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The Analysis of Control Systems with a High Potential for Robust Stability on the Control Object Output

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Abstract

This paper considers control systems with an increased potential for robust stability in the output of an object in the class of one-parameter structurally stable mappings from catastrophe theory. The study of the dynamic compensator with a high potential for robust stability is performed by the gradient-velocity method of Lyapunov vector functions. The area of robust stability of the control system for the object output is obtained in the form of a system of the simplest inequalities for the matrix of controller parameters and the monitoring device.

Keywords: *Control Systems, Closed-Loop Control Systems, Lyapunov Vector Function, Gradient-Speed Method*

1. INTRODUCTION

The modern control systems are characterized by the ever-increasing complexity of the control objects as well as the requirements for high efficiency. In these systems, uncertainty can be caused both by the presence of uncontrolled disturbances acting on the control object [1], the lack of knowledge of the true values of the parameters of control objects and unpredictable changes in time [1,2,3,4].

The actual problem is the design of control systems that provide in some sense the best protection against uncertainty in the knowledge of the properties of an object and the instability of control systems.

The ability of a control system to maintain stability under parametric or non-parametric uncertainty is understood as the robustness of the system. In the general formulation of the study of the system for robust stability, it is necessary to indicate the restrictions applied to the fluctuation of uncertain parameters of the control system, under which stability is maintained. A large number of papers have been devoted to the problem of studying robust stability of control systems.

These works mainly investigated the robust stability of polynomials, matrices within the framework of the linear stability principle [2, 3, 4].

It should be noted that the instability is determined by the output of uncertain system parameters beyond the boundaries of robust stability. Known approaches do not allow to expand the area of stability of the system based on the choice of the control law.

In practical tasks related to the development and creation of control systems under conditions of substantial parametric uncertainty, an increase in the robust stability potential [5,6,7,8,9] is one of the key factors guaranteeing the control system from the instability mode. In this regard, under conditions of uncertainty, this paper proposes methods for the synthesis and research of a control system with a guaranteed wide area of robust stability, called control systems with an increased potential for robust stability [5, 6].

The concept of building a control system with a high potential for robust stability is based on the applied results of catastrophe theories [10,11].

This analysis is devoted to building and researching control systems with an increased potential for robust stability with respect to the output of a dynamic volume with uncertain parameters, with an approach to building control systems in the class of one-parameter structurally stable mappings [5, 6, 7], which make it possible to maximally increase the potential of robust control system stability.

2. MATERIAL AND METHODS

The actual problem is the creation of control systems for the object's output, when in practice it is not the state vector that is available for measurement, but the object's output. In this case, the control law does not use the object state variables themselves, but their estimates obtained using an observing device [12,13,14,15] and it is required to build a control system for the output of the object in the form of a dynamic compensator [12] with a high robust potential sustainability.

The paper also proposes a method for the study of stability and synthesis of control systems for the output of an object based on the gradient-velocity method of Lyapunov vector functions [5,8,9,16,17].

The study of the stability of a closed-loop control system for the output of an object and the solution of the problem of regulator synthesis (determining the elements of the amplification matrix) and the observer (calculating the elements of the matrix of the observing device) are based on the direct method of Lyapunov [18,19,20].

The proposed gradient-velocity method of Lyapunov vector functions in the study of the output control system of the object eliminates complex and ambiguous calculations and canonical transformations and allows one to determine the region of choice of controller parameters and the observer, providing the desired transition characteristics of a closed system.

3. RESULTS

To simplify the system, you can transform the equation of state. For this, we use the error of estimation $\varepsilon(t) = x(t) - \hat{x}(t),$ and (1) - (3) can be converted to the form:

$$\frac{dx}{dt} = Ax + Bu, x(t_0) = x_0, \tag{1}$$

$$u = -\hat{x}^3 + k\hat{x} \tag{2}$$

$$\frac{d\varepsilon}{dt} = (A - LC)\varepsilon, \varepsilon(t_0) = x_0 - \hat{x}_0 \tag{3}$$

Here, the control law is specified in the form of one-parameter structurally stable mappings with respect to the state vector $x(t)$ и and the error vector $\varepsilon(t)$ $\hat{x}(t) = x(t) - \varepsilon(t)$, and $u(t) = -x^3 + kx - \varepsilon^3 + k\varepsilon - 3x\varepsilon(x+\varepsilon)$.

