpha_1(L_\mu(G)) + \beta_1(L_\mu(G)) = 2q+1$, we have $\alpha_1(L_\mu(G)) \leq 2\left\lceil \frac{q}{2} \right\rceil + 1$. Now, if $L(G)$ contains a spanning odd cycle C_q : $e_1e_2e_3...e_qe_1; q = 2k+1, k \geq 1$, then the subset N of $E(L_\mu(G))$, where $N = \{e_1e_2', e_2e_3', ..., e_{q-1}e_q', e_qe_1'\}$ also forms an edge cover of $L_\mu(G)$ with $|N| = q$. In this case, $q < 2\alpha_1(L(G))$. Also, $L_\mu(G)$ has atleast q independent edges of $E(L_\mu(G))$ to cover these independent vertices in $L_\mu(G)$.
Therefore, $\alpha_1(L_\mu(G))$ cannot be less than $q+1$. Hence $q+1 \leq \alpha_1(L_\mu(G))$.
Thus we have,
$q+1 \leq \alpha_1(L_\mu(G)) \leq 2\left\lceil \frac{q}{2} \right\rceil + 1$ and
by Gallai's result, we obtain
$2\left\lfloor \frac{q}{2} \right\rfloor + 1 \leq \beta_1(L_\mu(G)) \leq q+1$. □

5 Domination number and Chromatic number of line Mycielskian graph of a graph

A set D of vertices in a graph G is a *dominating set* if every vertex in $V(G) - D$ is adjacent to atleast one vertex of D. A dominating set D is said to be a *minimal dominating set*, if no proper subset of D is a dominating set. The minimum cardinality of a dominating set is called the *domination number* of G and is denoted by γ(G). A set S of edges of a graph G is an edge dominating set if every edge in $E(G) - S$ is adjacent to atleast one edge of S.

A *coloring* of a graph is an assignment of colors to its vertices(or edges) so that no two adjacent vertices(or adjacent edges) have the same color. The *chromatic number* $\chi(G)$ of G is the minimum number of colors required to assign to the vertices of G in such a way that no two adjacent vertices of G receive the same colour. The *edge chromatic number* $\chi^{'}(G)$ of a graph G is the minimum number of colors required to assign to the edges of G in such a way that no two adjacent edges of G receive the same color.

In this section, we derive an expression for domination number of line Mycielskian graph of a graph $\gamma(L_\mu(G))$ and chromatic number of line Mycielskian graph of a graph $\chi(L_\mu(G))$.

Theorem 5.1. *If G is a (p, q) connected graph with $p \geq 3$, then*

$$\gamma(L_\mu(G)) = \gamma(L(G)) + 1.$$

Proof. Since $V[L_\mu(G)] = E(G) \cup E_1(G) \cup \{e\}$
Let D be a minimum edge dominating set of G. Since the root vertex e is adjacent to every vertex of $E_1(G)$ in $L_\mu(G)$, the set $D \cup \{e\}$ is a minimum dominating set of $L_\mu(G)$.
Hence,

$$\gamma(L_\mu(G)) = \gamma(L(G)) + 1.$$

□

Theorem 5.2. *For a connected (p, q) graph G with order $p \geq 3$, the chromatic number of line Mycielskian graph of a graph is*

$$\chi(L_\mu(G)) = \chi(L(G)) + 1.$$

Proof. Let G be a connected (p, q) graph. For each edge e_i in G, let e_i' be the new vertex chosen in the construction of $L_\mu(G)$. Since $\chi\,'(G) = \chi(L(G))$, $\chi\,'(G)$ coloring of G can be extended to $\chi(L(G))$ coloring of $L(G)$. Also, we can assign the same color to an edge e_i and e_i' in $L_\mu(G)$.
Hence,

$$\chi(L_\mu(G)) = \chi(L(G)) + 1$$

□

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