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Calculating Optimum Gear Ratios of Mechanical Driven Systems Using Worm-Helical Gearbox and Chain Drive

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Abstract

This paper presents a study on the determination of the optimum gear ratios of mechanical driven systems using a worm-helical gearbox and a chain drive. In the study, an optimization problem was performed to find the optimum gear ratios. In addition, the length of the mechanical system was selected as the objective function of the optimization problem. The effects of the input factors including the total transmission ratio of the system, the calculation coefficient of the worm diameter coefficient, the coefficient of wheel face width and the allowable contact stress of helical gear unit, and the output torque were investigated. To evaluate the influences of these factors on the optimum gear ratios, a simulation experiment was designed and conducted by computer programs. Significantly, several models to determine the optimum gear ratios were proposed. Using these models, the determination of optimum gear ratios is accurate and simple.

Keywords: Optimum gearbox design; gear ratio; optimum gear ratio; worm-helical gearbox.

1. INTRODUCTION

In optimum design of a gearbox or a mechanical driven system, the determination of the optimum gear ratios has a very important role. This is due to the large dependence of the dimention, the mass and, therefore, their cost on the gear ratios of the gearbox of the system. Heretofore, several studies have been done to identify the optimum gear ratios.

The optimum gear ratios can be found by using different methods. The methods can be the graph method [1, 2], the practical method [3] or the model method [3-15]. Besides, the gear ratios have been found for different types of gearboxes. Concerning helical gearboxes, the optimum gear ratios were determined for two step gearboxes [3-7], three step gearboxes [8-14] and four step gearboxes [12-18]. In addition, the gear ratios have been found for different objectives such as the minimum gear mass [9, 12, 13, 15, 17], the minimum area of gearbox cross section [8, 11] and the minimum gearbox length [7, 10].

Regarding worm gearboxes, considerable attention has been paid to this area. For a two step worm gearbox, the gear ratios can be determined by a pratical equation [3] or calculated to get the reasonable housing structure by the following equation [22]:

$$u_1 \approx u_2 \approx \left(u_h\right)^{1/2} \qquad (1)$$

Furthermore, for this type of gearbox, the optimum gear ratio of the second step was determined by $u_2 \approx 30.97$ [23].

For worm-helical gearboxes, the optimum gear ratio of the helical gear unit is found by the following practical model [22]:

$$u_2 = \left(0.03 \div 0.06\right) u_h \qquad (2)$$

Also, to obtain a good condition for oil lubrication for both steps, the gear ratio of the worm gear unit is determined by a graph [2] or the optimum gear ratio of the helical gear unit u_2 is calculated by [22]:

$$u_2 = 6.86 \cdot \psi_{ba2}^{1/2} \qquad (3)$$

In which, $\psi_{ba2} = 0.3 \ldots 0.4$ is the wheel face width coefficient of the helical gear unit.

For worm-helical gearbox, the optimum gear ratio of the second step u_2 is found as $u_2 = \left[u_{2\max}\right] = 8 \ldots 10$ [24].

In regard to mechanical driven systems, several studies have been proposed on calculating the optimum gear ratios of systems containing a gearbox and a V-belt drive [19, 20] or a chain drive [21].

In this paper, an optimization study on obtaining the optimum gear ratios of mechanical driven systems using a worm-helical gearbox and a chain drive was introduced. The objective of the optimization problem in the study is to attain the minimum system length. Besides, the influences of the input factors including the total gearbox ratio, the calculation coefficient of the worm diameter coefficient, the coefficient of wheel face width and the allowable contact stress of helical gear unit, and

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the output torque were explored. To assess these effects, a simulation experiment was planned and conducted by computer programming. Notably, equations to calculate the optimum gear ratios were proposed. Using these equations, the optimum gear ratios will be achieved accurately and simply as they are explicit.

2. OPTIMIZATION PROBLEM

The length of a mechanical driven system using a worm-helical gearbox and a chain drive is calculated by (Figure 1):

$$L = \max\left(L_g, L_{cg}\right) \qquad (1)$$

In which, L_g and L_{cg} are determined by the following equations (see Figure 1):

$$L_g = d_{w21}/2 + a_{w2} + d_{w22}/2 \qquad (2)$$

$$L_{cg} = d_{w22}/2 + a_c + d_2/2 \qquad (3)$$

In the above equations, a_{w2} and d_{w22} are the center distance and the pitch diameter of the driven gear of the helical gear unit. d_{w21} the pitch diameter of the worm unit; d_{w21} is calculated by the following equation [26]:

$$d_{w21} = m \cdot z_2 = 2 \cdot a_{w1} \cdot z_2 / \left(z_2 + q\right) \qquad (4)$$

In which, q is the coefficient of the worm pitch diameter; q is determined by [26]:

$$q = k_q \cdot z_2 \qquad (5)$$

Where, k_q is the coefficient for obtaining q;
$k_q = 0.25 \ldots 0.3$ [25].

