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Estimation of Technical Efficiency of a Normal Exponential Stochastic Frontier Production Model- An Application in Measuring Technical Efficiency of Schools in Coimbatore and Tirupur districts

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Abstract

The present study focuses on the estimation of technical efficiency of a normal exponential stochastic frontier production and the application of it in the estimation of technical efficiency of schools of three different sectors namely public, private and aided schools in Coimbatore and Tirupur districts. The analysis was done based on the results from a survey report on Mathematics subject at their X and XII standard levels. The Translog Normal Exponential Stochastic Frontier Production Model depicted a maximum technical efficiency score of 99.23 % and minimum technical efficiency score of 87.13% at their XII standard level and a maximum technical efficiency of 99.45% and a minimum of 87.39% at their X standard level. The Cobb-Douglas Normal Exponential Stochastic Frontier Production Model gave a maximum Technical Efficiency score of 99.10% and minimum technical efficiency score of 82.19% and a maximum technical efficiency score of 99.09% and a minimum technical efficiency score of 82.39% at their X standard level. The correlation coefficient value is observed as r=0.584, 0.579 at their X and XII standard levels respectively while analyzing using Translog Normal Exponential Stochastic Frontier Production Model and r= 0.786, 0.793 at their X and XII standard levels respectively while analyzing using Cobb-Douglas Normal Exponential Stochastic Frontier Production Model. The Chi-Square values were obtained as 1.3632 and 1.4592 at their X and XII standard levels respectively in case of efficiency estimation using Translog Normal Exponential Stochastic Frontier Production Model and the Chi-square values were observed as 1.1171 and 1.9896 at their X and XII standard levels in case of efficiency estimation using Cobb-Douglas Normal Exponential Stochastic Frontier Production Model.

Keywords: Technical Efficiency, Normal Exponential Stochastic Production Frontier Model, Translog Production Function, Cobb-Douglas Production Function, Mathematics

INTRODUCTION

Stochastic Frontier Analysis

Stochastic Frontier Analysis is a method of mathematical modelling and is widely used probabilistic model to estimate the individual efficiency scores. The Stochastic Frontier Analysis was first proposed by Aigner et al (1977) [1] and Meeusen and Van de Broeck(1977)[2]. The purpose of Stochastic Frontier Analysis is to measure how efficient a firm is with the given observations of input and output by using two error terms, u and v.

The parametric estimation is based on the pioneering work of Aigner and Chu(1986)[3],. The Technical Efficiency of a producer is given by

$$TE_i = \frac{y_i}{f(x_i,\beta)exp\{v_i\}}$$

where y_i- scalar output of producer i ; $f(x_i,\alpha)$- production frontier, α – vector of parameters to be estimated and v_i-non-negative technical inefficiency component.

which defines Technical Efficiency as the ratio of observed output to the maximum feasible output, conditional on $exp\{v_i\}$ as discussed by Kumbhakar and Lovell (2003)[4]. Technical Efficiency TE_i can be attained by the exponential conditional expectation of u given the composed error term ϵ , which is given by

$TE_i = exp\left[-E\left({}^{u_i}/_{\epsilon_i}\right)\right]$ as suggested by Johndrow et al(1982)[5].

This paper involves four sections namely

Section I: Derivation of the Normal-Exponential Stochastic Production Frontier Model.

Section II: Estimation of the parameters of the Normal-Exponential Stochastic Production Frontier Model.

Section III: Measurement of the Technical Efficiency of the Normal-Exponential Stochastic Production Frontier Model .

Section IV: Estimation of Technical Efficiency of Schools in Coimbatore and Tirupur districts with regard to their Mathematics subject score at their secondary and higher secondary levels using Normal Exponential Stochastic Production Frontier Model.

DATA

The primary data was collected from about 900 students during the year 2013-2017 in Coimbatore and Tirupur districts with the aid of student-oriented survey methodology. The questionnaire invokes various input factors for analysis, among which seven input factors pertaining to the score in Mathematics subject at their secondary and higher secondary levels were chosen for analysis using Translog and Cobb-Douglas Normal Exponential Stochastic Frontier Production Models.

