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A New View on Fuzzy Rough Approach to $\wp$-open Sets

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Abstract

The main aim of this paper is to discuss the basic ideas and concepts of the so called "$\wp$-open sets" and to give a brief survey of the history and of some trends in recent development of mathematics and its applications in the context of fuzzy sets. And this paper aim to extend the concepts of fuzzy rough set, fuzzy rough $\wp$-open sets, fuzzy rough $\wp$-closed sets, fuzzy rough $\wp$-interiors and fuzzy rough $\wp$-closures are investigated. Also, some interesting properties of them are established.

Keywords: Fuzzy rough set, fuzzy rough $\wp$-open set, fuzzy rough $\wp$-closed set, fuzzy rough $\wp$-interior and fuzzy rough $\wp$-closure.

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1. INTRODUCTION

In 1965, Zadeh [5] the concept of fuzzy sets had been investigated to meet the challenges due to lack of solving the problems involved in our day to day life. It play a vital role in the study of fuzzy topological space which had been introduced by Chang [1] in 1968. In this way Z.Pawlak [4] introduce the concept of rough set in 1982, we compare this concept with that of fuzzy set, and we show that these two notion aim to different purpose. It may be seen as an extension of classical set theory and has been successfully applied to machine learning, intelligent systems, inductive reasoning, pattern recognition, image processing, signal analysis, knowledge discovery, decision analysis, expert systems and many other field. S.Nanda and S.Majumdar [3] studied the concept of fuzzy rough sets in 1992. The concepts of fuzzy rough topological space was introduced by S.Padmapriya, M.K.Uma and E.Roja [4]. In this paper, we introduce the concept of fuzzy rough $\wp$-open sets, fuzzy rough $\wp$-closed sets, fuzzy rough $\wp$-interiors and fuzzy rough $\wp$-closures are established some of its properties in detail.

2. PRELIMINARIES

Let U be any non empty set and let $\mathcal{B}$ be a complete subalgebra of the Boolean algebra $\mathcal{P}(U)$ of subsets of U. The pair $(U, \mathcal{B})$ is called rough universe. Consider a rough set

$$X = (X_L, X_U) \in \mathcal{B}^2 \text{ with } X_L \subset X_U.$$

Definition:2.1.[2]

A fuzzy rough set $A = (A_L, A_U)$ in X is characterized by a pair of maps $A_L: X_L \longrightarrow I$ and $A_U: X_U \longrightarrow I$ with $A_L(x) \leq A_U(x)$ for every $x \in X_U$. The collection of all **fuzzy rough sets** in X is denoted by FRS($\boldsymbol{X}$).

Example:2.1

Let X={a,b,c} be a non-empty set.

$A_L(a) = 0.2$ $A_L(b) = 0.3$ $A_L(c) = 0.5$

$A_U(a) = 0.3$ $A_U(b) = 0.4$ $A_U(c) = 0.6$ with $A_L(a) \leq A_U(a)$ for every $a \in X_U$.

Then $A = (A_L, A_U)$ is fuzzy rough set in X.

Definition:2.2.[2]

For any two fuzzy rough set $A=(A_L,A_U)$ and $B=(B_L,B_U)$ in X.

(i)A=B iff

$A_L(x) = \mu_{BL}(x)$ for every $x \in X_L$ and

$A_U(x) = \mu_{BU}(x)$ for every $x \in X_U$.

(ii)A⊆Biff

$A_L(x) \leq \mu_{BL}(x)$ for every $x \in X_L$ and

$A_U(x) \leq \mu_{BU}(x)$ for every $x \in X_U$.

If $\{A_i : i \in J\}$ be any family of fuzzy rough set in X, where $A = (A_L, A_U)$ then $E = \cup_i A_i$ iff

$E_L(x) = sup_{i \in J} A_{L_i}(x)$ for every $x \in X_L$ and

$E_U(x) = sup_{i \in J} A_{U_i}(x)$ for every $x \in X_U$.

Similarly, F=$\cap_i A_i$ iff

$F_L(x) = inf_{i\in J} A_{L_i}(x)$ for every $x\in X_L$ and

$F_U(x) = inf_{i\in J} A_{U_i}(x)$ for every $x\in X_U$.