For helical gear unit, the driven diameter d_{w22} is found by [25]:

$$d_{w22} = 2 \cdot a_{w2} \cdot u_2 / \left(u_2 + 1\right) \qquad (6)$$

Therefore, the optimization problem can be defined as:

$$\text{minimize } L \qquad (7)$$

With the following constraints

$$1 \le u_c \le 6$$

$$8 \le u_1 \le 80 \qquad (8)$$

$$1 \le u_2 \le 9$$

It is certain that to solve the above optimization problem, it is required to determine the center distances and the pitch diameters of the worm gear unit and the helical gear unit.

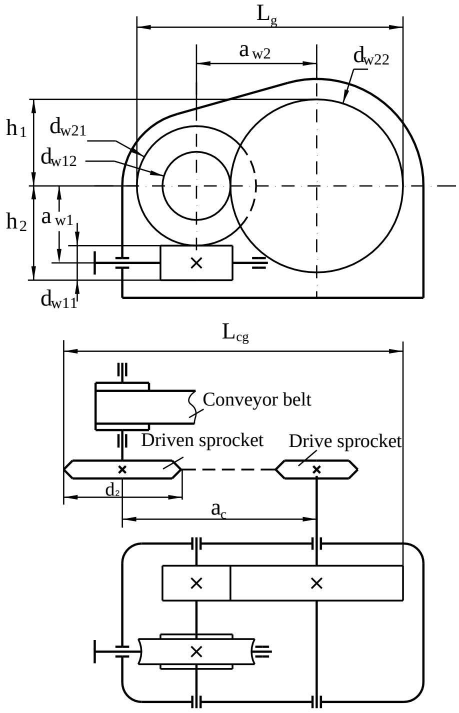

Figure 1. Calculation schema

2.1 Determining the center distance and the pitch diameters of the worm gear set

The center distance of the worm gear unit (mm) is calculated by [25]:

$$a_{w1} = K_a \cdot \left(K_{HV} \cdot K_{H\beta} \cdot T_{21} / \left[\sigma_{H1}\right]^2\right)^{1/3} \qquad (9)$$

In which, K_a is a coefficient; $K_a = 610$ [25]; K_{hV} is the internal dynamic coefficient; $K_{hV} = 1 \ldots 1.3$ [25]; $K_{h\beta}$ is the coefficient of the load concentration; $K_{h\beta} = 1 \ldots 1.3$ [25]; T_{21} is the wheel torque (Nmm); With the worm-helical gearbox, T_{21} is determined by the following equation:

$$T_{21} = T_{out} / \left(u_2 \cdot u_c \cdot \eta_{br} \cdot \eta_c \cdot \eta_o^2\right) \qquad (10)$$

Where, T_{out} is the system output torque (Nmm); η_{hg} is the transmission efficiency of the helical gear unit ($\eta_{hg} = 0.96 \ldots 0.98$ [26]); η_c is the transmission efficiency of the chain drive ($\eta_c = 0.95 \ldots 0.97$ [26]); η_b is the transmission efficiency of a rolling bearing pair ($\eta_b = 0.99 \ldots 0.995$ [26]). Choosing $\eta_{hg} = 0.97$, $\eta_c = 0.96$, $\eta_b = 0.992$ and substituting them into (10) gets:

$$T_{21} = 1.0913 \cdot T_{out} / \left(u_2 \cdot u_c\right) \qquad (11)$$

Choosing $K_{hV} = 1.2$, $K_{h\beta} = 1.2$ and substituting them, K_a and T_{21} into (9) gives:

$$a_{w1} = 709.1945 \cdot \left[T_{21} / \left(u_2 \cdot u_c \cdot \left[\sigma_{H1} \right]^2 \right) \right]^{1/3} \quad (12)$$