The input variables considered in this study can be listed out as follows

(i) Student-Teacher Ratio (x_1)

(ii) School facilities (x_2)

(iii) Socio-Economic Status (x_3)

(iv) Syllabus (x_4)

(v) Teaching Related Factors (x_5)

(vi) Learning disability (x_6)

(vii) Extra Tuition Classes (x_7)

STOCHATIC PRODUCTION FUNCTIONS USED:

The stochastic frontier model in terms of a general production function for the i^th production unit is $y_i = f(x_i, \beta)exp\{v_i - u_i\}$ where v_i is the two-sided noise component and u_i is the non-negative technical inefficiency component of the error term.

TRANSLOG STOCHASTIC PRODUCTION FUNCTION

The Empirical Model of the study can be formulated as below

The technical efficiency of the students regarding their Mathematics learning at their school level with respect to the input factors considered was estimated. The empirical formulation with the aid of Translog production function can be specified as

$$\ln y = \beta_0 + \sum_{i=1}^{7} \beta_i \ln x_i + \frac{1}{2}\sum_{i=1}^{7}\sum_{i=1}^{7} \beta_{ii} (\ln x_i)^2 + \sum_{i=1}^{7}\sum_{j=1}^{7} \beta_{ij} (\ln x_i) * (\ln x_i)$$

COBB-DOUGLAS STOCHACTIC PRODUCTION FUNCTION

The general form of Cobb-Douglas Production function is

$$lny_i = \alpha_0 + \sum_{n} \alpha_n ln\, x_{ni} - u_i$$

I.ESTIMATION OF NORMAL EXPONENTIAL STOCHASTIC PRODUCTION FRONTIER MODEL

In this paper in the derivation of Normal-Exponential Stochastic Production Frontier Model the following distributional assumptions were made.

(i) The error term represents the statistical noise $v_i \sim iid\ N(0, \sigma_n{}^2)$

(ii) The error term representing the technical efficiency $u_i \sim iid\ N^+(0, \sigma_e{}^2)$

(i.e exponential).

(iii) v_i and u_i are distributed independently of each other and of the regressors.

The probability density function of v is

$$f(v) = \frac{1}{\sqrt{2\pi}\sigma_n} exp\left\{\frac{-v^2}{2\sigma_n{}^2}\right\} \quad (1)$$

The probability density function of u is

$$f(u) = \frac{1}{\sigma_e} exp\left\{\frac{-u}{\sigma_e}\right\} \quad (2)$$

$$f(u, v) = f(u). f(v) \quad (3)$$

$$f(u, v) = \frac{1}{\sqrt{2\pi}\sigma_n} exp\left\{\frac{-v^2}{2\sigma_n{}^2}\right\}. \frac{1}{\sigma_e} exp\left\{\frac{-u}{\sigma_e}\right\} \quad (4)$$

$$f(u, v) = \frac{1}{\sqrt{2\pi}\sigma_n\sigma_e} exp\left\{-\frac{v^2}{2\sigma_n{}^2} - \frac{u}{\sigma_e}\right\} \quad (5)$$

Let $v = u + \epsilon$

$$f(u, \epsilon) = \frac{1}{\sqrt{2\pi}\sigma_n\sigma_e} exp\left\{-\frac{(u+\epsilon)^2}{2\sigma_n{}^2} - \frac{u}{\sigma_e}\right\} \quad (6)$$

$$f(u, \epsilon) = \frac{1}{\sqrt{2\pi}\sigma_n\sigma_e} exp\left\{-\frac{u}{\sigma_e} - \frac{(u^2+\epsilon^2+2u\epsilon)}{2\sigma_n{}^2}\right\} \quad (7)$$

$$f(u, \epsilon) = \frac{1}{\sqrt{2\pi}\sigma_n\sigma_e} exp\left\{-\frac{u}{\sigma_e} - \frac{u^2}{2\sigma_n{}^2} - \frac{\epsilon^2}{2\sigma_n{}^2} - \frac{2u\epsilon}{2\sigma_n{}^2}\right\} \quad (8)$$

