Thermal Diffusion and Radiation Effects on Steady MHD Free Convective Poiseuille Flow between Two Vertical Porous Plates

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Abstract
An analysis is performed to study the effects of thermal diffusion and thermal radiation on a steady free convective flow of electrically conducting viscous fluid through a porous medium bounded by two infinite vertical porous plates. The plates are subjected to constant normal suction/injection velocity. The dimensionless governing equations are solved by perturbation technique with Eckert number as perturbation parameter. The expressions for velocity, temperature and concentration field with skin friction (shear stress), heat flux (in terms of Nusselt number) and mass flux (in terms of Sherwood number) have been obtained. The behaviour of velocity and shear stress at both the plates are demonstrated graphically for different parameters involved in the problem namely Hartmann number, Soret number, Schmidt number, Radiation parameter etc.

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Key words: Thermal diffusion, Thermal radiation, Free convective, Poiseuille flow, Porous medium.

1: INTRODUCTION
The study of problems of MHD (Magneto-hydrodynamics) free convective flow with heat and mass transfer have attracted the attention of a number of scholars due to the importance of such problems of science and technology. There are numerous examples of application of MHD principles including MHD generator, MHD pumps, MHD flow meters etc. Problems of natural convection of electrically conducting fluid in presence of transverse magnetic field have got much importance because of its wide applications of Missile technology, Plasma physics, Astrophysics, Geophysics etc. The present form of MHD is due to the pioneer contribution of several notable authors like Alfven (1942), Cowling (1957), Shercliff (1965), Ferraro and Plumpton (1966) and Cramer and Pai (1978).

For natural convection the existence of the large difference between surface temperature and ambient temperature causes the radiation effect to be important. At high temperature thermal radiation can significantly affect the heat transfer and temperature in boundary layer flow of participating fluid. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various population devices for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effect of thermal radiation and mass diffusion.

Due to the importance of the above physical aspects, several authors have carried out model studies on the problems of free convective flows of incompressible viscous fluid under different flow geometries taking into account of thermal radiation. Some of them are Ganesan and Loganathen (2002), Mbelegdouge et. al. (2007), Makinde (2005) and Ahmed (2012). Ahmed and Sarma (2009) studied the thermal radiation effect on a transient MHD flow with mass transfer past an impulsively fixed infinite vertical plate. Recently Das et. al. (2011) have studied the radiation effect on natural flow of an optically thin viscous incompressible fluid near a suddenly moving vertical plate with ramped wall temperature by adopting Cogley- Vincentinue-Gilles equilibrium model (1968).

The objective of the present work is to study the effects of thermal diffusion, thermal radiation and transverse magnetic field on a steady flow with mass transfer of an electrically viscous electrically conducting non-gray fluid between two porous plates. This is an extension to the work of Kalita and Ahmed (2011) to consider the effect of thermal radiation.

2: MATHEMATICAL FORMULATION
We consider the flow of an incompressible viscous electrically conducting optically thin non-Grey fluid through a porous medium bounded by two infinite vertical porous plates separated by a distance h in presence of transverse magnetic field by making the following assumption.

(1) All fluid properties are constant except the influence of the variation in density in the buoyancy force term.

(2) The viscous and Ohmic dissipations of energy are negligible.

(3) The magnetic Reynolds number is so small for the induced magnetic field can be neglected in comparison to the applied magnetic

(4) The plates are electrically non-conducting.

(5) The radiation heat flux in the direction of the plate velocity is considered negligible in comparison to that in the normal direction.
We introduce a coordinate system \( (\vec{x},\vec{y},\vec{z}) \) with x-axis vertically upwards along a plate, y-axis perpendicular to it directed to the fluid region and z-axis along the width of the either plates. Let \( \mathbf{q} = (u,v,0) \) be the fluid velocity at the point \((\vec{x},\vec{y},\vec{z})\) and \( \mathbf{B} = B_0 \hat{j} \) be the applied magnetic field, where \( \hat{i} \) and \( \hat{j} \) being the unit vectors along x-axis and y-axis respectively. Since the plates are infinite in length therefore all the physical quantities except possibly the pressure \( p \) are independent of \( \vec{x} \).