Definition:2.3.[2]

Let $A = (A_L, A_U)$ be a fuzzy rough sets in X. Then the complement A' of A is defined by ordered pairs (A'_L, A'_U) of membership functions where

$A'_L(x) = 1 - A_L(x)$ for every $x \in X_L$ and

$A'_U(x) = 1 - A_U(x)$ for every $x \in X_U$.

Definition:2.4.[2]

The null fuzzy rough set and whole fuzzy set in X are defined by $\tilde{0} = (0_L, 0_U)$ and $\tilde{1} = (1_L, 1_U)$.

Definition:2.5.[4]

A fuzzy rough topology on a set X is a family τ of fuzzy sets in X which satisfies the following conditions

(i) $\tilde{0}, \tilde{1} \in \tau$.

(ii) A,B $\in \tau$ then A $\cap$ B $\in \tau$ and

(iii) If $A_j \in \tau$ for all j $\in$ J then $\cup A_{j\in J} \in \tau$.

Then τ is called a fuzzy rough topology on X and the pair (X, τ) is a **fuzzy rough topological space** (in short $FRTS$). Every members of τ is called **fuzzy rough open set** (in short FOS).

Proposition:2.1.[2]

If A,B,C,D and B_i i∈J are FRS in X, then

(i) A⊂B and C⊂D implies A∪C⊂B∪D,

(ii) A⊂B and B⊂C implies A⊂C,

(iii) A∩B⊂A, A∪B⊃B,

(iv) $A\cup(\cap_i B_i) = \cap_i (A\cup B_i)$ and $A\cap(\cup_i B_i) = \cup_i (A\cap B_i)$,

(v) A⊂B⟹ A'⊂B',

(vi) $(\cup_i B_i)' = \cap_i B_i'$ and $(\cap_i B_i)' = \cup_i B_i'$.

Proposition:2.2.[2]

If A is any fuzzy rough set in X, then $\tilde{0} \subset A \subset \tilde{1}$.

3. FUZZY ROUGH 𝓅-OPEN SETS:

In this section the concepts of fuzzy rough 𝓅- open sets, fuzzy rough 𝓅-closed sets, fuzzy rough 𝓅- interior sets, fuzzy rough 𝓅-closure sets are introduced and some of its propositions are also discussed.

3.1.Notation:

(i) Fuzzy rough interior of A is denoted by **FRint(A)**.

(ii) Fuzzy rough closure of A is denoted by **FRcl(A)**.

(iii) Fuzzy rough open set of A is denoted by **FROS(A)**.

(iv) Fuzzy rough closed set of A is denoted by **FRCS(A)**.

Definition:3.1

Let (X,τ) be a fuzzy rough topological space. Let A = (A_L, A_U) be a fuzzy rough set in X. Then A said to be ***fuzzy rough 𝓅-open set***, if

A⊆ FRint (FRcl(A)).

Example:3.1

Let X={a,b,c} and A, B be fuzzy rough sets of X defined as

$A_L(a) = 0.2$ $A_L(b) = 0.3$ $A_L(c) = 0.5$

$A_U(a) = 0.3$ $A_U(b) = 0.4$ $A_U(c) = 0.6$

$B_L(a) = 0.3$ $B_L(b) = 0.4$ $B_L(c) = 0.6$

$B_U(a) = 0.3$ $B_U(b) = 0.5$ $B_U(c) = 0.7$

Let $\tau = \{\tilde{0}, \tilde{1}, A, B\}$ be a fuzzy rough topology on X.

Then (X, τ) is a fuzzy rough topological space.

Let C be a fuzzy rough set defined as

$C_L(a) = 0$ $C_L(b) = 0.1$ $C_L(c) = 0.2$

$C_U(a) = 0.4$ $C_U(b) = 0.4$ $C_U(c) = 0.3$

Then C is a fuzzy rough $\wp$-open set in X.

Definition:3.2

The complement of a fuzzy rough $\wp$-open set is said to be **fuzzy rough $\wp$-closed set**, if

$$A \supseteq FRcl\,(FRint(A)).$$

Example:3.2

Let X={c,d} and C=(C_L,C_U), D=(D_L,D_U) be fuzzy rough sets of X defined as

$C_L(c) = 0.2$ $C_L(d) = 0.5$

$C_U(c) = 0.5$ $C_U(d) = 0.5$

$D_L(c) = 0.5$ $D_L(d) = 0.5$

$D_U(c) = 0.5$ $D_U(d) = 0.5$

Let $\tau = \{\tilde{0}, \tilde{1}, C, D\}$ be a fuzzy rough topology on X.