$$f(u, \epsilon) = \frac{1}{\sqrt{2\pi}\sigma_n\sigma_e} exp\left\{-\frac{u}{\sigma_e} - \frac{u^2}{2\sigma_n{}^2} - \frac{\epsilon^2}{2\sigma_n{}^2} - \frac{u\epsilon}{\sigma_n{}^2}\right\} \quad (9)$$

$$f(u, \epsilon) = \frac{1}{\sqrt{2\pi}\sigma_n\sigma_e} exp\left\{-\frac{1}{2}\left[\frac{u^2}{\sigma_n{}^2} + 2\left(\frac{u\epsilon}{\sigma_n{}^2} + \frac{u}{\sigma_e}\right) + \frac{\epsilon^2}{\sigma_n{}^2}\right]\right\} \quad (10)$$

$$f(u, \epsilon) = \frac{1}{\sqrt{2\pi}\sigma_n\sigma_e} exp\left\{-\frac{1}{2}\left[\frac{u^2}{\sigma_n{}^2} + 2u\left(\frac{\epsilon}{\sigma_n{}^2} + \frac{1}{\sigma_e}\right) + \frac{\epsilon^2}{\sigma_n{}^2}\right]\right\} \quad (11)$$

$$f(u, \epsilon) = \frac{1}{\sqrt{2\pi}\sigma_n\sigma_e} exp\left\{-\frac{1}{2}\left[\frac{u^2}{\sigma_n{}^2} + \frac{2u\sigma_n}{\sigma_n}\left(\frac{\epsilon}{\sigma_n{}^2} + \frac{1}{\sigma_e}\right) + \frac{\epsilon^2}{\sigma_n{}^2}\right]\right\} \quad (12)$$

$$f(u, \epsilon) = \frac{1}{\sqrt{2\pi}\sigma_n\sigma_e} exp\left\{-\frac{1}{2}\left[\frac{u^2}{\sigma_n{}^2} + \frac{2u}{\sigma_n}\left(\frac{\epsilon}{\sigma_n} + \frac{\sigma_n}{\sigma_e}\right) + \frac{\epsilon^2}{\sigma_n{}^2}\right]\right\} \quad (13)$$

$$f(u, \epsilon) = \frac{1}{\sqrt{2\pi}\sigma_n\sigma_e} exp\left\{-\frac{1}{2}\left[\frac{u^2}{\sigma_n{}^2} + \frac{2u}{\sigma_n}\left(\frac{\epsilon}{\sigma_n} + \frac{1}{\lambda}\right) + \frac{\epsilon^2}{\sigma_n{}^2}\right]\right\} \quad (14)$$

$$f(u,\epsilon)=\frac{1}{\sqrt{2\pi}\sigma_n\sigma_e}exp\left\{-\frac{1}{2}\left[\frac{u^2}{{\sigma_n}^2}+\frac{2u}{\sigma_n}\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)+\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)^2-\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)^2+\frac{\epsilon^2}{{\sigma_n}^2}\right]\right\} \quad (15)$$

$$f(u,\epsilon)=\frac{1}{\sqrt{2\pi}\sigma_n\sigma_e}exp\left\{-\frac{1}{2}\left[\left(\frac{u}{\sigma_n}+\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)\right)^2-\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)^2+\frac{\epsilon^2}{{\sigma_n}^2}\right]\right\} \quad (16)$$

$$f(u,\epsilon)=\frac{1}{\sqrt{2\pi}\sigma_n\sigma_e}exp\left\{-\frac{1}{2}\left[\left(\frac{u}{\sigma_n}+\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)\right)^2\right]\right\}exp\left\{-\frac{1}{2}\left[-\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)^2+\frac{\epsilon^2}{{\sigma_n}^2}\right]\right\} \quad (17)$$

$$f(u,\epsilon)=\frac{1}{\sqrt{2\pi}\sigma_n\sigma_e}exp\left\{-\frac{1}{2}\left[\left(\frac{u}{\sigma_n}+\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)\right)^2\right]\right\}exp\left\{-\frac{1}{2}\left[-\frac{\epsilon^2}{{\sigma_n}^2}-\frac{1}{\lambda^2}-\frac{2\epsilon}{\lambda\sigma_n}+\frac{\epsilon^2}{{\sigma_n}^2}\right]\right\} \quad (18)$$