The equations governing the flow, heat and mass transfer are

**Equation of continuity**

\[
\frac{\partial u}{\partial y} = 0 \Rightarrow \dot{v} = -v_0 \text{ (constant suction/injection)} \quad (2.1)
\]

**Momentum equation**

\[
-v_0 \frac{d\dot{u}}{dy} = \nu \frac{d^2 \dot{u}}{dy^2} + g\beta (\overline{T} - T_s)
\]

\[
+ \frac{\nu}{\rho} \frac{d}{dy} \left( \frac{\dot{u}}{\rho} \right) - \frac{\sigma B_0^2 \dot{u}}{k} \quad (2.2)
\]

**Energy equation**

\[
-v_0 \frac{d\overline{T}}{dy} = \frac{\lambda}{\rho c_p} \frac{d^2 \overline{T}}{dy^2} + \frac{\nu}{c_p} \left( \frac{d\dot{u}}{dy} \right)^2 - \frac{1}{\rho c_p} \frac{d}{dy} \left( \frac{\dot{u}}{\rho} \right) \quad (2.3)
\]

We assume that the medium is optically thin with relatively low density. Following the Cogly-Vincentine-gilles equilibrium model, we have

\[
\frac{\partial q_r}{\partial y} = 4 \left( \overline{T} - T_s \right) \int_0^\infty K_s \left( \frac{\partial c_s}{\partial T} \right) d\lambda = 4 I \left( \overline{T} - T_s \right) \quad (2.5)
\]

Thus, with the help of equation (2.5), the energy equation (2.3) becomes,

\[
-v_0 \frac{d\overline{T}}{dy} = \frac{\lambda}{\rho c_p} \frac{d^2 \overline{T}}{dy^2} + \frac{\nu}{c_p} \left( \frac{d\dot{u}}{dy} \right)^2 - 4 I \left( \overline{T} - T_s \right) \frac{1}{\rho c_p} \quad (2.6)
\]

The relevant boundary conditions are

\[
\overline{y} = 0; \dot{u} = 0, \overline{T} = T_0, \overline{C} = C_0 \bigg|_{\overline{y} = 0}
\]

\[
\overline{y} = h; \dot{u} = u_0, \overline{T} = T_1, \overline{C} = C_1 \bigg|_{\overline{y} = h}
\]

where \( \nu \) is the kinematic viscosity, \( g \) is the acceleration due to gravity, \( v_0 \) is the constant suction/injection velocity, \( \beta \) is the coefficient of volume expansion for heat transfer, \( \tilde{\beta} \) is the coefficient of volume expansion for mass transfer, \( D_M \) is the chemical molecular diffusivity, \( D_T \) is the chemical thermal diffusivity, \( k \) is the permeability of porous medium, \( B_0 \) is the strength of applied magnetic field, \( \overline{T} \) is the temperature of the fluid, \( \overline{T}_s \) is the temperature at static condition, \( \overline{C} \) is the species concentration, \( \overline{C}_s \) is the concentration at static condition, \( \mu \) is the coefficient of viscosity, \( \mathbf{B} \) is the magnetic induction vector, \( \sigma \) is the electrical conductivity, \( \mathbf{C}_p \) is the specific heat at constant pressure, \( \rho \) is the density of fluid, \( \mathbf{K}_w \) is the absorption coefficient, \( q_r \) is the relative heat flux, \( c_0 \) is the Planck function and other symbols have their usual meanings.

Now to reduce the governing equations in non-dimensional form, we introduce the following non-dimensional quantities:

\[
\overline{y} = \frac{y}{h} \text{ (distance)}, \quad u = \frac{u}{v_0} \quad (\text{fluid velocity}), \quad \theta = \frac{T - T_s}{T_0 - T_s} \quad (\text{fluid temperature}), \quad \phi = \frac{C - C_s}{C_0 - C_s} \quad (\text{Species concentration}),
\]

\[
g_c = \frac{h g \beta (T_0 - T_s)}{v_0^2} \quad (\text{Grashoff number for heat transfer})
\]

\[
g_m = \frac{h g \beta (C_0 - C_s)}{v_0^2} \quad (\text{Grashoff number for mass transfer}),
\]

\[
E = \frac{v_0^2}{c_p (T_0 - T_s)} \quad (\text{Eckert number}), \quad P = \frac{\mu c_p}{\lambda} \quad (\text{Prandtl number}), \quad M = \frac{\sigma B_0^2 h^2}{\rho_D} \quad (\text{Hartmann number}), \quad R = \frac{v_0 h}{\nu} \quad (\text{Reynolds number}), \quad S_c = \frac{\nu}{D_M} \quad (\text{Schmidt number}),
\]