Then (X, τ) is a fuzzy rough topological space.

Let E=(E_L,E_U) be a fuzzy rough set defined as

$E_L(c) = 0.6$ $E_L(d) = 0.5$

$E_U(c) = 0.5$ $E_U(d) = 0.5$

Then E is a fuzzy rough $\wp$-closed set in X.

Definition:3.3

Let (X,τ) be a fuzzy rough topological space. Let A = (A_L, A_U) be a fuzzy rough set in X. The **fuzzy rough $\wp$-interoir** of A is denoted by FR $\wp$-int(A) and is defined by

$$FR\ \wp\text{-int}(A) = \bigcup\{B = (B_L, B_U) \text{ is a fuzzy rough } \wp\text{-open set in X and } B \subseteq A\}.$$

Definition:3.4

Let (X,τ) be a fuzzy rough topological space. Let A = (A_L, A_U) be a fuzzy rough set in X. The **fuzzy rough $\wp$-closure** of A is denoted by FR $\wp$-cl(A) and is defined by

$$FR\ \wp\text{-cl}(A) = \bigcap\{B = (B_L, B_U) \text{ is a fuzzy rough } \wp\text{-open set in X and } A \subseteq B\}.$$

Proposition:3.1

Let (X,τ) be a fuzzy rough topological space. For any two fuzzy rough sets A = (A_L, A_U) and B = (B_L, B_U) of an fuzzy rough topological space(X,τ) then the following statements are true.

(i) $FR\wp - cl(0_\sim) = 0_\sim$.

(ii) $A \subseteq B \Longrightarrow FR\wp - cl(A) \subseteq FR\wp - cl(B)$.

(iii) $FR\wp - cl[FR\wp - cl(A)] = FR\wp - cl(A)$.

(iv) $FR\wp - cl(A \cup B) \supseteq [FR\wp - cl(A)] \cup [FR\wp - cl(B)]$.

(v) $FR\wp - cl(A \cap B) \subseteq [FR\wp - cl(A)] \cap [FR\wp - cl(B)]$.

Proof:

(i)Since $0_\sim$ itself is a fuzzy rough $\wp$ closed set.

$$FR\wp - cl(0_\sim) = 0_\sim$$

(ii) $A \subseteq B$

$FR\wp - cl(A) = \cap\{K; K = (K_L, K_U)$ is a fuzzy rough $\wp$ closed set $in\ X\ and\ A \subseteq K\}$

$\subseteq \cap\{K; K = (K_L, K_U)$ is a fuzzy rough $\wp$ closed set $in\ X\ and\ B \subseteq K\}$

$\subseteq FR\wp - cl(B)$

(iii) Since $FR\wp - cl(A)$ is a fuzzy rough $\wp$ closed set in X

$$FR\wp - cl[FR\wp - cl(A)] = FR\wp - cl(A)$$

(iv) $FR\wp - cl(A \cup B)$

$= \cap\{K; K = (K_L, K_U)$ is a fuzzy rough $\wp$ closed $set\ in\ X\ and\ (A \cup B) \subseteq K\}$

$\supseteq [\cap\{K; K = (K_L, K_U)$ is a fuzzy rough $\wp$ closed set in X and $A \subseteq K\}] \cup$
$[\cap\{K; K = (K_L, K_U)$ is a fuzzy rough $\wp$ closed set in X and $B \subseteq K\}]$

$\supseteq [FR\wp - cl(A)] \cup [FR\wp - cl(B)]$.

(v) $FR\wp - cl(A \cap B)$

$= \cap\{C; C = (C_L, C_U)$ is a fuzzy rough $\wp$ closed set in X and $(A \cap B) \subseteq C\}$

$\subseteq [\cap \{C; C = (C_L, C_U)$ is a fuzzy rough $\wp$ closed set in X and $A \subseteq C\}] \cap$
$[\cap \{C; C = (C_L, C_U)$ is a fuzzy rough $\wp$ closed set in X and $B \subseteq C\}]$
$\subseteq [FR\wp - cl(A)] \cap [FR\wp - cl(B)]$

Proposition:3.2

Let (X, τ) be a fuzzy rough topological space. Let $A = (A_L, A_U)$ and $B = (B_L, B_U)$ are fuzzy rough sets in fuzzy rough topological space (X, τ). Then the following statements are true.