$$f(u,\epsilon)=\frac{1}{\sqrt{2\pi}\sigma_n\sigma_e}exp\left\{-\frac{1}{2}\left[\left(\frac{u}{\sigma_n}+\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)\right)^2\right]\right\}exp\left\{\frac{1}{2}\left[\frac{1}{\lambda^2}+\frac{2\epsilon}{\lambda\sigma_n}\right]\right\} \quad (19)$$

$$f(u,\epsilon)=\frac{1}{\sqrt{2\pi}\sigma_n\sigma_e}exp\left\{\frac{1}{2}\left[\frac{1}{\lambda^2}+\frac{2\epsilon}{\lambda\sigma_n}\right]\right\}exp\left\{-\frac{1}{2}\left[\left(\frac{u}{\sigma_n}+\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)\right)^2\right]\right\} \quad (20)$$

The marginal density function of ϵ is given by

$$f(\epsilon)=\int_0^\infty f(u,\epsilon)\,du \quad (21)$$

$$f(\epsilon)=\int_0^\infty\frac{1}{\sqrt{2\pi}\sigma_n\sigma_e}exp\left\{\frac{1}{2}\left[\frac{1}{\lambda^2}+\frac{2\epsilon}{\lambda\sigma_n}\right]\right\}exp\left\{-\frac{1}{2}\left[\left(\frac{u}{\sigma_n}+\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)\right)^2\right]\right\}du \quad (22)$$

$$f(\epsilon)=\frac{1}{\sqrt{2\pi}\sigma_n\sigma_e}exp\left\{\frac{1}{2}\left[\frac{1}{\lambda^2}+\frac{2\epsilon}{\lambda\sigma_n}\right]\right\}\int_0^\infty exp\left\{-\frac{1}{2}\left[\left(\frac{u}{\sigma_n}+\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)\right)^2\right]\right\}du \quad (23)$$

Let $x=\frac{u}{\sigma_n}+\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)$

$dx=\frac{du}{\sigma_n}$

$du=\sigma_n dx$

Limits

U	0	∞
X	$\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}$	∞

$$f(\epsilon)=\frac{1}{\sqrt{2\pi}\sigma_n\sigma_e}exp\left\{\frac{1}{2}\left[\frac{1}{\lambda^2}+\frac{2\epsilon}{\lambda\sigma_n}\right]\right\}\int_{\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}}^\infty exp\left\{-\frac{x^2}{2}\right\}\sigma_n\,dx \quad (24)$$

$$f(\epsilon)=\frac{1}{\sqrt{2\pi}\sigma_e}exp\left\{\frac{1}{2}\left[\frac{1}{\lambda^2}+\frac{2\epsilon}{\lambda\sigma_n}\right]\right\}\int_{\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}}^\infty exp\left\{-\frac{x^2}{2}\right\}dx \quad (25)$$

$$f(\epsilon)=\frac{1}{\sigma_e}exp\left\{\frac{1}{2}\left[\frac{1}{\lambda^2}+\frac{2\epsilon}{\lambda\sigma_n}\right]\right\}\frac{1}{\sqrt{2\pi}}\int_{\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}}^\infty exp\left\{-\frac{x^2}{2}\right\}dx \quad (26)$$

$$f(\epsilon)=\frac{1}{\sigma_e}exp\left\{\frac{1}{2}\left[\frac{1}{\lambda^2}+\frac{2\epsilon}{\lambda\sigma_n}\right]\right\}\left[1-\Phi\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)\right] \quad (27)$$

MEAN

$$E(\epsilon)=E(v-u)=E(v)-E(u)=0-E(u) \quad (28)$$

$$E(\epsilon)=-E(u) \quad (29)$$

$$E(u) = \int_0^\infty u\, f(u) du \tag{30}$$

$$E(u) = \int_0^\infty u\, \frac{1}{\sigma_e} exp\left\{\frac{-u}{\sigma_e}\right\} du \tag{31}$$