\[
m = \frac{\overline{T}_1 - \overline{T}_s}{\overline{T}_0 - \overline{T}_s} \quad (\text{non dimensional temperature at the plate } \overline{y} = h), \quad n = \frac{\overline{C}_1 - \overline{C}_s}{\overline{C}_0 - \overline{C}_s} \quad (\text{non-dimensional concentration at the plate } \overline{y} = h), \quad S_o = \frac{D_T (T_0 - T_s)}{v_0 (C_0 - C_s)} \quad (\text{Soret number}), \quad F = \frac{4 I v_0^2}{\lambda v_0^2} \quad (\text{Radiation parameter}), \quad \alpha = \frac{k}{h^2} \quad (\text{permeability of porous medium}).
\]
The non-dimensional equations with boundary conditions are

\[
\frac{du}{dy} = \frac{1}{R} \frac{d^2u}{dy^2} + G_i \theta + G_m \phi - \frac{u}{R\alpha} - MRu \tag{2.7}
\]

\[
\frac{d\phi}{dy} = \frac{1}{S_c R} \frac{d^2\phi}{dy^2} + \frac{S_0}{R} \frac{d^2\theta}{dy^2} \tag{2.9}
\]

Subjected to boundary conditions

\[
y = 0; u = 0, \theta = 1, \phi = 1 \bigg| \begin{array}{l}
y = 1; u = 0, \theta = m, \phi = n \end{array} \tag{2.10}
\]

**Figure 1:** Flow configuration of the problem
3: SOLUTION OF THE PROBLEM

To solve the equations (2.7), (2.8), and (2.9) subject to boundary conditions (2.10) we assume the solutions of the equations to be of the form

\[ u = u_0(y) + E u_1(y) + O(E^2) \]
\[ \theta = \theta_0(y) + E \theta_1(y) + O(E^2) \]
\[ \phi = \phi_0(y) + E \phi_1(y) + O(E^2) \]  

Substituting from the equations (3.1) in (2.7), (2.8), and (2.9) and by equating the coefficient of similar powers of E and neglecting the higher powers of E, the following equations are obtained

\[ u_0'' + Ru_1' - Au_0 = -G_1 R \theta_0 - G_m R \phi_0 , \]
where \( A = \frac{1}{\alpha} + MR^2 \)  

The solutions of the equations (3.2) to (3.7) subject to boundary conditions (3.8) are obtained and shown below

