International Journal of Applied Engineering Research ISSN 0973-4562 Volume 14, Number 1 (2019) pp. 180-186

Total $S-$ Irregularity in Cartesian Product of S-Valued Graphs

M.Sundar[1] **and M.Chandramouleeswaran**[2]

[1] *Sree Sowdambika College of Engineering, Aruppukottai - 626101. India.*

[2] *Sri Ramanas College of Arts and Science for Women , Aruppukottai - 626134. India.*

Abstract :
In this paper we consider the total $S-$ irregularity of $S-$ valued graphs under some graph operations. We present sharp upper bounds for disjoint union, sum and Cartesian product of $S-$ valued graphs.

Keywords: Semiring, $S-$ valued graphs, Graph operations, Irregularity

1. INTRODUCTION

All graphs we consider are simple and finite. A $S-$ valued graph G^S is said to be $S-$ regular if all its vertices have the same degree, otherwise it is called $S-$ irregular. However, in many applications and problems it is of big importance to know how irregular a given graph is. Several graph topological indices have been proposed for that purpose. In [5], [6], the authors have defined the $S-$ imbalance of an edge in G^S by considering the weights and degrees separately: weight $S-$ imbalance and degree $S-$ imbalance. In weight $S-$ imbalance, the imbalance of an edge is discussed only when the degrees of the end vertices are equal. On the otherhand, degree $S-$ imbalance is discussed if the weights of the end vertices are equal. In this paper, we have modified this notion and redefine the concepts $S-$ imbalance of an edge and $S-$ irregularity of a $S-$ valued graph G^S irrespective of the equality in any component. Further, we introduce the notion of total $S-$ irregularity of G^S and prove some results.

2. PRELIMINARIES

In this section, we recall some basic definitions that are needed for our sequel

Definition 2.1 *[1] A semiring $(S, +, \cdot)$ is an algebraic system with a non-empty set S together with two binary operations $+$ and $\cdot$ such that*

1. $(S, +)$ *is a monoid.*
2. $(S, \cdot)$ *is a semigroup.*
3. *For all* $a, b, c \in S, a \cdot (b + c) = a \cdot b + a \cdot c$ *and* $(a + b) \cdot c = a \cdot c + b \cdot c$
4. $0 \cdot x = x \cdot 0 = 0, \ \forall x \in S.$

The element $0 \in S$ is the additive identity as well as the zero of the semiring S.

Definition 2.2 *[1] Let $(S, +, \cdot)$ be a semiring. $\preceq$ is said to be a canonical pre-order if for $a, b \in S, a \preceq b$ if and only if there exists $c \in S$ such that $a + c = b$.*

Definition 2.3 *[2] Let $G = (V, E \subset V \times V)$ be a given graph with $V, E \neq \phi$. For any semiring $(S, +, \cdot)$, a semiring valued graph (or a S-valued graph) G^S is defined to be the graph $G^S = (V, E, \sigma, \psi)$ where $\sigma : V \to S$ and $\psi : E \to S$ is defined by*
$$\psi(v_i, v_j) = \psi(e_i^j) = \begin{cases} min\{\sigma(v_i), \sigma(v_j)\} & if\ \sigma(v_i) \preceq \sigma(v_j)\ or\ \sigma(v_j) \preceq \sigma(v_i) \\ 0 & otherwise \end{cases}$$
for every unordered pair $(v_i, v_j) \in E \subset V \times V$. We call σ, a $S-$ vertex set and ψ, a $S-$ edge set of the $S-$ valued graph G^S.

Definition 2.4 *[3] The open neighbourhood of v_i in G^S is defined as*
$$N_S(v_i) = \{(v_j, \sigma(v_j)) \mid (v_i, v_j) \in E, \ \psi(v_i, v_j) \in S\}.$$
The closed neighbourhood of v_i in G^S is defined as $N_S[v_i] = N_S(v_i) \cup \{(v_i, \sigma(v_i))\}$.