Let $\frac{u}{\sigma_e} = y \quad du = \sigma_e\, dy$

Limits

U	0	∞
Y	0	∞

$$E(u) = \int_0^\infty y\sigma_e \frac{1}{\sigma_e} exp\{-y\}\sigma_e dy \tag{32}$$

$$E(u) = \sigma_e \int_0^\infty yexp\{-y\}dy \tag{33}$$

$$E(u) = \sigma_e[y\, exp\{-y\}(-1) - exp\{-y\}]_0^\infty \tag{34}$$

$$E(u) = \sigma_e \tag{35}$$

$$E(u^2) = \int_0^\infty u^2\, f(u) du \tag{36}$$

$$E(u^2) = \int_0^\infty u^2\, \frac{1}{\sigma_e} exp\left\{\frac{-u}{\sigma_e}\right\} du \tag{37}$$

Let $\frac{u}{\sigma_e} = y \quad du = \sigma_e\, dy$

Limits

U	0	∞
Y	0	∞

$$E(u^2) = \int_0^\infty y^2{\sigma_e}^2 \frac{1}{\sigma_e} exp\{-y\}\sigma_e dy \tag{38}$$

$$E(u^2) = {\sigma_e}^2 \int_0^\infty y^2 exp\{-y\}dy \tag{39}$$

$$E(u^2) = 2{\sigma_e}^2 \tag{40}$$

VARIANCE

$$Var(u) = E(u^2) - [E(u)]^2 \tag{41}$$

$$Var(u) = 2{\sigma_e}^2 - {\sigma_e}^2 \tag{42}$$

$$Var(u) = {\sigma_e}^2 \tag{43}$$

$$Var(\epsilon) = Var(v) - Var(u) \tag{44}$$

$$Var(\epsilon) = {\sigma_n}^2 - {\sigma_e}^2 \tag{45}$$

II. ESTIMATION OF THE PARAMETERS

$$L(sample) = \prod_{i=1}^{p} f(\epsilon_i) \quad (46)$$

Let $\sigma^2 = \sigma_e{}^2 + \sigma_n{}^2$, $\lambda = \frac{\sigma_e}{\sigma_n}$, $\sigma_e{}^2 =$, $\sigma_n{}^2 = \frac{\sigma^2}{(1+\lambda^2)}$

The log-likelihood function of the Normal Exponential Stochastic production frontier model is given by

$$\ln L = -\frac{p}{2}(\ln \sigma_e{}^2) + \sum_{i=1}^{p} \ln\left[1 - \Phi\left(\frac{\epsilon_i}{\sigma_n} + \frac{1}{\lambda}\right)\right] + \frac{p}{2\lambda^2} + \sum_{i=1}^{p} \frac{\epsilon_i}{\lambda\sigma_n} \quad (47)$$

Let $\epsilon_i = y_i - \alpha' x_i$ and x_i being [1xK] vectors.

$$\ln L = -\frac{p}{2}(\ln \sigma_e{}^2) + \sum_{i=1}^{p} \ln\left[1 - \Phi\left(\frac{(y_i - \alpha' x_i)}{\sigma(1+\lambda^2)^{-1/2}} + \frac{1}{\lambda}\right)\right] + \frac{p}{2\lambda^2} + \sum_{i=1}^{p} \frac{(y_i - \alpha' x_i)}{\lambda\sigma(1+\lambda^2)^{-1/2}} \quad (48)$$

$$\ln L = -\frac{p}{2}\left(\ln \frac{\sigma^2}{(1+\lambda^{-2})}\right) + \sum_{i=1}^{p} \ln\left[1 - \Phi\left(\frac{(y_i - \alpha' x_i)}{\sigma_n} + \frac{1}{\lambda}\right)\right] + \frac{p}{2\lambda^2} + \sum_{i=1}^{p} \frac{(y_i - \alpha' x_i)}{\lambda\sigma_n} \quad (49)$$