(i) $FR\wp\text{-}int(A)$ is the largest fuzzy rough $\wp$ open set contained in A.
(ii) If A is a fuzzy rough $\wp$ open set then $A = FR\wp\text{-}int(A)$.
(iii) If A is a fuzzy rough $\wp$ open set then $FR\beta\text{-}int[FR\wp\text{-}int(A)] = FR\wp\text{-}int(A)$.
(iv) $1_\sim\text{-}FR\wp\text{-}int(A) = FR\wp\text{-}cl(1_\sim\text{-A})$.
(v) $1_\sim\text{-}FR\wp\text{-}cl(A) = FR\wp\text{-}int(1_\sim\text{-A})$.
(vi) If $A \subseteq B \Longrightarrow FR\wp - int(A) \subseteq FR\wp - int(B)$.
(vii) $[FR\wp - int(A)] \cup [FR\beta - int(B)] \subseteq FR\wp - int(A \cup B)$
(viii) $[FR\wp - int(A)] \cap [FR\wp - int(B)] \supseteq FR\wp - int(A \cap B)$.

Proof:

The proof of the (i) and (ii) are trivial.
The proof of the (iii) follows from (i) and (ii).

(iv) $1_\sim\text{-}FR\wp\text{-}int(A)$
$=1_\sim - \cup \{B; B = (B_L, B_U)$ is a fuzzy rough $\wp$ open set in X and $B \subseteq A\}$
$=\cap \{B; B = (B_L, B_U)$ is a fuzzy rough $\wp$ closed set in X and $B \supseteq 1_\sim$-A$\}$
$= FR\wp\text{-}cl(1_\sim\text{-A})$.

(v) $1_\sim\text{-}FR\wp\text{-}cl(A)$
$= 1_\sim - \cap \{B; B = (B_L, B_U)$ is a fuzzy rough $\wp$ open set in X and $B \supseteq A\}$
$=\cup \{B; B = (B_L, B_U)$ is a fuzzy rough $\wp$ open set in X and $B \subseteq 1_\sim$-A$\}$
$= FR\wp\text{-}int(1_\sim\text{-A})$.

(vi) $A \subseteq B$
$FR\wp - int(A) = \cup \{C: C = (C_L, C_U)$ is a fuzzy rough $\wp$ open set in X and $C \subseteq A\}$
$\subseteq \cup \{C: C = (C_L, C_U)$ is a fuzzy rough $\wp$ open set in X and $C \subseteq B\}$
$\subseteq FR\wp - int(B)$.

(vii) $FR\wp - int(A \cup B)$
$= \cup \{C: C = (C_L, C_U)$ is a fuzzy rough $\wp$ open set in X and $C \subseteq (A \cup B)\}$
$\supseteq [\cup \{C: C = (C_L, C_U)$ is a fuzzy rough $\wp$ open set in X and $C \subseteq A\}] \cup$
$[\cup \{C: C = (C_L, C_U)$ is a fuzzy rough $\wp$ open set in X and $C \subseteq B\}]$
$\supseteq [FR\wp - int(A)] \cup [FR\wp - int(B)]$.

(viii) $FR\wp - int(A \cap B)$
$= \{C: C = (C_L, C_U)$ is a fuzzy rough $\wp$ open set in X and $C \subseteq (A \cap B)\}$
$\subseteq [\cup \{C: C = (C_L, C_U)$ is a fuzzy rough $\wp$ open set in X and $C \subseteq A\}] \cap$
$[\cup \{C: C = (C_L, C_U)$ is a fuzzyrough $\wp$ open set in X and $C \subseteq B\}]$
$\subseteq [FR\wp - int(A)] \cap [FR\wp - int(B)]$.

CONCLUSION:

It is known that various types of functions play a significant role in the theory of classical point set topology and engineering, economics etc. A great number of papers dealing with such functions have appeared and a good many of them have been extended to the fuzzy topological spaces, rough topological spaces and fuzzy rough topological spaces by workers. The purpose of the present paper is to define $\wp$-open sets in fuzzy rough topological spaces

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