Definition 2.5 *[3] The degree of a vertex v_i of the $S-$ valued graph G^S is defined as $deg_S(v_i) = (\sum_{v_j \in N_S(v_i)} \psi(v_i, v_j), l)$ where l is the number of edges incident with v_i.*

Definition 2.6 *[3] A $S-$ valued graph G^S is said to be*

1. *$S-$ vertex regular if $\sigma(v) = a, \ \forall \ v \in V$ and for some $a \in S$*
2. *$S-$ edge regular if $\psi(u, v) = a, \ \forall (u, v) \in E$ and for some $a \in S$.*
3. *$S-$ regular if it is both vertex as well as edge regular.*

Definition 2.7 *[4] Let $G_1^S = (V_1, E_1, \sigma_1, \psi_1)$ and $G_2^S = (V_2, E_2, \sigma_2, \psi_2)$ be two $S-$ valued graphs with $V_1 \cap V_2 = \phi$. Then their disjoint union is defined as $G_1^S \cup G_2^S = (V, E, \sigma, \psi)$ where $V = V_1 \cup V_2; \ E = E_1 \cup E_2$.*

$$For \ v \in V, \ \sigma(v) = \begin{cases} \sigma_1(v) & if \ v \in V_1 \\ \sigma_2(v) & if \ v \in V_2 \end{cases}$$

For $(v_i, v_j) \in E = E_1 \cup E_2, \psi(v_i, v_j) = \begin{cases} \psi_1(v_i, v_j) & if \ (v_i, v_j) \in E_1 \\ \psi_2(v_i, v_j) & if \ (v_i, v_j) \in E_2 \end{cases}$

Definition 2.8 *[4] Let $G_1^S = (V_1, E_1, \sigma_1, \psi_1)$ and $G_2^S = (V_2, E_2, \sigma_2, \psi_2)$ be two $S-$ valued graphs. Then their sum $G_1^S + G_2^S = (V, E, \sigma, \psi)$ where $V = V_1 \cup V_2$ and $V_1 \cap V_2 = \phi, E = E_1 \cup E_2 \cup \{(v_i, v_j) \mid v_i \in V_1 \ and \ v_j \in V_2\}$,*

$$For \ v \in V, \ \ \sigma(v) = \begin{cases} \sigma_1(v) & if \ v \in V_1 \\ \sigma_2(v) & if \ v \in V_2 \end{cases}$$

and for $(v_i, v_j) \in E$,

$$\psi(v_i, v_j) = \begin{cases} \psi_1(v_i, v_j) & if \ (v_i, v_j) \in E_1 \\ \psi_2(v_i, v_j) & if \ (v_i, v_j) \in E_2 \\ min\{\sigma_1(v_i), \sigma_2(v_j)\} & if \ v_i \in V_1 \ and \ v_j \in V_2 \end{cases}$$

Definition 2.9 *[5] Let $G_1^S = (V_1, E_1, \sigma_1, \psi_1)$ where $V_1 = \{v_i \mid i = 1, 2, .., p_1\}$,
$E_1 \subseteq V_1 \times V_1$ and $G_2^S = (V_2, E_2, \sigma_2, \psi_2)$ where $V_2 = \{u_j \mid j = 1, 2, ..., p_2\}$,
$E_2 \subseteq V_2 \times V_2$ be two given $S-$ valued graphs.
The Cartesian product of two $S-$ valued graphs G_1^S and G_2^S is a graph defined as $G_\square^S = G_1^S \square G_2^S = (V = V_1 \times V_2, E = E_1 \times E_2, \sigma = \sigma_1 \times \sigma_2, \psi = \psi_1 \times \psi_2)$, where $V = V_1 \times V_2 = \{w_{ij} = (v_i, u_j) \mid v_i \in V_1 \ and \ u_j \in V_2, i = 1, 2, ..., p_1; j = 1, 2, ..., p_2\}$ and $E = E_1 \times E_2 \subseteq (V_1 \times V_2) \times (V_1 \times V_2)$ such that two vertices w_{ij} and w_{kl} are adjacent if $i = k$ and $u_j u_l \in E_2$ or $j = l$ and $v_i v_k \in E_1$.
Define $\sigma : V \to S$ by $\sigma(w_{ij}) = \min\{\sigma_1(v_i), \sigma_2(u_j)\}$*

$$\psi : E \to S \ by \ \psi(e_{ij}^{kl}) = \psi((v_i, u_j)(v_k, u_l)) = \begin{cases} min\{\sigma_1(v_i), \psi_2(u_j, u_l)\} & if \ i = k \ and \ u_j u_l \in E_2 \\ min\{\psi_1(v_i, v_k), \sigma_2(u_j)\} & if \ j = l \ and \ v_i v_k \in E_1 \end{cases}$$