(1)-(3) present in the form:

$$\begin{aligned}\frac{dx}{dt} &= Ax + B(-x^3 + kx) + B(-\varepsilon^3 + K\varepsilon) - \\ &- 3Bx\varepsilon(x+\varepsilon), x(t_0) = x_0\end{aligned} \tag{4}$$

$$\frac{d\varepsilon}{dt} = A\varepsilon - LC\varepsilon, \varepsilon(t_0) = \varepsilon_0 \tag{5}$$

Let's consider a system with one input and one output, respectively, the system has a matrix of the form:

$$A = \begin{Vmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{Vmatrix}, \; , C = \| c_1, c_2, \dots, c_n \|$$

$$B = \begin{Vmatrix} 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \\ b_n & b_{n-1} & b_{n-2} & \dots & b_1 \end{Vmatrix}, L = \begin{Vmatrix} 0 \\ 0 \\ \dots \\ 0 \\ l_n \end{Vmatrix}$$

System (4), (5) in the expanded form we will present in the form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = -(a_n + b_n k_1)x_1 - (a_{n-1} + b_n k_2)x_2 - \\ -(a_{n-2} + b_n k_3)x_3 -, \dots, -(a_1 + b_n k_n)x_n + \\ + b_n k_1 \varepsilon_1 + + b_n k_2 \varepsilon_2 + b_n k_3 \varepsilon_3, \dots, + b_n k_n \varepsilon_n \\ \varepsilon_1 = \varepsilon_2 \\ \varepsilon_2 = \varepsilon_3 \\ \dots \\ \varepsilon_{n-1} = \varepsilon_n \\ \dot{\varepsilon}_n = -(a_n + l_n c_1)\varepsilon_1 - (a_{n-1} + l_n c_2)\varepsilon_2 - \\ -(a_{n-2} + l_n c_3)\varepsilon_3 -, \dots, -(a_1 + l_n c_n)\varepsilon_n \end{cases} \tag{6}$$

In the absence of external influences, the processes in system (4) and (5) should asymptotically approach the processes in the system with a regulator as it was, as if the closed-loop control system of the state vector was exposed to damped disturbances. The role of these disturbances is played by the component dictated by equation (6). The error must be subside and the error subsidence rate is determined during the observer synthesis. The main property of system (4), (5) or (6) is asymptotic stability. Find the steady state of the system (6):

$$x_{1s} = 0, x_{2s} = 0, \dots, x_{ns} = 0 \text{ и } \varepsilon_{1s} = 0, \varepsilon_{2s} = 0, \dots, \varepsilon_{ns} = 0 \tag{7}$$

and

$$x_{is}^{1,2} = \pm\sqrt{\frac{a_{n-i+1}}{b_{n-i+1}} + k_i}, \varepsilon_{is} = 0, i = 1, \dots, n \tag{8}$$

We study the robust stability of steady-state states of system (6) by the gradient-velocity method of Lyapunov vector functions [5, 7, 16]. The steady state (7) is oberved from the beginning. The next step is to find the gradient vector of the vector Lyapunov functions from the equation of state (6) of the components $V(x,\varepsilon)=(V_1(x,\varepsilon),V_2(x,\varepsilon),...,V_{2n}(x,\varepsilon))$:

$$\begin{cases}
\frac{\partial V_1(x,\varepsilon)}{\partial x_1}=0,\frac{\partial V_1(x,\varepsilon)}{\partial x_2}=-x_2,\frac{\partial V_1(x,\varepsilon)}{\partial x_3}=0,...,\frac{\partial V_1(x,\varepsilon)}{\partial x_n}=0,\\
\frac{\partial V_1(x,\varepsilon)}{\partial \varepsilon_1}=0,\frac{\partial V_1(x,\varepsilon)}{\partial \varepsilon_2}=0,\frac{\partial V_1(x,\varepsilon)}{\partial \varepsilon_3}=0,...,\frac{\partial V_1(x,\varepsilon)}{\partial \varepsilon_n}=0\\
\frac{\partial V_2(x,\varepsilon)}{\partial x_1}=0,\frac{\partial V_2(x,\varepsilon)}{\partial x_2}=0,\frac{\partial V_2(x,\varepsilon)}{\partial x_3}=-x_3,...,\frac{\partial V_2(x,\varepsilon)}{\partial x_n}=0,\\
\frac{\partial V_2(x,\varepsilon)}{\partial \varepsilon_1}=0,\frac{\partial V_2(x,\varepsilon)}{\partial \varepsilon_2}=0,\frac{\partial V_2(x,\varepsilon)}{\partial \varepsilon_3}=0,...,\frac{\partial V_2(x,\varepsilon)}{\partial \varepsilon_n}=0...\\
\frac{\partial V_n(x,\varepsilon)}{\partial x_1}=b_nx_1^3-(a_n+b_nk_1)x_1,\frac{\partial V_n(x,\varepsilon)}{\partial x_2}=b_{n-1}x_2^3-(a_{n-1}+b_{n-1}k_2)x_2,\\
\frac{\partial V_n(x,\varepsilon)}{\partial x_3}=b_{n-2}x_3^3-(a_{n-2}+b_{n-2}k_3)x_3,...,\frac{\partial V_n(x,\varepsilon)}{\partial x_n}=b_1x_n^3-(a_1+b_1k_n)x_n,\\
\frac{\partial V_n(x,\varepsilon)}{\partial \varepsilon_1}=b_n\varepsilon_1^3-b_nk_1\varepsilon_1-3b_nx_1\varepsilon_1(x_1+\varepsilon_1),\\
\frac{\partial V_n(x,\varepsilon)}{\partial \varepsilon_2}=b_{n-1}\varepsilon_2^3-b_{n-1}k_2\varepsilon_2-3b_{n-1}x_2\varepsilon_2(x_2+\varepsilon_2),\\
\frac{\partial V_n(x,\varepsilon)}{\partial \varepsilon_3}=-b_{n-2}\varepsilon_3^3-b_{n-2}k_3\varepsilon_3-3b_{n-2}x_3\varepsilon_3(x_3+\varepsilon_3),...,\\
\frac{\partial V_n(x,\varepsilon)}{\partial \varepsilon_n}=-b_1\varepsilon_n^3-b_1k_n\varepsilon_n-3b_1x_n\varepsilon_n(x_n-\varepsilon_n),...,\\
\frac{\partial V_{n+1}(x,\varepsilon)}{\partial \varepsilon_1}=0,\frac{\partial V_{n+1}(x,\varepsilon)}{\partial \varepsilon_2}=-\varepsilon_2,\frac{\partial V_{n+1}(x,\varepsilon)}{\partial \varepsilon_3}=0,...,\frac{\partial V_{n+1}(x,\varepsilon)}{\partial \varepsilon_n}=0,\\
\frac{\partial V_{n+1}(x,\varepsilon)}{\partial x_i}=0,i=1,...,n;\\
\frac{\partial V_{n+2}(x,\varepsilon)}{\partial \varepsilon_1}=0,\frac{\partial V_{n+2}(x,\varepsilon)}{\partial \varepsilon_2}=0,\frac{\partial V_{n+2}(x,\varepsilon)}{\partial \varepsilon_3}=-\varepsilon_3,...,\frac{\partial V_{n+2}(x,\varepsilon)}{\partial \varepsilon_n}=0,..........\\
\frac{\partial V_{2n}(x,\varepsilon)}{\partial x_i}=0,i=1,...,n;\\
\frac{\partial V_{2n}(x,\varepsilon)}{\partial \varepsilon_1}=-(a_n-l_nc_1)\varepsilon_1,\frac{\partial V_{2n}(x,\varepsilon)}{\partial \varepsilon_2}=-(a_{n-1}-l_nc_2)\varepsilon_2,\\
\frac{\partial V_{2n}(x,\varepsilon)}{\partial \varepsilon_3}=-(a_{n-2}-l_nc_3)\varepsilon_3,...,\frac{\partial V_{2n}(x,\varepsilon)}{\partial \varepsilon_n}=-(a_1-l_nc_n)\varepsilon_n
\end{cases} \quad (9)$$

From (6) the decomposition of the components of the velocity vector can be represented as:

$$\begin{cases} \left(\frac{dx_1}{dt}\right)_{x_2} = x_2, \left(\frac{dx_2}{dt}\right)_{x_3} = x_3, \\ \ldots \\ \left(\frac{dx_n}{dt}\right)_{x_1} = -b_n x_1^3 + (a_n + b_n k_1)x_1, \left(\frac{dx_n}{dt}\right)_{x_2} = -b_{n-1}x_2^3 + (a_{n-1} + b_{n-1}k_2)x_2, \\ \left(\frac{dx_n}{dt}\right)_{x_3} = -b_{n-2}x_3^3 + (a_{n-2} + b_{n-2}k_3)x_3, \ldots, \left(\frac{dx_n}{dt}\right)_{x_n} = -b_1 x_n^3 + (a_1 + b_1 k_n)x_n, \\ \left(\frac{dx_n}{dt}\right)_{\varepsilon_1} = -b_n\varepsilon_1^3 + b_n k_1\varepsilon_1 - 3b_n x_1\varepsilon_1(x_1 + \varepsilon_1), \\ \left(\frac{dx_n}{dt}\right)_{\varepsilon_2} = -b_{n-1}\varepsilon_2^3 + b_{n-1}k_2\varepsilon_2 - 3b_{n-1}x_2\varepsilon_2(x_2 + \varepsilon_2), \\ \left(\frac{dx_n}{dt}\right)_{\varepsilon_3} = -b_{n-2}\varepsilon_3^3 + b_{n-2}k_3\varepsilon_3 - 3b_{n-2}x_3\varepsilon_3(x_3 + \varepsilon_3), \ldots, \\ \left(\frac{dx_n}{dt}\right)_{\varepsilon_n} = -b_1\varepsilon_n^3 - b_1 k_n\varepsilon_n - 3b_1 x_n\varepsilon_n(x_n - \varepsilon_n), \\ \left(\frac{d\varepsilon_1}{dt}\right)_{\varepsilon_2} = \varepsilon_2, \left(\frac{d\varepsilon_2}{dt}\right)_{\varepsilon_3} = \varepsilon_3, \left(\frac{d\varepsilon_n}{dt}\right)_{\varepsilon_1} = (a_n - l_n c_1)\varepsilon_1, \\ \left(\frac{d\varepsilon_n}{dt}\right)_{\varepsilon_2} = (a_{n-1} - l_n c_2)\varepsilon_2, \left(\frac{d\varepsilon_n}{dt}\right)_{\varepsilon_3} = (a_{n-2} - l_n c_3)\varepsilon_3, ,\ldots, \left(\frac{d\varepsilon_n}{dt}\right)_{\varepsilon_n} = (a_1 - l_n c_n)\varepsilon_n \end{cases} \quad (10)$$

The total time derivative of the Lyapunov function, defined as the scalar product of the gradient vector (9) and the velocity vector (10):

$$\begin{aligned} &\frac{dV(x,\varepsilon)}{dt} = \sum_{i=1}^{n}\sum_{k=1}^{n}\frac{\partial V_i(x,\varepsilon)}{\partial x_k}\left(\frac{dx_i}{dt}\right)_{x_k} + \sum_{i=n+1}^{2n}\sum_{k=1}^{n}\frac{\partial V_i(x,\varepsilon)}{\partial \varepsilon_k}\left(\frac{d\varepsilon_i}{dt}\right)_{\varepsilon_k} = \\ &= -x_2^2 - x_3^2, \ldots, -x_n^2 - \\ &-(b_n x_1^3 - (a_n - b_n k_1)x_1)^2 - (b_{n-1}x_2^3 - (a_{n-1} - b_{n-1}k_2)x_2)^2, - \\ &-(b_{n-2}x_3^3 - (a_{n-2} + b_{n-2}k_3)x_3)^2 -, \ldots, - \\ &-(b_n\varepsilon_1^3 - b_n k_1\varepsilon_1 - b_n x_1\varepsilon_1(x_1 + \varepsilon_1))^2 - \\ &-(b_{n-1}\varepsilon_2^3 + b_{n-1}k_2\varepsilon_2 - 3b_{n-1}x_2\varepsilon_2(x_2 - \varepsilon_2))^2 - \\ &-(b_{n-2}\varepsilon_3^3 - b_{n-2}k_3\varepsilon_3 - b_{n-2}x_3\varepsilon_3(x_3 + \varepsilon_3))^2, \ldots, \\ &-(b_1\varepsilon_n^3 - b_1 k_n\varepsilon_n - b_1 x_n\varepsilon_n(x_n - \varepsilon_n))^2, -\varepsilon_2^2 - \varepsilon_3^2 - \ldots - \\ &-\varepsilon_n^2 - ((a_n - l_n c_1)\varepsilon_1)^2 - ((a_{n-1} - l_n c_2)\varepsilon_2)^2, -((a_{n-2} - l_n c_3)\varepsilon_3)^2, \ldots, -(a_1 + l_n c_n)\varepsilon_n)^2 \end{aligned} \quad (11)$$

From (11) it follows that the total time derivative of the vector of Lyapunov function is a sign-negative function.