The parameters σ^2, λ and α are estimated using first order conditions of the likelihood function as follows

$$\frac{\partial \ln L}{\partial \sigma^2} = -\frac{p}{2}\frac{1}{\sigma^2} - \frac{1}{2\sigma^3}\sum_{i=1}^{p} \frac{\phi\left(\frac{(y_i - \alpha' x_i)}{\sigma(1+\lambda^2)^{-\frac{1}{2}}} + \frac{1}{\lambda}\right)}{1 - \Phi\left(\frac{(y_i - \alpha' x_i)}{\sigma(1+\lambda^2)^{-\frac{1}{2}}} + \frac{1}{\lambda}\right)} \frac{(y_i - \alpha' x_i)}{(1+\lambda^2)^{-\frac{1}{2}}} - \sum_{i=1}^{p} \frac{(y_i - \alpha' x_i)}{2\sigma^3\lambda(1+\lambda^2)^{-\frac{1}{2}}} = 0 \quad (50)$$

$$\frac{\partial lnL}{\partial \lambda} = \frac{p(1+\lambda^{-2})}{\lambda^3} + \sum_{i=1}^{p} \frac{\phi\left(\frac{(y_i - \alpha' x_i)}{\sigma(1+\lambda^2)^{-\frac{1}{2}}} - \frac{1}{\lambda^2}\right)}{1 - \Phi\left(\frac{(y_i - \alpha' x_i)}{\sigma(1+\lambda^2)^{-\frac{1}{2}}} + \frac{1}{\lambda}\right)} \left(\frac{(y_i - \alpha' x_i)\lambda}{\sigma(1+\lambda^2)^{-\frac{1}{2}}} - \frac{1}{\lambda^2}\right) - \frac{p}{\lambda^3} + \sum_{i=1}^{p} \frac{(y_i - \alpha' x_i)}{\sigma}\left[-\frac{1}{\lambda^2}\frac{1}{(1+\lambda^2)^{-\frac{1}{2}}} - \frac{1}{(1+\lambda^2)^{\frac{1}{2}}}\right] = 0 \quad (51)$$

$$\frac{\partial \ln L}{\partial \alpha} = \sum_{i=1}^{p} \frac{\phi\left(\frac{(y_i - \alpha' x_i)}{\sigma(1+\lambda^2)^{-\frac{1}{2}}} + \frac{1}{\lambda}\right)}{1 - \Phi\left(\frac{(y_i - \alpha' x_i)}{\sigma(1+\lambda^2)^{-\frac{1}{2}}} + \frac{1}{\lambda}\right)} \left[\frac{x_i}{\sigma(1+\lambda^2)^{-\frac{1}{2}}}\right] + \sum_{i=1}^{p} \left[\frac{x_i}{\sigma(1+\lambda^2)^{-\frac{1}{2}}\lambda}\right] = 0 \quad (52)$$

Let A be an pxk matrix, B be an px1 vector, **ε** be an px1 vector$(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_p)$,

$$\gamma_i = \frac{\phi\left(\frac{(y_i - \alpha' x_i)}{\sigma(1+\lambda^2)^{-\frac{1}{2}}} + \frac{1}{\lambda}\right)}{1 - \Phi\left(\frac{(y_i - \alpha' x_i)}{\sigma(1+\lambda^2)^{-\frac{1}{2}}} + \frac{1}{\lambda}\right)} \quad (53)$$

$$-\frac{p}{2}\frac{1}{\sigma^2} - \frac{1}{2\sigma^3}\frac{(\gamma_i B - \gamma_i \alpha' A)}{(1+\lambda^2)^{-\frac{1}{2}}} - \frac{1}{2\sigma^3}\frac{(B - \alpha' A)}{\lambda(1+\lambda^2)^{-\frac{1}{2}}} = 0 \quad (54)$$

$$p + \frac{1}{\sigma(1+\lambda^2)^{-\frac{1}{2}}}\left[\gamma_i B - \gamma_i \alpha' A - \frac{(B - \alpha' A)}{\lambda}\right] = 0 \quad (55)$$

$$p + \frac{1}{\sigma\lambda(1+\lambda^2)^{-\frac{1}{2}}}[\gamma_i B\lambda - \gamma_i \alpha' A\lambda - (B - \alpha' A)] = 0 \quad (56)$$

$$\frac{1}{\sigma\lambda(1+\lambda^2)^{-\frac{1}{2}}}[\gamma_i B\lambda - \gamma_i \alpha' A\lambda - (B - \alpha' A)] = -p \quad (57)$$

$$\frac{1}{-p\lambda(1+\lambda^2)^{-\frac{1}{2}}}[\gamma_i B\lambda - \gamma_i \alpha' A\lambda - (B-\alpha' A)] = \sigma \tag{58}$$