Definition 2.10 *[7] Any $S-$ valued grpah G^S which is not $S-$ regular is called weight $S-$ irregular graph.*

Definition 2.11 *[7] A $S-$ valued graph G^S is said to be a weight $S-$ vertex irregular if for every vertex $v \in V, \sigma(v) \neq \sigma(u), u \in N_S(v)$.*

3. TOTAL $S-$ IRREGULARITY IN CARTESIAN PRODUCT OF TWO $S-$ VALUED GRAPHS

In this section, we redefine the concept of $S-$ imbalance of an edge and $S-$ irregularity of G^S and prove some results related to total $S-$ irregularity of some $S-$ valued graphs.

Definition 3.1 *Consider a $S-$ valued graph $G^S = (V, E, \sigma, \psi)$. Let e_i^j denotes the edge $(v_i, v_j) \in E$. Then the $S-$ edge imbalance of e_i^j, denoted by $Imb_S(e_i^j)$, is defined as*

$$Imb_S(e_i^j) = (\psi(e_i^j), \mid d_G(v_i) - d_G(v_j) \mid).$$

Definition 3.2 *Let* $G^S = (V, E, \sigma, \psi)$ *be a* $S-$ *valued graph.Then the* $S-$ *irregularity of* G^S *is defined by*

$$Irr_S(G^S) = \Sigma_{e_i^j \in E(G^S)} Imb_S(e_i^j).$$

Definition 3.3 *The total* $S-$ *irregularity of a* $S-$ *valued graph is defined by*

$$Tirr_S(G^S) = \Sigma_{\substack{v_i, v_j \in V(G^S) \\ i < j}} (min\{\sigma(v_i), \sigma(v_j)\}, \mid d_G(v_i) - d_G(v_j) \mid),$$

where the sum runs over each pair of vertices $v_i, v_j \in V(G^S), i < j$.

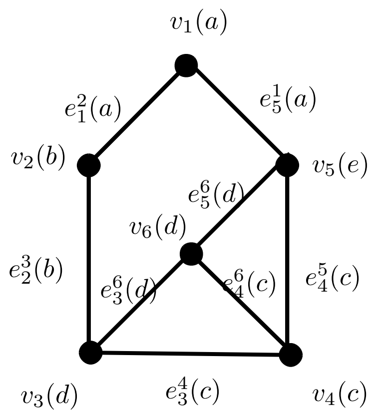
Example 3.4 Consider the semiring $S = (\{0, a, b, c, d, e\}, +, \cdot)$ with the binary operations "+" and " $\cdot$ " defined by the following cayley tables.

$+$	0	a	b	c	d	e
0	0	a	b	c	d	e
a	a	a	b	c	d	e
b	b	b	b	d	d	e
c	c	c	d	c	d	e
d	d	d	d	d	d	e
e	e	e	e	e	e	e

$\bullet$	0	a	b	c	d	e
0	0	0	0	0	0	0
a	0	0	0	a	a	b
b	0	a	b	a	b	b
c	0	0	0	c	c	e
d	0	a	b	c	d	e
e	0	c	e	c	e	e

$\preceq$	$elements\ of\ S$
0	$0, a, b, c, d, e$
a	a, b, c, d, e
b	b, d, e
c	c, d, e
d	d, e
e	e

$\mathbf{G^S}$



$Figure.1$

$$Imb_S(e_1^2) = (\psi(e_1^2), |d(v_1) - d(v_2)|) = (a, 0), Imb_S(e_2^3) = (b, 1), Imb_S(e_3^4) = (c, 0),$$

$$Imb_S(e_3^6) = (d, 0), Imb_S(e_4^5) = (c, 0), Imb_S(e_4^6) = (c, 0), Imb_S(e_5^1) = (a, 1), Imb_S(e_5^6) = (d, 0)$$