The Lyapunov function with respect to the components of the gradient vector from (9) can be represented in the form:

$$\begin{aligned} V(x,\varepsilon) &= \frac{1}{4}b_n x_1^4 - \frac{1}{2}(a_n - b_n k_1)x_1^2 + \frac{1}{4}b_{n-1}x_2^4 - \frac{1}{2}(a_{n-1} - b_{n-1}k_2 + 1)x_2^2 + \\ &+ \frac{1}{4}b_{n-2}x_3^4 - \frac{1}{2}(a_{n-2} + b_{n-2}k_3 + 1)x_3^2 +,\dots,+ \\ &+ \frac{1}{4}b_1 x_m^4 - \frac{1}{2}(a_1 + b_n k_n + 1)x_n^2 + \frac{1}{4}b_n\varepsilon_1^4 - \frac{1}{2}b_n k_1\varepsilon_1^2 - \frac{1}{2}b_n x_1^2\varepsilon_1^2 + \frac{1}{3}b_n x_1\varepsilon_1^3 + \\ &+ \frac{1}{4}b_{n-1}\varepsilon_2^4 - \frac{1}{2}b_{n-1}k_2\varepsilon_2^2 - \frac{1}{2}b_{n-1}x_2^2\varepsilon_2^2 + \frac{1}{3}b_{n-1}x_2\varepsilon_2^3 \\ &+ \frac{1}{4}b_{n-2}\varepsilon_3^4 - \frac{1}{2}b_{n-2}k_3\varepsilon_3^2 - \frac{1}{2}b_{n-2}x_3^2\varepsilon_3^2 + \frac{1}{3}b_{n-2}x_3\varepsilon_3^3 + \dots + \\ &+ \frac{1}{4}b_1\varepsilon_n^4 - \frac{1}{2}b_1 k_n\varepsilon_n^2 - \frac{1}{3}b_1 x_n^2\varepsilon_n^2 - \frac{1}{2}(a_n - l_n c_1)\varepsilon_1^2 - \frac{1}{2}(a_{n-1} - l_n c_2 + 1)\varepsilon_2^2 - \\ &- \frac{1}{2}(a_{n-2} - l_n c_3 + 1)\varepsilon_3^2 -,\dots,- \frac{1}{2}(a_1 - l_n c_n + 1)\varepsilon_n^2, \end{aligned} \tag{12}$$

The positive definiteness of functions (12) is not obvious, so we use the Morse lemma from the theories of catastrophes [10, 11]. The function (12) in the vicinity of the stationary state (7) satisfies all the conditions of the Morse Lemma. This allows functions (12) to be represented in the neighborhood of the steady state (7) as the following quadratic form:

$$\begin{aligned} V(x,\varepsilon) &= -\frac{1}{2}(a_n - b_n k_1)x_1^2 - \\ &- \frac{1}{2}(a_{n-1} + b_{n-1}k_2 + 1)x_2^2 - \\ &- \frac{1}{2}(a_{n-2} - b_{n-2}k_3 + 1)x_3^2 -,\dots, \\ &- \frac{1}{2}(a_1 - b_1 k_n + 1)x_n^2 - \\ &- \frac{1}{2}(a_n - l_n c_1 + b_n k_1)\varepsilon_1^2 - \\ &- \frac{1}{2}(a_{n-1} - l_n c_2 + b_{n-1}k_2 + 1)\varepsilon_2^2 - \\ &- \frac{1}{2}(a_{n-2} - l_n c_3 + b_{n-2}k_3 + 1)\varepsilon_3^2 -,\dots,- \\ &- \frac{1}{2}(a_1 - l_n c_n + b_1 k_n + 1)\varepsilon_n^2, \end{aligned} \tag{13}$$

The condition of existence of the Lyapunov function vector is determined by the positive definiteness of the quadratic form (13):

$$\begin{cases} a_n + b_n k_1 < 0 \\ a_{n-1} + b_{n-1}k_2 + 1 < 0 \\ a_{n-2} + b_{n-2}k_3 + 1 < 0 \\ \dots \\ a_1 + b_1 k_n + 1 < 0 \end{cases} \tag{14}$$

$$\begin{cases} a_n - l_n c_1 - b_n k_1 < 0 \\ a_{n-1} - l_n c_2 - b_{n-1}k_2 + 1 < 0 \\ a_{n-2} - l_n c_3 - b_{n-2}k_3 + 1 < 0 \\ \dots \\ a_1 - l_n c_n - b_1 k_n + 1 < 0 \end{cases} \tag{15}$$