Wherein, $\left[\sigma_{H1} \right]$ is the allowable contact stress of the worm unit (N/mm^2) which depends on the wheel material. If the wheel material is tinless bronze or soft grey iron, from the data in [26], $\left[\sigma_{H1} \right]$ can be determined by the following regression equation (with the determination coefficient $R^2 = 0.9906$):

$$\left[\sigma_{H1} \right] = 5.0515 \cdot v_{sl}^2 - 49.742 \cdot v_{sl} + 189.9 \quad (13)$$

In which, v_{sl} is the slip velocity which can be calculated as [26]:

$$v_{sl} = 0.0088 \cdot \left(P_1 \cdot u \cdot n_1^2 \right)^{1/3} \quad (14)$$

If the wheel material is tin bronze, $\left[\sigma_{H1} \right]$ is determined by [26]:

$$\left[\sigma_{H1} \right] = K_{HL} \cdot v_{sl} \cdot \left[\sigma_{H0} \right] \quad (15)$$

Where, $\left[\sigma_{H0} \right]$ is the allowable contact stress when the stress change cycle is 10^7:

$$\left[\sigma_{H0} \right] = \left(0.7 \ldots 0.9 \right) \cdot \sigma_t \quad (16)$$

Here, σ_t is the tensile stress (N/mm^2) which depends on v_{sl} ; $\sigma_t = 260$ if $v_{sl} = 5 \ldots 8$; $\sigma_t = 230$ if $v_{sl} = 8 \ldots 12$ and $\sigma_t = 285$ if $v_{sl} = 8 \ldots 25$ [26].

K_{HL} is the service life ratio which is determined as [26]:

$$K_{HL} = \left(10^7 / N_{HE} \right)^{1/8} \quad (17)$$

In which, N_{HE} is the equivalent loading cycle number for the wheel teeth:

$$N_{HE} = 60 \cdot n_2 \cdot t_\Sigma \quad (18)$$

Where, t_Σ is the service lifetime (h); n_2 is the wheel rotational speed (rpm).

2.2 Determining the center distance and the pitch diameters of the helical gear set

The center distance of the helical gear unit a_{w2} is calculated as [26]:

$$a_{w2} = K_a \cdot \left(u_2 + 1 \right) \cdot \left[T_{12} \cdot k_{H\beta} / \left(\left[\sigma_H \right]^2 \cdot u_2 \cdot \psi_{ba2} \right) \right]^{1/3} \quad (19)$$

Where, k_a =43 is the mateial coefficient [26]; $k_{H\beta} = 1.1$ is the coefficient of the load concentration for the helical gear unit [26]; T_{12} is the torque on the drive gear which is obtained by:

$$T_{12} = T_{out} / \left(\eta_{hg} \cdot \eta_c \cdot \eta_b^2 \cdot u_2 \cdot u_c \right) \quad (20)$$

Substituting $\eta_{hg} = 0.97$, $\eta_c = 0.96$ and $\eta_b = 0.992$ (as in section 1.1) into (20) provides

$$T_{12} = 1.0913 \cdot T_{out} / \left(u_2 \cdot u_c \right) \quad (21)$$

Substituting k_a =43 and $k_{H\beta} = 1.1$ and (21) into (19) gives:

$$a_{w1} \approx 45,6998 \cdot \left(u_2 + 1 \right) \cdot \left[T_{out} / \left(u_2^2 \cdot u_c \cdot \left[\sigma_H \right]^2 \cdot \psi_{ba2} \right) \right]^{1/3} \quad (22)$$

2.3 Determining the driven sprocket diameter and the center distance of the chain drive

The pitch diameter of the driven sprocket d_2 can be found by [26]:

$$d_2 = d_1 \cdot u_c \quad (23)$$

In which, d_1 is the pitch diameter of the drive sprocket; d_1 is calculated as [26]:

$$d_1 = p / \sin\left(\pi / z_1 \right) \quad (24)$$

Where, z_1 is the number of teeth of the drive sprocket which can be determined by [21]:

$$z_1 = 32.4 - 2.4 \cdot u_c \quad (25)$$

With p is the chain pitch (mm); p can be found from the design power capacity P calculated as [26]:

$$P = P_1 \cdot k \cdot k_z \cdot k_n \quad (26)$$

Wherein, P_1 is the power rating (kW):

$$P_1 = T_1 \cdot n_1 / \left(9.55 \cdot 10^6 \right) \quad (27)$$

Wherein, n_1 is the drive sprocket revolution (rpm):

$$n_1 = n_m / u_g \quad (28)$$

$$T_1 = T_{out} \cdot \eta_c \cdot \eta_b \quad (29)$$

Where, η_c is the efficiency of the chain drive (η_c =0.95÷0.97 [26]); η_b is the efficiency of a bearing pair (η_b =0.99÷0.995 [26]); T_1 and T_{out} are the drive torque and the output torque, respectively (Nmm).