Then the $S-$ irregularity of G^S is given by

$$Irr_S(G^S) = \Sigma_{e_i^j \in E(G^S)} Imb_S(e_i^j) = (d, 2)$$

And the total $S-$ irregularity is given by

$$\begin{aligned} Tirr_S(G^S) &= \Sigma_{\substack{v_i, v_j \in V(G^S) \\ i < j}} (min\{\sigma(v_i), \sigma(v_j)\}, \mid d_G(v_i) - d_G(v_j) \mid) \\ &= (d, 8) \end{aligned}$$

Theorem 3.5 *For any* $S-$ *valued graph* G^S, $Irr_S(G^S) \preceq Tirr_S(G^S)$.

Proof

$$\begin{aligned} Irr_S(G^S) &= \Sigma_{e_i^j \in E(G^S)} Imb_S(e_i^j) \\ &= \Sigma_{e_i^j \in E(G^S)} (\psi(e_i^j), \mid d_G(v_i) - d_G(v_j) \mid). \\ &\preceq \Sigma_{v_i, v_j \in E(G^S)} (min\{\psi(v_i), \psi(v_j)\}, \mid d_G(v_i) - d_G(v_j) \mid) \\ &\quad + \Sigma_{v_i, v_j \in V(G^S) \setminus E(G^S)} (min\{\psi(v_i), \psi(v_j)\}, \mid d_G(v_i) - d_G(v_j) \mid) \\ &= \Sigma_{v_i, v_j \in V(G^S)} (min\{\sigma(v_i), \sigma(v_j)\}, \mid d_G(v_i) - d_G(v_j) \mid) \\ &= Tirr_S(G^S). \end{aligned}$$

Hence $Irr_S(G^S) \preceq Tirr_S(G^S)$.

Example 3.6 Consider the semiring given in example 3.4.
Consider the $S-$ valued graphs G_1^S, G_2^S and their disjoint union of the graphs.

G_1^S $\qquad$ G_2^S

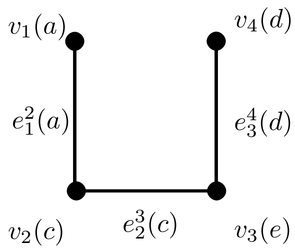


Figure.1

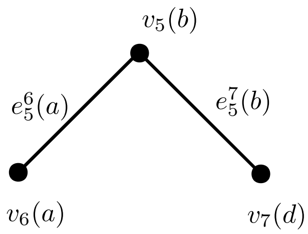
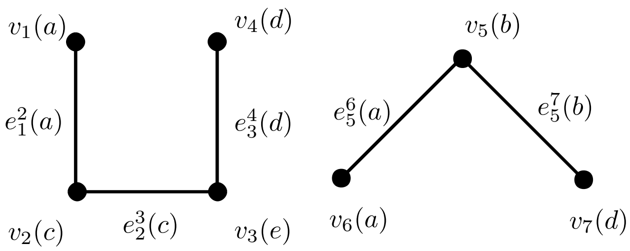


Figure.2

$G_1^S \cup G_2^S$



$$Imb_S(e_1^2) = (a,1), Imb_S(e_2^3) = (c,0), Imb_S(e_3^4) = (d,1), Imb_S(e_5^6) = (a,1), Imb_S(e_5^7) = (b,1).$$

$Irr_S(G_1^S) = \Sigma_{e_i^j \in E(G_1^S)} Imb_S(e_i^j) = (d,2)$.
$Irr_S(G_2^S) = \Sigma_{e_i^j \in E(G_2^S)} Imb_S(e_i^j) = (b,2)$.
$Irr_S(G_1^S) + Irr_S(G_2^S) = (d,4)$.
$Irr_S(G_1^S \cup G_2^S) = \Sigma_{e_i^j \in E(G_1^S \cup G_2^S)} Imb_S(e_i^j) = (d,4)$.
Thus $Irr_S(G_1^S \cup G_2^S) = Irr_S(G_1^S) + Irr_S(G_2^S)$.
$Tirr_S(G_1^S) = \Sigma_{\substack{v_i, v_j \in V(G_1^S) \\ i < j}} (min\{\sigma(v_i), \sigma(v_j)\}, \mid d_G(v_i) - d_G(v_j) \mid) = (d,4)$
$Tirr_S(G_2^S) = \Sigma_{\substack{v_i, v_j \in V(G_2^S) \\ i < j}} (min\{\sigma(v_i), \sigma(v_j)\}, \mid d_G(v_i) - d_G(v_j) \mid) = (b,2)$