The equation of the system state (6) in deviations relative to the steady state (8) is written as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = -b_n x_1^3 + 3\sqrt{a_n + b_n k_1} x_1^2 - (a_n + b_n k_1) x_1 - b_n \varepsilon_1^3 + b_n k_1 \varepsilon_1 - b_n x_1^2 \varepsilon_1 + b_n x_1 \varepsilon_1^2 - \\ - b_{n-1} x_2^3 + 3\sqrt{a_{n-1} + b_{n-1} k_2} x_2^2 - (a_{n-1} + b_{n-1} k_2) x_2 - b_{n-1} \varepsilon_2^3 + b_{n-1} k_2 \varepsilon_2 - 3b_{n-1} x_2^2 \varepsilon_2 - 3b_{n-1} x_2 \varepsilon_2^2 - \\ - b_{n-2} x_3^3 + 3\sqrt{a_{n-2} + b_{n-2} k_3} x_3^2 - (a_{n-2} + b_{n-2} k_3) x_3 - b_{n-2} \varepsilon_3^3 - 3b_{n-2} k_3 \varepsilon_3 - 3b_{n-2} x_3^2 \varepsilon_3 - 3b_{n-2} x_3 \varepsilon_3^2 - \dots - \\ - b_1 x_n^3 + 3\sqrt{a_1 - b_1 k_n} x_n^2 - (a_1 + b_1 k_n) x_n - b_1 \varepsilon_n^3 + b_1 k_n \varepsilon_n - 3b_1 x_n^2 \varepsilon_n - 3b_1 x_n \varepsilon_n^2 \\ \dot{\varepsilon}_1 = \varepsilon_2 \\ \dot{\varepsilon}_2 = \varepsilon_3 \\ \dots \\ \dot{\varepsilon}_{n-1} = \varepsilon_n \\ \dot{\varepsilon}_n = (a_n - l_n c_1)\varepsilon_1 + (a_{n-1} - l_n c_2)\varepsilon_2 + (a_{n-2} - l_n c_3)\varepsilon_3 +,\dots,+ (a_1 - l_n c_n)\varepsilon_n \end{cases} \quad (16)$$

The stability of the system (16) is investigated by the gradient-velocity method of Lyapunov vector functions [5, 7, 16, 17].

The total time derivative of the Lyapunov function vector is found in the form:

$$\begin{aligned} \frac{dV(x,\varepsilon)}{dt} &= -x_2^2 - x_3^2,\dots,-x_n^2 - (b_n x_1^3 - 3\sqrt{a_n + b_n k_1} x_1^2 + (a_n + b_n k_1) x_1)^2 - \\ &- (b_{n-1} x_2^3 - 3\sqrt{a_{n-1} + b_{n-1} k_2} x_2^2 + \\ &+ (a_{n-1} + b_{n-1} k_2) x_2)^2 - (b_{n-2} x_3^3 - 3\sqrt{a_{n-2} + b_{n-2} k_3} x_3^2 + (a_{n-2} + b_{n-2} k_3) x_3)^2 -,\dots,- \\ &- (b_1 x_n^3 - 3\sqrt{a_1 - b_1 k_n} x_n^2 + (a_1 + b_1 k_n) x_n)^2 - \\ &- (b_n \varepsilon_1^3 - b_n k_1 \varepsilon_1 - b_n x_1^2 \varepsilon_1 - b_n x_1 \varepsilon_1^2)^2 - (b_{n-1} \varepsilon_2^3 - b_{n-1} k_2 \varepsilon_2 - b_{n-1} x_2^2 \varepsilon_2 - b_{n-1} x_2 \varepsilon_2^2)^2 - \\ &- (b_{n-2} \varepsilon_3^3 - b_{n-2} k_3 \varepsilon_3 - b_{n-2} x_3^2 \varepsilon_3 - b_{n-2} x_3 \varepsilon_3^2)^2 - \dots - \\ &- (b_1 \varepsilon_n^3 - b_1 k_n \varepsilon_n - b_1 x_n^2 \varepsilon_n - b_1 x_n \varepsilon_n^2)^2 - \varepsilon_2^2 - \varepsilon_3^2 - \dots - \\ &- \varepsilon_n^2 - ((a_n \mp l_n c_1)^2 \varepsilon_1^2 - ((a_{n-1} \mp l_n c_2)^2 \varepsilon_2^2 - ((a_{n-2} \mp l_n c_3)^2 \varepsilon_3^2 -,\dots,- (a_1 \mp l_n c_n)^2 \varepsilon_n^2 \end{aligned} \quad (17)$$

From (17) it follows that the total time derivative of a Lyapunov vector function is guaranteed to be a sign-negative function.