\[ \theta_0(y) = A_1 e^{\lambda_1 y} + A_2 e^{\lambda_2 y} \]  
\[ \phi_0(y) = A_3 + A_4 e^{-\lambda_1 y} + A_2 e^{\lambda_1 y} + A_4 e^{\lambda_2 y} + A_3 \]  
\[ u_0(y) = A_{14} e^{\lambda_1 y} + A_{13} e^{\lambda_1 y} + A_3 e^{\lambda_2 y} + A_4 e^{\lambda_3 y} + A_9 + A_{16} e^{-\lambda_3 y} \]  
\[ \theta_1(y) = A_{61} e^{\lambda_1 y} + A_{62} e^{\lambda_2 y} + A_{38} e^{\lambda_3 y} + A_{39} e^{\lambda_4 y} + A_{40} e^{\lambda_5 y} + A_{41} e^{\lambda_6 y} + A_{42} e^{\lambda_7 y} + A_{43} e^{\lambda_8 y} + A_{44} e^{\lambda_9 y} + A_{45} e^{\lambda_{10} y} + A_{46} e^{\lambda_{11} y} + A_{47} e^{\lambda_{12} y} + A_{48} e^{\lambda_{13} y} + A_{49} e^{\lambda_{14} y} + A_{50} e^{\lambda_{15} y} \]  
\[ + A_{51} e^{\lambda_{16} y} + A_{52} e^{\lambda_{17} y} + A_{53} e^{\lambda_{18} y} + A_{54} e^{\lambda_{19} y} + A_{55} e^{\lambda_{20} y} + A_{56} e^{\lambda_{21} y} \]  
\[ \phi_1(y) = A_{86} + A_{87} e^{\lambda_{22} y} + A_{88} e^{\lambda_{23} y} + A_{89} e^{\lambda_{24} y} + A_{90} e^{\lambda_{25} y} + A_{91} e^{\lambda_{26} y} + A_{92} e^{\lambda_{27} y} + A_{93} e^{\lambda_{28} y} + A_{94} e^{\lambda_{29} y} \]  
\[ + A_{95} e^{\lambda_{30} y} + A_{96} e^{\lambda_{31} y} + A_{97} e^{\lambda_{32} y} + A_{98} e^{\lambda_{33} y} + A_{99} e^{\lambda_{34} y} + A_{100} e^{\lambda_{35} y} + A_{101} e^{\lambda_{36} y} + A_{102} e^{\lambda_{37} y} \]  
\[ + A_{103} e^{\lambda_{38} y} + A_{104} e^{\lambda_{39} y} + A_{105} e^{\lambda_{40} y} + A_{106} e^{\lambda_{41} y} + A_{107} e^{\lambda_{42} y} + A_{108} e^{\lambda_{43} y} + A_{109} e^{\lambda_{44} y} + A_{110} e^{\lambda_{45} y} \]  
\[ + A_{111} e^{\lambda_{46} y} + A_{112} e^{\lambda_{47} y} + A_{113} e^{\lambda_{48} y} + A_{114} e^{\lambda_{49} y} + A_{115} e^{\lambda_{50} y} + A_{116} e^{\lambda_{51} y} \]  
\[ + A_{117} e^{\lambda_{52} y} + A_{118} e^{\lambda_{53} y} + A_{119} e^{\lambda_{54} y} + A_{120} e^{\lambda_{55} y} + A_{121} e^{\lambda_{56} y} + A_{122} e^{\lambda_{57} y} \]  
\[ + A_{123} e^{\lambda_{58} y} + A_{124} e^{\lambda_{59} y} + A_{125} e^{\lambda_{60} y} + A_{126} e^{\lambda_{61} y} + A_{127} e^{\lambda_{62} y} + A_{128} e^{\lambda_{63} y} + A_{129} e^{\lambda_{64} y} + A_{130} e^{\lambda_{65} y} \]  
\[ + A_{131} e^{\lambda_{66} y} \]  

Where, \( \lambda_1 = \frac{-RP + R\sqrt{P^2 + 4F}}{2} \) , \( \lambda_2 = \frac{-RP - R\sqrt{P^2 + 4F}}{2} \) , \( \lambda_3 = \frac{-R + \sqrt{R^2 + 4A}}{2} \) , \( \lambda_4 = \frac{-R - \sqrt{R^2 + 4A}}{2} \).

The other constants \( A_1, A_2, \ldots, A_{134} \) are not shown for the sake of brevity.
The expressions for non-dimensional velocity, temperature and species concentration in the following form:
\[ u(y) = u_0(y) + E u_1(y), \quad \theta(y) = \theta_0(y) + E \theta_1(y), \]
\[ \phi(y) = \phi_0(y) + E \phi_1(y) \]

4: COEFFICIENT OF SKIN-FRICTION

The skin-friction in the non-dimensional form on the plate \( y = 0 \) and \( y = 1 \) are respectively given by
\[ \tau_0 = \frac{1}{R} \left[ \frac{du}{dy} \right]_{y=0} = \frac{1}{R} \left[ u_0'(0) + E u_1'(0) \right] \quad \text{and} \]
\[ \tau_1 = \frac{1}{R} \left[ \frac{du}{dy} \right]_{y=1} = \frac{1}{R} \left[ u_0'(1) + E u_1'(1) \right] \]

5: RATE OF HEAT-TRANSFER

The (Nusselt number) rate of heat transfer from the plates to the fluid in the non-dimensional form on the plate \( y = 0 \) and \( y = 1 \) are respectively given by
\[ N_n = \left( \frac{\partial \theta}{\partial y} \right)_{y=0} = \theta_0'(0) + E \theta_1'(0) \quad \text{and} \]
\[ N_u = \left( \frac{\partial \theta}{\partial y} \right)_{y=1} = \theta_0'(1) + E \theta_1'(1) \]

6: RATE OF MASS-TRANSFER

The (Sherwood number) rate of mass transfer between the fluid and the plates \( y=0 \) and \( y=1 \) are respectively given by
\[ Sh_0 = \left( \frac{\partial \phi}{\partial y} \right)_{y=0} = \left[ \phi_0'(0) + E \phi_1'(0) \right] \quad \text{and} \]
\[ Sh_1 = \left( \frac{\partial \phi}{\partial y} \right)_{y=1} = \left[ \phi_0'(1) + E \phi_1'(1) \right] \]

7: RESULTS AND DISCUSSION

In order to get physical insight into the problem the numerical values of velocity distribution and skin friction have been obtained and they are demonstrated graphically.