$$Tirr_S(G_1^S \cup G_2^S) = \Sigma_{\substack{v_i, v_j \in V(G_1^S \cup G_2^S) \\ i < j}} (min\{\sigma(v_i), \sigma(v_j)\}, \mid d_G(v_i) - d_G(v_j) \mid) = (d,12)$$

Here $Tirr_S(G_1^S \cup G_2^s) = (d,12) \succeq (d,6) = Tirr_S(G_1^S) + Tirr_S(G_2^S)$.
For best possible bound, consider the size of the graphs;

$q_S(G_1^S) = \Sigma_{e_i^j \in E(G_1^S)}(\psi(e_i^j), \mid E(G_1^S) \mid) = (a + c + d, 3) = (d, 3)$
$q_S(G_2^S) = \Sigma_{e_i^j \in E(G_2^S)}(\psi(e_i^j), \mid E(G_2^S) \mid) = (a + b, 2) = (b, 2).$
$Tirr_S(G_1^S) + Tirr_S(G_2 S) + q_S(G_1^S) \cdot q_S(G_2^S) = (d, 6) + (b, 6) = (d, 12).$
Thus we obseve that, $Tirr_S(G_1^S \cup G_2^S) = Tirr_S(G_1^S) + Tirr_S(G_2 S) + q_S(G_1^S) \cdot q_S(G_2^S).$

Theorem 3.7 *Let G_1^S and G_2^S be two $S-$ valued graphs, then $Irr_S(G_1^S \cup G_2^S) = Irr_S(G_1^S) + Irr_S(G_2^S).$*
Proof:

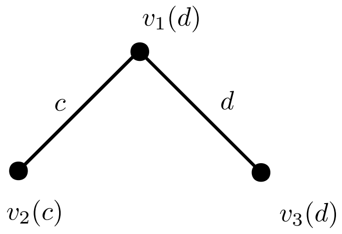
$$\begin{aligned} Irr_S(G_1^S \cup G_2^S) &= \Sigma_{e_i^j \in E(G_1^S \cup G_2^S)} Imb_S(e_i^j) \\ &= \Sigma_{e_i^j \in E(G_1^S) + E(G_2^S)} Imb_S(e_i^j) \\ &= \Sigma_{e_i^j \in E(G_1^S)} Imb_S(e_i^j) + \Sigma_{e_i^j \in E(G_2^S)} Imb_S(e_i^j) \\ &= Irr_S(G_1^S) + Irr_S(G_2^S) \end{aligned}$$

Thus $Irr_S(G_1^S \cup G_2^S) = Irr_S(G_1^S) + Irr_S(G_2^S).$

Corollary 3.8 $Tirr_S(G_1^S \cup G_2^S) \succeq Tirr_S(G_1^S) + Tirr_S(G_2^S).$

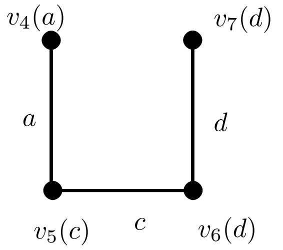
Example 3.9 Consider the semiring given in example 3.4 .
Consider the two $S-$ valued graphs G_1^S, G_2^S and their sum of the graphs.