k, k_z and k_n are coefficients determined by [26]:

$$k = k_d \cdot k_p \cdot k_c \cdot k_{adj} \cdot k_{\text{lub}} \cdot k_{con} \tag{30}$$

$$k_z = 25 / z_1 \tag{31}$$

$$k_n = n_{01} / n_1 \tag{32}$$

In which, k_d, k_p, k_c, k_{adj}, k_{lub} and k_{con} are the coefficients refering to the effects of parameters including the shock factor, the drive position, the center distance, the possibility of center distance adjusting, the lubrication and operating conditions, respectively; n_{01} is tabulated number of the teeth of the drive sprocket.

2.4 Experimental work

Table 1. Input parameters

Factor	Code	Unit	Low	High
Total transmission ratio of the system	u_t	-	50	250
Coefficient for calculation q	K_q	-	0.25	0.3
Coefficient of wheel face width of stage 2	x_{ba2}	-	0.35	0.4
Allowable contact stress of stage 2	AS_2	MPa	350	420
Output torque	T_{out}	Nmm	10^5	10^6

To explore the effects of the input parameters on the optimum gear ratios, a simulation experiment was constructed and conducted. A 2-level full factorial design was selected for the experiment. In this experiment, 5 input parameters were selected for the exploration (Table 1). Subsequently, the experiment was performed with $2^5 = 32$ numbers of runs. Aslo, a computer program was built based on equations (10) and (11) to accomplish the experiment. Table 2 presents different levels of the input parameters and the results of the output response (the optimum gear ratio of the chain drive u_c and the worm gear set u_1).

3. RESULTS AND DISCUSSIONS

3.1. Effect of input parameters on the optimum gear ratio of the chain drive

Figure 2 shows the graph of the main effects for the optimum gear ratio of the chain drive u_c to assess the influence of the input parameters on u_c. From the figure, it is noted that the optimum gear ratio u_c increases considerably with the growth of the output torque T_{out}. However, it grows moderately when the total ratio u_t expands and it rises insignificantly when the allowable contact stress of the helical gear unit AS_2 goes up. Moreover, u_c does not depend on the calculation coefficient of the worm diameter coefficient k_q and the coefficient of wheel face width of the helical gear unit ψ_{ba2}.

Table 2. Experimental plans and output response.

StdOrder	RunOrder	CenterPt	Blocks	u_g	K_q	Xba2	AS_2 (MPa)	Tout (Nm)	u_c	u_2
2	1	1	1	250	0.25	0.35	350	100000	3.1	1.93
16	2	1	1	250	0.3	0.4	420	100000	3.1	1.85
21	3	1	1	50	0.25	0.4	350	1000000	4.6	1.36
23	4	1	1	50	0.3	0.4	350	1000000	4.6	1.36
19	5	1	1	50	0.3	0.35	350	1000000	4.5	1.39
17	6	1	1	50	0.25	0.35	350	1000000	4.5	1.39
…										
9	31	1	1	50	0.25	0.35	420	100000	3.1	1.87
20	32	1	1	250	0.3	0.35	350	1000000	5.7	1.77