$$\frac{-\gamma_i A}{\sigma(1+\lambda^2)^{-\frac{1}{2}}} - \frac{A}{\sigma\lambda(1+\lambda^2)^{-\frac{1}{2}}} = 0 \tag{59}$$

$$\frac{A}{\lambda} = -\gamma_i A \tag{60}$$

$$\lambda = -\frac{A}{\gamma_i A} \tag{61}$$

$$-p - \frac{(\gamma_i B - \gamma_i \alpha' A)}{\sigma(1+\lambda^2)^{-\frac{1}{2}}} - \frac{(B-\alpha' A)}{\lambda\sigma(1+\lambda^2)^{-\frac{1}{2}}} = 0 \tag{62}$$

Post multiplying by A^{-1}

$$-p\,\sigma(1+\lambda^2)^{-\frac{1}{2}}\lambda A^{-1} - \gamma_i B A^{-1}\lambda + \gamma_i \alpha'\lambda - BA^{-1} + \alpha' = 0 \tag{63}$$

$$p\,\sigma(1+\lambda^2)^{-\frac{1}{2}}\lambda A^{-1} + \gamma_i B A^{-1}\lambda + BA^{-1} = \gamma_i \alpha'\lambda + \alpha' \tag{64}$$

$$p\,\sigma(1+\lambda^2)^{-\frac{1}{2}}\lambda A^{-1} + \gamma_i B A^{-1}\lambda + BA^{-1} = (\gamma_i\lambda + 1)\alpha' \tag{65}$$

III. ESTIMATION OF TECHNICAL EFFICIENCY

$$f(u|\epsilon) = \frac{f(u,\epsilon)}{f(\epsilon)} \tag{66}$$

$$f(u|\epsilon) = \frac{\frac{1}{\sqrt{2\pi}\sigma_n\sigma_e} exp\left\{\frac{1}{2}\left[\frac{1}{\lambda^2}+\frac{2\epsilon}{\lambda\sigma_n}\right]\right\} exp\left\{-\frac{1}{2}\left[\left(\frac{u}{\sigma_n}+\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)\right)^2\right]\right\}}{\frac{1}{\sigma_e} exp\left\{\frac{1}{2}\left[\frac{1}{\lambda^2}+\frac{2\epsilon}{\lambda\sigma_n}\right]\right\}\left[1-\Phi\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)\right]} \tag{67}$$

$$f(u|\epsilon) = \frac{\frac{1}{\sqrt{2\pi}\sigma_n} exp\left\{-\frac{1}{2}\left[\left(\frac{u}{\sigma_n}+\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)\right)^2\right]\right\}}{\left[1-\Phi\left(\frac{\epsilon}{\sigma_n}+\frac{1}{\lambda}\right)\right]} \tag{68}$$

$$f(u|\epsilon) = \frac{\frac{1}{\sqrt{2\pi}\sigma_n} exp\left\{-\frac{1}{2}\left[\left(\frac{u}{\sigma_n}+\left(\frac{\epsilon}{\sigma_n}+\frac{\sigma_n}{\sigma_e}\right)\right)^2\right]\right\}}{\left[1-\Phi\left(\frac{\epsilon}{\sigma_n}+\frac{\sigma_n}{\sigma_e}\right)\right]} \tag{69}$$

$$f(u|\epsilon) = \frac{\frac{1}{\sqrt{2\pi}\sigma_n} exp\left\{-\frac{1}{2}\left[\left(\frac{u}{\sigma_n}+\left(\frac{\epsilon}{\sigma_n}+\frac{\sigma_n{}^2}{\sigma_e\sigma_n}\right)\right)^2\right]\right\}}{\left[1-\Phi\left(\frac{\epsilon}{\sigma_n}+\frac{\sigma_n{}^2}{\sigma_e\sigma_n}\right)\right]} \tag{70}$$

$$f(u|\epsilon) = \frac{\frac{1}{\sqrt{2\pi}\sigma_n} exp\left\{-\frac{1}{2\sigma_n}\left[\left(u+\left(\epsilon+\frac{\sigma_n{}^2}{\sigma_e}\right)\right)^2\right]\right\}}{\left[1-\Phi\left(\frac{\epsilon}{\sigma_n}+\frac{\sigma_n{}^2}{\sigma_e\sigma_n}\right)\right]} \tag{71}$$