G_1^S



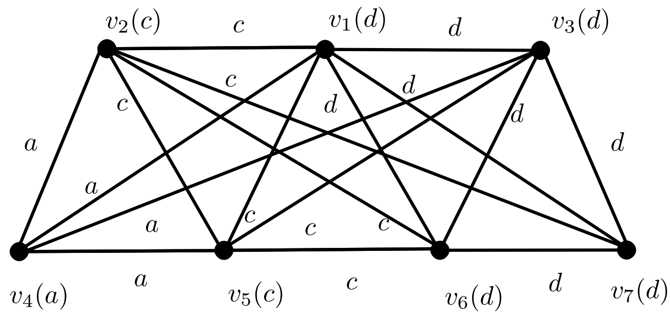
$Figure.1$

G_2^S



$Figure.2$

$G_1^S + G_2^S$



$Figure.3$

$$Imb_S(e_1^2) = (\psi(e_1^2), |d(v_1) - d(v_2)|) = (c, 1), Imb_S(e_1^3) = (d, 1), Imb_S(e_4^5) = (a, 1), Imb_S(e_5^7) = (c, 0),$$
$$Imb_S(e_6^7) = (d, 0), Imb_S(e_1^4) = (a, 2), Imb_S(e_1^5) = (c, 1), Imb_S(e_1^6) = (d, 1), Imb_S(e_1^7) = (d, 2),$$
$$Imb_S(e_2^4) = (a, 1), Imb_S(e_2^5) = (c, 0), Imb_S(e_2^6) = (c, 0), Imb_S(e_2^7) = (c, 1), Imb_S(e_3^4) = (a, 1)$$

$Imb_S(e_3^5) = (c, 0), Imb_S(e_3^6) = (d, 0), Imb_S(e_3^7) = (d, 1)$
$Irr_S(G_1^S) = (d, 2); Irr_S(G_2^S) = (d, 2)$ *and* $Irr_S(G_1^S) + Irr_S(G_2^S) = (d, 4)$
$Irr_S(G_1^S + G_2^S) = (d, 10)$
Here $Irr_S(G_1^S + G_2^S) \succeq Irr_S(G_1^S) + Irr_S(G_2^S)$
Now $q_S(G_1^S) = (d, 2); q_S(G_2^S) = (d, 3)$
Therefore we observe that $Irr_S(G_1^S + G_2^S) = Irr_S(G_1^S) \cdot q_S(G_1^S) + Irr_S(G_2^S) \cdot q_S(G_2^S)$ $Tirr_S(G_1^S) = \Sigma_{\substack{v_i, v_j \in V(G_1^S) \\ i < j}} (min\{\sigma(v_i), \sigma(v_j)\}, \mid d_G(v_i) - d_G(v_j) \mid) = (d, 2).$ $Tirr_S(G_2^S) = (d, 4);$ $Tirr_S(G_1^S) + Tirr_S(G_2^S) = (d, 2) + (d, 4) = (d, 6)$

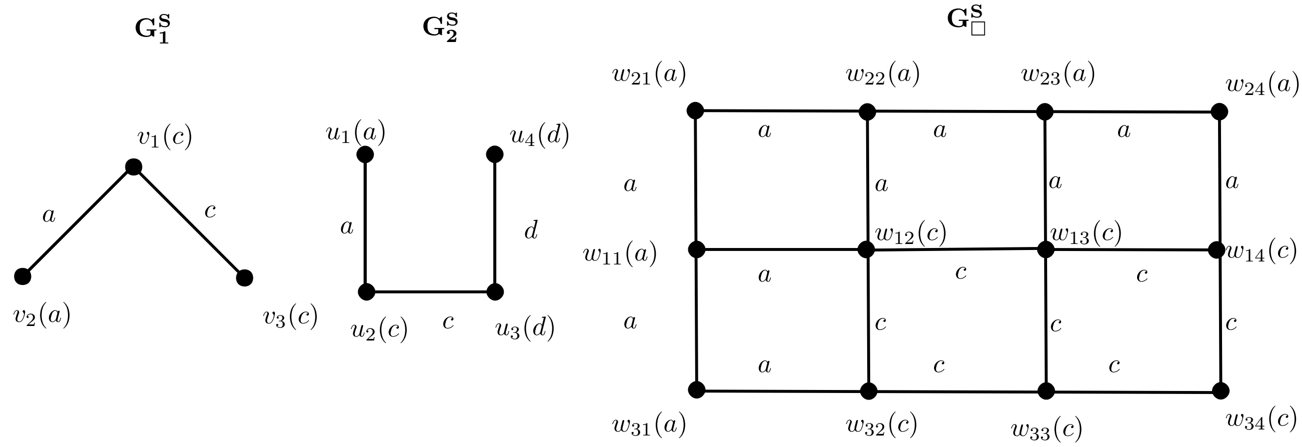
$$Tirr_S(G_1^S + G_2^S) = \Sigma_{\substack{v_i, v_j \in V(G_1^S + G_2^S) \\ i < j}} (min\{\sigma(v_i), \sigma(v_j)\}, \mid d_G(v_i) - d_G(v_j) \mid) = (d, 15)$$

