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A New Class of Strongly e^* Continuous Functions in Topological Spaces

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Abstract

In this paper we introduce some new class of functions called strongly e* continuous functions in generalized topological spaces by using e*-open sets, e-continuous function and e*-continuous functions and some interesting properties are investigated.

Keywords: e*-open set, e-continuous function, e*-continuous function, strongly e* continuous function and e^*- Irresolute.

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1. INTRODUCTION

Topology is the mathematical study of the properties that are preserved through deformations, twisting and stretching's of objects. Topology began with the study of curves, surfaces and other objects in the plane and their space. The story of generalized topology goes back to 1963. That year, N. Levine in the paper "Semi-open Sets and Semi-continuity in Topological Spaces" tried to generalize a topology by replacing open sets with semi-open sets [5]. The concept of "generalized topology" was revised by him in 2002. In 1960, N. Levine introduced strong continuity in topological spaces [6]. Strongly α* continuous functions in topological spaces is introduced by S. Pious Missier and P. Anbarasi Rodrigo [8]. In this project and a new class of functions called strongly e^* - continuous functions is studied. Also e^* open and e^* closed functions and their relations with various functions are studied and investigated their properties.

2. PRELIMINARIES

Throughout this paper (X, τ), (Y, σ) and (Z, η) or X, Y, Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively.

DEFINITION 2.1 [3]

A function f : X → Y is said to be an **open map** if for each open set O of (X, τ), the set f(O) is open in (Y, σ).

DEFINITION 2.2 [4]

Let X be a non-empty set. A generalized topology on the set X is a collection μ of subsets of P(X) having the following properties:

(a) ϕ is in μ.
(b) Any union of the elements of μ belongs to μ.

Clearly, μ is a generalized topology on X. the ordered pair (X, μ) is called a generalized topological space.

DEFINITION 2.3 [2]

A subset A of X is said to be **regular open** in (X, τ) if **A = int(cl(A)).**

DEFINITION 2.4 [2]

A subset A of X is said to be ***e* - open** if **A** $\subseteq$ **cl(*δ*-int(A))** ∪ **int(*δ*-cl(A)).**

DEFINITION 2.5 [2]

A subset A of X is said to be **e^*- open** in (X, τ) if **A ⊆ cl(int(*δ*-cl(A))).**

DEFINITION 2.6 [1]

A subset A of a X is said to be **generalized open (g- open)** in (X, τ) if **int(A) ⊇ U** whenever **A ⊇ U** and U is closed in (X, τ).

DEFINITION 2.7 [5]

A function f : (X, τ) → (Y, σ) is said to be **e^*-Irresolute** if f^{-1}(O) is e^*- open in (X, τ) for every e^*- open set O in (Y, σ).

DEFINITION 2.8 [6]

A function f : (X, τ) → (Y, σ) is called a **strongly continuous** if

f^{-1}(O) is both open and closed in (X, τ) for each subset O in (Y, σ).

DEFINITION 2.9 [5]

A function f : (X, τ) → (Y, σ) is said to be ***e* – continuous** if f^{-1}(O) is e - open in (X, τ) for every open set O in (Y, σ).

DEFINITION 2.10 [5]

A function f : (X, τ) → (Y, σ) is said to be ***e****- continuous** if f^{-1}(O) is e^*- open of (X, τ) for every open set O of (Y, σ).

PROPOSITION 2.1 [2]

Every open set is e^*- open set and every closed set is e^*- closed set.

3. SOME PROPERTIES OF STRONGLY e^* CONTINUOUS FUNCTIONS:

In this section, the concepts of strongly e*-continuous functions, e*-continuous functions, e*-irresolute functions are introduced and some of its properties are also established.

DEFINITION 3.1

A function f: (X, μ_1) → (Y, μ_2) is called a **strongly *e****- continuous** if the inverse image of every e^*-open set in (Y, μ_2) is open in (X, μ_1).

DEFINITION 3.2

A function f : (X, μ_1) → (Y, μ_2) is said to be ***e****- continuous** if f^{-1}(O) is e^*- open of (X, μ_1) for every open set O of (Y, μ_2).

DEFINITION 3.3

A function f : (X, μ_1) → (Y, μ_2) is said to be ***e****- irresolute** if f^{-1}(O) is e^*-open in (X, μ_1) for every e^*-open set O in (Y,μ_2).

PROPOSITION 3.1

A function f : (X, μ_1) → (Y, μ_2) from a generalized topological spaces (X, μ_1) into a generalized topological spaces (Y, μ_2) is strongly e^*- continuous if and only if the inverse image of every e^*- closed set in (Y, μ_2) is closed in (X, μ_1).