The Lyapunov function can be represented by a scalar form in the form:

$$\begin{aligned} V(x,\varepsilon) &= \frac{1}{4} x_1^4 - \sqrt{a_n + b_n k_1} x_1^3 + \frac{1}{2}(a_n + b_n k_1) x_1^2 + \frac{1}{4} b_{n-1} x_2^4 - \sqrt{a_{n-1} + b_{n-1} k_2} x_2^3 + \\ &+ \frac{1}{2}(a_{n-1} + b_{n-1} k_2 - 1) x_2^2 + \frac{1}{4} b_{n-2} x_3^4 - \sqrt{a_{n-2} + b_{n-2} k_3} x_3^4 - \frac{1}{2}(a_{n-2} + b_{n-2} k_3 - 1) x_3^2 +,\dots,+ \\ &+ b_1 x_n^4 - \sqrt{a_1 - b_1 k_n} x_n^3 + \frac{1}{2}(a_1 + b_1 k_n - 1) x_n^2 + \\ &+ \frac{1}{4} b_n \varepsilon_1^4 - \frac{1}{2} b_n k_1 \varepsilon_1^2 - \frac{1}{2} b_n x_1^2 \varepsilon_1^2 - \frac{1}{3} b_n x_1 \varepsilon_1^2 + \frac{1}{4} b_{n-1} \varepsilon_2^4 - \frac{1}{2} b_{n-1} k_2 \varepsilon_2^2 - \frac{1}{2} b_{n-1} x_2^2 \varepsilon_2^2 - \frac{1}{3} b_{n-1} x_2 \varepsilon_2^3 + \\ &+ \frac{1}{4} b_{n-2} \varepsilon_3^n - \frac{1}{2} b_{n-2} k_3 \varepsilon_3^2 - \frac{1}{2} b_{n-2} x_3^2 \varepsilon_3^2 - \frac{1}{3} b_{n-2} x_3 \varepsilon_3^3 + \dots + \frac{1}{4} b_1 \varepsilon_n^3 - \frac{1}{2} b_1 k_n \varepsilon_n^2 - \frac{1}{2} b_1 x_n^2 \varepsilon_n^2 - \frac{1}{3} x_n \varepsilon_n^3 \\ &- \frac{1}{2}(a_n - l_n c_1 - 1)\varepsilon_1^2 - \frac{1}{2}(a_{n-1} - l_n c_2 - 1)\varepsilon_2^2 - \frac{1}{2}((a_{n-2} - l_n c_3 - 1)\varepsilon_3^2 + \dots,+ \frac{1}{2}(a_1 - l_n c_n - 1)\varepsilon_n^2 \end{aligned} \quad (18)$$

Function (18) in the vicinity of the stationary state (8) satisfies all the conditions of the Morse lemma from the theories of catastrophes. This allows functions (18) in the vicinity of the steady state (8) in the form of the following quadratic form.

$$V(x,\varepsilon)=\frac{1}{2}(a_n+b_nk_1)x_1^2+\frac{1}{2}(a_{n-1}+b_{n-1}k_2-1)x_2^2+\frac{1}{2}(a_{n-2}+b_{n-2}k_3-1)x_3^2+,...,$$
$$+\frac{1}{2}(a_1-b_1k_n-1)x_n^2-\frac{1}{2}(a_n-l_nc_1-b_nk_1)\varepsilon_1^2\mp \qquad (19)$$
$$-\frac{1}{2}(a_{n-1}-l_nc_2-b_{n-1}k_2-1)\varepsilon_2^2-\frac{1}{2}(a_{n-2}-l_nc_3-b_{n-2}k_3-1)\varepsilon_3^2,...,-\frac{1}{2}(a_1-l_nc_n-b_1k_n-1)\varepsilon_n^2,$$

From this, from (19) we obtain the condition for the existence of a Lyapunov vector function in the form:

$$\begin{cases} a_n+b_nk_1>0 \\ a_{n-1}+b_{n-1}k_2-1>0 \\ a_{n-2}+b_{n-2}k_3-1>0 \\ ... \\ a_1+b_1k_n-1>0 \end{cases} \qquad (20)$$

$$\begin{cases} a_n-l_nc_1-b_nk_1<0 \\ a_{n-1}-l_nc_2-b_{n-1}k_2-1<0 \\ a_{n-2}-l_nc_3-b_{n-2}k_3-1<0 \\ ... \\ a_1-l_nc_n-b_1k_n-1<0 \end{cases} \qquad (21)$$

System (4) and (5) is a dynamic compensator with an increased potential for operational stability. The steady state (7) exists and is stable when the uncertain parameters of the system change in region (14) and (15), and the steady state (8) appears and exists when the steady state (7) is unstable, and they do not exist at the same time. The steady state (8) will be stable if the system of inequalities (20) and (21) is fulfilled.