Here $Tirr_S(G_1^S + G_2^S) \succeq Tirr_S(G_1^S) + Tirr_S(G_2^S)$

Theorem 3.10 *Cartesian product of two $S-$ irregular graphs is again $S-$ irregular.*

Proof: Let $G_1^S = (V_1, E_1, \sigma_1, \psi_1)$ and $G_2^S = (V_2, E_2, \sigma_2, \psi_2)$ be two $S-$ irregular graphs.
Then $\sigma_1(v_i) \neq \sigma_1(v_k)$, for some i and k.
Therefore either $\sigma_1(v_i) \preceq \sigma_1(v_k)$ *or* $\sigma_1(v_k) \preceq \sigma_1(v_i)$ and $\sigma_2(u_j) \neq \sigma_2(u_l)$
That is either $\sigma_2(u_j) \preceq \sigma_2(u_l)$ *or* $\sigma_2(u_l) \preceq \sigma_2(u_j)$, for some j and l.
Since G_2^S is $S-$ irregular, there is a vertex $u_x \in V_2$ such that
$\sigma_1(v_i) \neq \sigma_2(u_x) \neq \sigma_2(u_j)$.
Now consider the cartesian product $G_1^S \Box G_2^S = (V, E, \sigma, \psi)$.
Now $\sigma(w_{ix}) = min\{\sigma_1(v_i), \sigma_2(u_x)\} = \sigma_1(v_1)$ *or* $\sigma_2(u_x)$
and $\sigma(w_{kj}) = min\{\sigma_1(v_k), \sigma_(u_j)\} = \sigma_1(v_k)$ *or* $\sigma_2(u_j)$
Here $w_{ix} \neq w_{kj}$, for some i, j, k, x.
Hence the Cartesian product of two $S-$ irregular graphs is again $S-$ irregular.

Example 3.11 Consider the semiring given in example 3.4 .
Consider the two $S-$ valued graphs G_1^S, G_2^S and their Cartesian product.



$Irr_S(G_1^S) = (c, 2); Irr_S(G_2^S) = (d, 2); Irr_S(G_\Box^S = G_1^S \Box G_2^S) = (c, 14)$
We have $Irr_S(G_\Box^S = G_1^S \Box G_2^S) \preceq Irr_S(G_1^S) + Irr_S(G_2^S)$.
$Tirr_S(G_1^S) = (c, 2); Tirr_S(G_2^S) = (d, 4); Tirr_S(G_\Box^S = G_1^S \Box G_2^S) = (c, 52)$
Now consider the two $S-$ valued graphs G_3^S, G_4^S and their cartesian product.