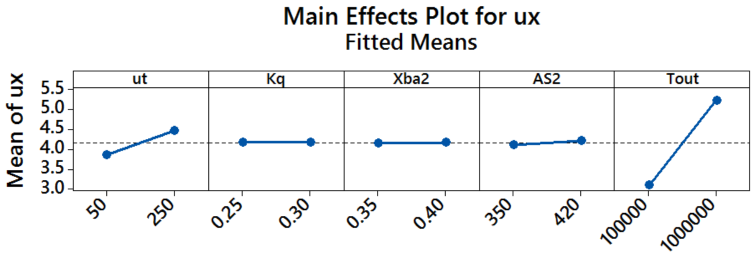

Figure 2. Main effects plot for optimum gear ratio of the chain drive

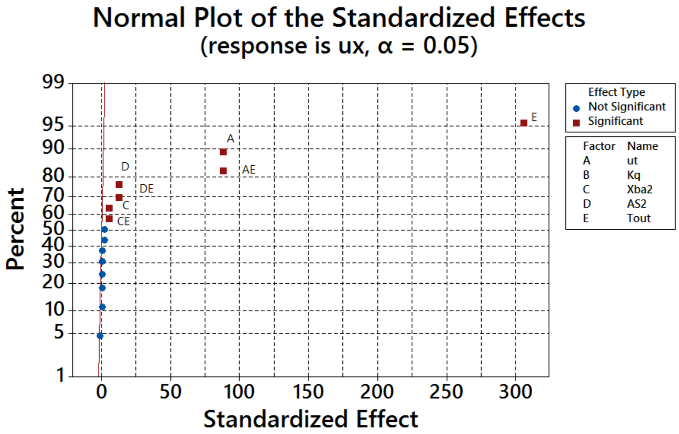

Figure 3. Normal Plot for u_c

Figure 3 shows the Normal Plot of the standardized effects. It can be seen from the figure that the output torque T_{out} (factor E) and then the total system ratio u_t and the interaction AE are the most significant factors for the optimum gear ratio of the chain drive. The factors with less influence are the allowable contact stress of helical gear unit AS_2 (factor D), the coefficient of wheel face width of the helical gear unit ψ_{ba2} (factor C) and the interactions DE and CE. In addition, all these affecting factors have positive standardized effects. If they increase, the optimum gear ratio of the chain drive grows.

Figure 4 presents the estimated effects and coefficients for u_c. It is observed from the figure that the factors which have a significant effect on a response (with P-values lower than 0.05) are the output torque T_{out}, the total system ratio u_t, the allowable contact stress of the helical gear set AS_2, and the interactions $u_t \cdot AS2$, $u_t \cdot T_{out}$, $\psi_{ba2} \cdot T_{out}$ and $AS_2 \cdot T_{out}$. Subsequently, the optimum gear ratio of the chain drive can be calculated by:

$$u_c = 3.134 - 0.000681 \cdot u_t - 0.167 \cdot \psi_{ba2} - 0.000278 \cdot AS_2 - 0.0000001 \cdot T_{out} + {}_{out} + 0.00000001 \cdot u_t \cdot T_{out} + 0.000002 \cdot \psi_{ba2} \cdot T_{out} + 0.0000000 \cdot AS_2 \cdot T_{out} \quad (33)$$

The equation (**33**) fits very well with the experimental data as all of the values of adj-R2 and pred-R2 are above 99.9% (Figure 4). Hence, this equation can be used to calculate the optimum gear ratio of the chain drive u_c.

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		4.16875	0.00361	1155.28	0.000	
ut	0.61250	0.30625	0.00361	84.87	0.000	1.00
Xba2	0.03750	0.01875	0.00361	5.20	0.000	1.00
AS2	0.08750	0.04375	0.00361	12.12	0.000	1.00
Tout	2.13750	1.06875	0.00361	296.18	0.000	1.00
ut*Tout	0.61250	0.30625	0.00361	84.87	0.000	1.00
Xba2*Tout	0.03750	0.01875	0.00361	5.20	0.000	1.00
AS2*Tout	0.08750	0.04375	0.00361	12.12	0.000	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0204124	99.98%	99.97%	99.96%

Figure 4. Estimated Effects and Coefficients for u_c.

3.2. Effect of input parameters on the optimum gear ratio of the helical gear unit

Figure 5 describes the main effect of each input parameter on the optimum gear ratio of the helical gear unit u_2. From the Figure it is reported that u_2 decreases significantly with the

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increase of the output torque T_{out}. Besides, the ratio increases with the growth of the total system ratio u_t. Also, the factors with less influence and in descending order to u_2 include the allowable contact stress AS_2 and the coefficient of wheel face width of the helical gear unit ψ_{ba2}. Moreover, it is observed that u_2 is not affected by the coefficient k_q.