4. CONCLUSION

The known methods of control on the exit of the object are based on the model control on the exit of the object. The choice of the elements of the controller and observer matrix requires canonical transformations and complex and ambiguous calculations of the roots of the characteristic equation of a closed system. The roots of the characteristic polynomial of a closed system are obtained by combining the roots of the system with a model controller and the eigenvalues of the state observer.

This paper has proposed an approach to determine the range of changes in the parameters of the object, the controller and the observer. The approach provides robust stability to the dynamic compensator for any changes in the uncertain parameters of the system and allows to control the instability modes in control systems.

The gradient-speed method of Lyapunov vector functions allows one to solve the problem of constructing automatic control systems with a high potential for robust stability in the output of an object directly from the elements of the matrix of the controller object and the observer.

REFRENCES:

[1] A. Korzhansky. Management and supervision in the conditions of uncertainty. Moscow, Science, 1978.

[2] B.T. Polyak, P.S. Shcherbakov. Robust stability and control. Moscow, Nauka, 2012, p. 303.

[3] Dorato P., Vedavalii Recent Advanced in Robust Control – New York: IEEE press, 1990

[4] V.M. Kuntsevich. Management in the face of uncertainty. Guaranteed results in management and identification tasks. 2007, p. 620.

[5] M.A. Beisenbi. Investigation of robust stability of automatic control systems by the method of functions A.M. Lyapunov. Astana, 2015. p. 204.

[6] M.A. Beisenbi. Methods of increasing the potential of robust stability of control systems. Astana, 2011, p. 292.

[7] Beisenbi M., Uskenbayeva G., Satybaldina D., Martsenyuk V., Shailhanova A. Robust stability of spacecraft traffic control system using Lyapunov functions. 16th International Conference on Control, Automation and System (ICCAS), IEEE 2016, pp. 743-748.

[8] Beisenbi M.A., Abdrakhmanova L.G. Research of dynamic properties of control systems with increased potential of robust stability in a class of two-parameter structurally stable maps by Lyapunov function.//International Conference on Computer, Network and Communication Engineering (ICCNCE 2013). – Published by Atlantis Press, 2013. – 201-203 p.

[9] Beisenbi M., Uskenbayeva J., The Research of Robust Stability in Dynamical System.// International Conference on Control, Engineering & Information Technology (CEIT), Sousse, Tunisia, 2013. – Proceedings of IPCO. – P.142-147.

[10] R. Gidmor. Applied Theory of Disasters. In 2 volumes. Vol.1. Moscow, Mir, 1984.

[11] T. Poston, I. Stuart. Theory of catastrophes and its applications. Moscow, Science, 2001, №6.

[12] Andrievsky BR, Fratkov AL Selected chapters of the theory of automatic control with examples in the language MATLAB - SPb .: Nauka, 2000. 475p.

[13] Kvakernah H., Sivan R. Linear optimal control systems. M .: Mir, 1986. 650 p.

[14] Andreev Yu.N. Managing finite-dimensional linear objects. M .: Nauka, 1976. 424 p.

[15] Rey U. Techniques for managing technological processes. M .: Mir, 1983.638 p.

[16] Beisenbi M., Uskenbayeva G. The New Approach of Design Robust Stability for Linear Control System. Proc. of the Intl. Conf. on Advances in Electronics and Electrical Technology—AEET 2014, pp.11-18.2014

[17] Beisenbi M., Yermekbayeva J. Construction of Lyapunov function to examine Robust Stability for Linear System. International Journal of Control. Energy and Electrical Engineering (CEEE), V (1) pp.17-22. Publisher Copyright – IPCO-2014.

[18] I.G. Malkin. The theory of motion stability. Moscow, Science, 1966, p. 534.

[19] E.A. Barbashin. Introduction to the theory of stability 3. Moscow, Nauka, 1967, p. 225.

[20] A.A. Voronov, V.M. Matrosov. Method of Lyapunov vector functions in the theory of stability. Moscow, Nauka, 1987, p. 252.

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