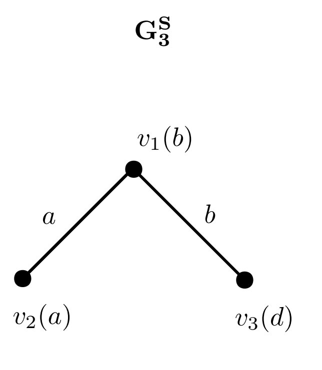


figure : **4**

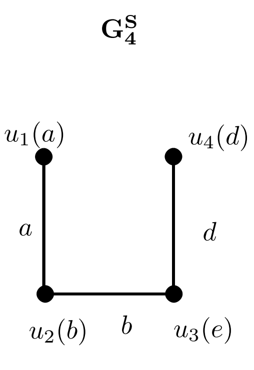


figure : **5**

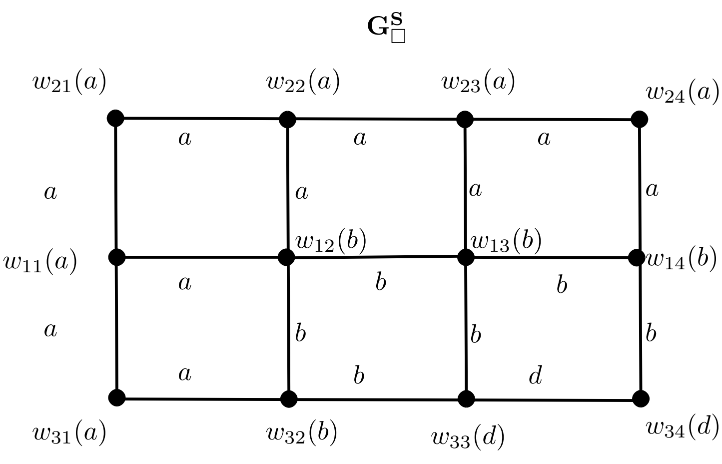


figure : **6**

$Irr_S(G_3^S) = (b, 2); Irr_S(G_4^S) = (d, 2); Irr_S(G_\Box^S = G_3^S \Box G_4^S) = (d, 14)$
We have $Irr_S(G_\Box^S = G_3^S \Box G_4^S) \preceq Irr_S(G_3^S) + Irr_S(G_4^S)$.

$Tirr_S(G_3^S) = (b, 2); Tirr_S(G_4^S) = (d, 4); Tirr_S(G_\square^S = G_3^S \square G_4^S) = (d, 52)$
$p_S(G_1^S) = (c, 3); p_S(G2^S) = (d, 4); p_S(G_3^S) = (d, 3); p_S(G_4^S) = (e, 4)$

Remark 3.12 From the above two examples, we observe the following:

1. $Irr_S(G_\square^S) \preceq Irr_S(G_1^S) \cdot (p_S(G_1^S)) + Irr_S(G_2^S) \cdot (p_S(G_2^S))$
2. $Tirr_S(G_\square^S) \preceq (p_S(G_2^S))^2 \cdot Tirr_S(G_1^S) + (p_S(G_1^S))^2 \cdot Tirr_S(G_2^S)$

These bounds can further be improved.

4. CONCLUSION

In this paper, we have discussed the total $S-$ irregularity of the cartesian product of two $S-$ valued graphs. One can extend this study to find a sharp bounds for $S-$ irregularity and total $S-$ irregularity.

REFERENCES

[1] **Jonathan S Golan** *Semirings and Their Applications*, Kluwer Academic Publishers, London.

[2] **Rajkumar. M, Jeyalakshmi. S. and Chandramouleeswaran. M** *Semiring Valued Graphs*, International Journal of Math.Sci. and Engg. Appls., Vol. 9 (3), 2015 pp. 141-152.

[3] **Jeyalakshmi. S, Rajkumar. M, and Chandramouleeswaran. M** *Regularity on S-valued graphs*, Global J. of Pure and Applied Maths., Vol 2(5), 2015, 2971 - 2978.

[4] **Rajkumar.M,Jeyalakshmi.S and Chandramouleeswaran.M** *Operations on S-VauedGraphs*, International Journal of Pure and Applied Mathematics, ISSN 1314-3395, Vol 105, No. 2 (2015), pp.153-162.

[5] **Sundar.M and Chandramouleeswaran. M** *Cartesian product of S-valued graphs*, Global Journal of Mathematical Sciences: Theory and Practical., Vol. 9(3), 2017, pp. 347-355.

[6] **Mangala Lavanya.S and Chandramouleeswaran.M** *Edge Irregularity on S-valued graphs*,Journal of Global Research in Mathematical Archives, Volume.4, No. 9, September 2017, pp. 19-23.

[7] **Rajkumar.M, and Chandramouleeswaran.M** *Total Weight Irregularity of S-valued Graphs*, Mathematical Sciences International Research Journal ISSN 2278-8697, Issue 1, Vol 6, 2017, pp. 124-127.