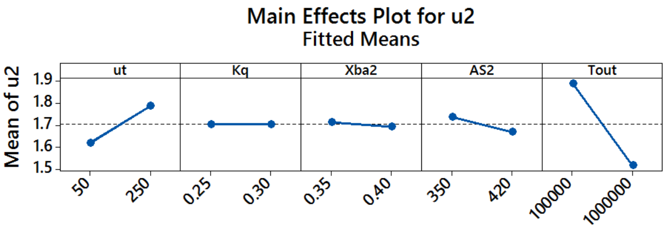

Figure 5. Main effects plot for optimum gear ratio of helical gear unit

The Normal Plot of the standardized effects is shown in Figure 6. It is revealed from the figure that the output torque (factor E), the total system ratio (factor A) and their interaction AE are the most significant factors for the optimum gear ratio of the helical gear unit u_2. Besides, the allowable contact stress AS_2 (factor D), the coefficient of wheel face width ψ_{ba2} (factor C) and the interactions AD and CD have moderate effect on u_2. Furthermore, factors which have a positive effect on u_2 are the total system ratio (factor A) and the interactions AE and CD. The other factors (E, D, C and AD) have a negative effect on u_2.

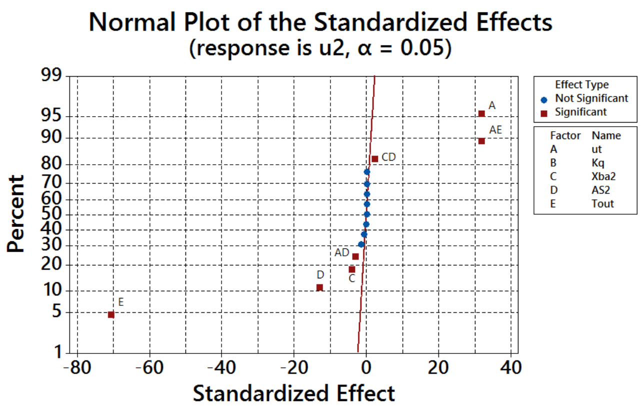

Figure 6. Normal Plot for u_2

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		1.70420	0.00235	726.16	0.000	
ut	0.16660	0.08330	0.00235	35.50	0.000	1.00
Xba2	-0.02127	-0.01064	0.00235	-4.53	0.000	1.00
AS2	-0.06850	-0.03425	0.00235	-14.59	0.000	1.00
Tout	-0.37160	-0.18580	0.00235	-79.17	0.000	1.00
ut*AS2	-0.01650	-0.00825	0.00235	-3.52	0.002	1.00
ut*Tout	0.16660	0.08330	0.00235	35.50	0.000	1.00
Xba2*AS2	0.01127	0.00564	0.00235	2.40	0.024	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0132758	99.74%	99.66%	99.53%

Figure 7. Estimated Effects and Coefficients for u_2

Figure 7 illustrates the estimated effects and coefficients for u_2. From the figure it can be realized that factors which have a significant effect on the response (with P-values lower than 0.05) are the total system ratio u_t, the coefficient of wheel face width ψ_{ba2}, the allowable contact stress AS_2, the output torque T_{out}, and the interactions $u_t \cdot AS2$, $u_t \cdot T_{out}$ and $\psi_{ba2} \cdot AS2$. Hence, the relation between u_2 and the significant effect factors can be described by the following equation:

$$u_2 = 3.289 + 0.000722 \cdot u_t - 2.91 \cdot \psi_{ba2} - 0.00304 \cdot AS_2 - 0.000001 \cdot T_{out} - \\ -0.000002 \cdot u_t \cdot AS_2 + 0.00000001 \cdot u_t \cdot T_{out} + 0.00644 \cdot \psi_{ba2} \cdot AS_2 \tag{24}$$

Equation (24) is used to determined the gear ratio of the helical gear unit u_2. After having u_c (Equation (23)) and u_2 (Equation (24)), the gear ratio of the worm unit can be calculated by:

$$u_1 = u_t / (u_2 \cdot u_c) \tag{25}$$

4. CONCLUSIONS

A study on calculating the optimum gear ratios of mechanical driven systems using a worm-helical gearbox and a chain drive was conducted. In order to do that, an optimization problem in which the length of the mechanical system was carefully chosen as the objective function of the problem was performed. Also, the influences of the input parameters including the total system ratio, the calculation coefficient of the worm diameter coefficient, the coefficient of wheel face width, the helical gear allowable contact stress, and the output torque were taken into account. In addtion, for estimating the effects of these parameters on the optimum gear ratios, a simulation experiment was designed and conducted by computer programming. Additionally, several equations to calculate the optimum gear ratios were suggested. As these equations are explicit, the calculation of optimum gear ratios has become simple and accurate.

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