PROOF:

Assume that f is strongly e^*- continuous. Let O be any e^*-closed set in (Y, μ_2) . Then, O^c is e^*- open set in (Y, μ_2) . Since f is strongly e^*continuous, f^{-1} (O^c) is open in (X, μ_1). But f^{-1} (O^c) = X - f^{-1} (O) and so f^{-1} (O) is closed in (X, μ_1).

Conversely, Assume that the inverse image of every e^*-closed set in (Y, μ_2) is closed in (X, μ_1).

Let O be any e^*- open set in(Y, μ_2). Then, O^c is e^*- closed set in (Y, μ_2). By assumption, f^{-1} (O^c) is closed in X. But f^{-1} (O^c) = X - f^{-1} (O) and f^{-1} (O) is open in (X, μ_1) .

Therefore, f is strongly e^*- continuous.

PROPOSITION 3.2

If a function f : (X, μ_1) → (Y, μ_2) is strongly e^*- continuous then it is e^*- continuous function.

PROOF:

Let O be an open set in (Y, μ_2). **By the proposition 2.1,** O is e^*- open in (Y, μ_2).

Since f is strongly e^* continuous, f^{-1} (O) is open in (X, μ_1) then, **by the proposition 2.1**, f^{-1} (O) is e^*- open in (X, μ_1) .

Therefore, f is e^*- continuous.

PROPOSITION 3.3

If a function f : (X, μ_1) → (Y, μ_2) is strongly e^*- continuous and the function g : (Y, μ_2) → (Z, μ_3) is e^*- continuous then $g \circ f$: (X, μ_1) → (Z,μ_3) is continuous.

PROOF:

Let O be any open set in (Z, μ_3). Since g ise^* - continuous, g^{-1} (O) is e^*- open in (Y, μ_2). Since f is strongly e^*-continuous, $f^{-1}(g^{-1}$(O)) is open in (X, μ_1). But $f^{-1}(g^{-1}$(O)) = $(g \circ f)^{-1}$(O)

Therefore, $g \circ f$ is continuous.

PROPOSITION 3.4

If a function f : (X, μ_1) → (Y, μ_2) is strongly e^*- continuous and the function g : (Y, μ_2) → (Z, μ_3) is e^*- irresolute then $g \circ f$: (X, μ_1) → (Z, μ_3) is strongly e^*- continuous.

PROOF:

Let O be e^*- open set in (Z, μ_3). Since g is e^*- irresolute, g^{-1} (O) is e^*- open in (Y, μ_2) . Since f is strongly e^*- continuous, $f^{-1}(g^{-1}$(O)) is open in (X, μ_1). But $f^{-1}(g^{-1}$(O)) = $(g \circ f)^{-1}$(O) is open in (X, μ_1).

Therefore, $g \circ f$ is strongly e^*-continuous.

PROPOSITION 3.5

If a function $f : (X, \mu_1) \rightarrow (Y, \mu_2)$ is strongly e^*- continuous and the function $g : (Y, \mu_2) \rightarrow (Z, \mu_3)$ is strongly e^*-continuous then $g \circ f : (X, \mu_1) \rightarrow (Z, \mu_3)$ is e^*- irresolute.

PROOF:

Let O be e^* - open set in (Z, μ_3). Since g is strongly e^*-continuous, g^{-1} (O) is open in (Y, μ_2). Since f is strongly e^*-continuous, $f^{-1}(g^{-1}(O))$ is e^* - open in (X, μ_1). But $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is e^* - open in (X, μ_1).

Therefore, $g \circ f$ is e^*- irresolute.

PROPOSITION 3.6

The composition of two strongly e^*- continuous functions is strongly e^*-continuous.

PROOF:

Let O be e^*-open set in (Z, μ_3). Since g is strongly e^* - continuous, g^{-1} (O) is open in (Y, μ_2). **By the proposition 2.1,** g^{-1} (O) is e^*- open in (Y, μ_2). Since f is strongly e^*-continuous, $f^{-1}(g^{-1}(O))$ is open in (X, μ_1). But $f^{-1}(g^{-1}(O)) = (g \circ f)^{-1}(O)$ is open in (X, μ_1).

Therefore, $g \circ f$ is strongly e^*-continuous.

CONCLUSION

Different types of generalizations of continuous functions were introduced and studied by various authors in the recent years. The purpose of the present paper is to define strongly e^*continuous functions. Also e^* open and e^* closed functions and their relations with various functions are studied and investigated their properties.

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