Throughout our investigation the Prandtl number \( P \) is taken to be equal to 0.71 which corresponds to the air at 20\(^\circ\)C. (In one graph the values of \( P \) are taken as 7 and 13.4 which corresponds to water at 20\(^\circ\)C and Sea water at 0\(^\circ\)C respectively). The values of Grashof number for heat transfer \( Gr \) is taken to be 5, the Grashof number for mass transfer \( Gm \) is considered to be 2 and the Eckert number \( E \) is assumed to be 0.05. The values of non-dimensional temperature \( m \) at the plate \( y=1 \), non-dimensional species concentration \( n \) at the plate \( y=1 \) are fixed at 2, 2 respectively. The values of other parameters namely the Hartmann number \( M \), the Reynolds number \( R \), the Soret number \( S_0 \), the permeability parameter \( \alpha \), the radiation parameter \( F \) and the Schmidt number \( Sc \) are chosen arbitrary. The values of Schmidt number \( Sc \) are chosen in such a way that they represent the diffusing chemical species of most common interest in air (for example the values of Schmidt number for He, H\(_2\)O, CO, H\(_2\)S, CO\(_2\) and SO\(_2\) are 0.22, 0.61, 0.77, 0.94, 1.0, 1.24 respectively). Here, Grashof number for heat transfer \( Gr > 0 \) corresponds to externally cooled plates.

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The velocity profiles against \( y \) are presented in figures 2-8 for different values of \( F, Sc, So, M, R, P \) and \( \alpha \). These figures indicate that the velocity profile is parabolic where the maximum value occurs at the centre of the maximum value occurs at the centre of the channel. Here it is seen that due to effects of thermal radiation, thermal diffusion, magnetic field fluid motion is retarded whereas fluid motion is accelerated due to increase of porosity of the medium and Prandtl number. From these figures we have also observed that due to high molar diffusivity and low viscosity fluid velocity is increased.
Figure 2: Velocity distribution $u$ versus $y$ when $P=0.71, \text{Sc}=0.61, M=1, R=5, S_0=5, \alpha=0.2$

Figure 4: Velocity distribution $u$ versus $y$ when $P=0.71, \text{Sc}=0.5, M=1, R=5, F=1, \alpha=0.2$

Figure 5: Velocity distribution $u$ versus $y$ when $\text{Sc}=0.61, S_0=5, M=1, R=5, F=1, \alpha=0.2$

Figure 6: Velocity distribution $u$ versus $y$ when $P=0.71, \text{Sc}=0.61, M=1, R=5, F=1, \alpha=0.2$

Figure 7: Velocity distribution $u$ versus $y$ when $P=0.71, \text{Sc}=0.61, S_0=5, M=1, R=5, F=1$
Figures 9-13 depict the behaviour of skin friction $\tau_0$ at the plate $y=0$ against $R$ for different values of $F$, $S_o$, $S_c$, $M$ and $\alpha$. It is inferred from these figures that radiation and magnetic field effects and porosity of the medium reduce the skin friction $\tau_0$ at the plate $y=0$. The same figures also indicate that due to high chemical molecular diffusivity the skin friction $\tau_0$ is increased. From figure 9 it is cleared that due to Soret effect $\tau_0$ is increased near the plate $y=0$ but it shows reverse effect far away from the plate.
Figures 14-18 exhibit the variation of skin friction $\tau_0$ at the plate $y=0$ against $R$ for different values of $M$, $S_o$, $S_c$, $F$ and $\alpha$. It is marked from these figures that due to effect of magnetic field and radiation skin friction $\tau_1$ increases but due to high chemical molecular diffusivity and porosity of the medium it is decreased. From figure 14 it is cleared that due to Soret effect skin friction $\tau_1$ is increased near the plate $y=1$ but it shows reverse effect far away from the plate.
8: CONCLUSIONS

1) Fluid motion is accelerated due to high molar diffusivity and low viscosity but due to effects of thermal radiation, thermal diffusion, magnetic field fluid motion is retarded.

2) Due to application of magnetic field viscous drag decreases at the plate y=0, while it is increased at plate y=1.

3) Due to radiation viscous drag decreases at the plate y=0, while it is increased at plate y=1.

REFERENCES