Let $\gamma' = \epsilon + \frac{\sigma_n{}^2}{\sigma_e}$

$$f(u|\epsilon) = \frac{\frac{1}{\sqrt{2\pi}\sigma_n} exp\left\{-\frac{1}{2\sigma_n}\left[(u+\gamma')^2\right]\right\}}{\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]} \quad (72)$$

$$E(u|\epsilon) = \int_0^{\infty} u\ f(u|\epsilon)\ du \quad (73)$$

$$E(u|\epsilon) = \int_0^{\infty} u \frac{\frac{1}{\sqrt{2\pi}\sigma_n} exp\left\{-\frac{1}{2\sigma_n}\left[(u+\gamma')^2\right]\right\}}{\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]}\ du \quad (74)$$

$$E(u|\epsilon) = \frac{\frac{1}{\sqrt{2\pi}\sigma_n}}{\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]} \int_0^{\infty} u\ exp\left\{-\frac{1}{2\sigma_n}(u+\gamma')^2\right\}\ du \quad (75)$$

Let $z = \frac{u+\gamma'}{\sigma_n}$

$u = \sigma_n z + \gamma'$

$du = \sigma_n dz$

Limits

U	0	∞
Z	$\frac{\gamma'}{\sigma_n}$	∞

$$E(u|\epsilon) = \frac{\frac{1}{\sqrt{2\pi}\sigma_n}}{\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]} \int_{\frac{\gamma'}{\sigma_n}}^{\infty} (\sigma_n z + \gamma')\, exp\left(-\frac{z^2}{2}\right)\ \sigma_n dz \quad (76)$$

$$E(u|\epsilon) = \frac{\frac{1}{\sqrt{2\pi}}}{\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]} \left[\int_{\frac{\gamma'}{\sigma_n}}^{\infty} \sigma_n z\ exp\left(-\frac{z^2}{2}\right)\ dz + \int_{\frac{\gamma'}{\sigma_n}}^{\infty} \gamma'\ exp\left(-\frac{z^2}{2}\right)\ dz\right] \quad (77)$$

$$E(u|\epsilon) = \frac{\frac{1}{\sqrt{2\pi}}}{\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]} \int_{\frac{\gamma'}{\sigma_n}}^{\infty} \sigma_n z\ exp\left(-\frac{z^2}{2}\right)\ dz + \frac{\frac{1}{\sqrt{2\pi}}}{\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]} \int_{\frac{\gamma'}{\sigma_n}}^{\infty} \gamma'\ exp\left(-\frac{z^2}{2}\right)\ dz \quad (78)$$

$$E(u|\epsilon) = \frac{\frac{\sigma_n}{\sqrt{2\pi}} exp\left(-\frac{\gamma'^2}{2\sigma_n^2}\right)}{\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]} + \frac{\frac{\gamma'}{\sqrt{2\pi}}}{\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]} \int_{\frac{\gamma'}{\sigma_n}}^{\infty}\ exp\left(-\frac{z^2}{2}\right)\ dz \quad (79)$$

$$E(u|\epsilon) = \frac{\sigma_n \phi\left(\frac{\gamma'}{\sigma_n}\right) + \gamma'\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]}{\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]} \quad (80)$$

$$E(u|\epsilon) = \gamma' + \frac{\sigma_n \phi\left(\frac{\gamma'}{\sigma_n}\right)}{\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]} \quad (81)$$

Technical Efficiency

$$TE = exp[-E(u_i|\epsilon_i)] \quad (82)$$

$$TE = exp\left[-\left\{\gamma' + \frac{\sigma_n \phi\left(\frac{\gamma'}{\sigma_n}\right)}{\left[1-\Phi\left(\frac{\gamma'}{\sigma_n}\right)\right]}\right\}\right] \quad (83)$$

IV. ESTIMATION OF TECHNICAL EFFICIENCY OF SCHOOLS IN COIMBATORE AND TIRUPUR DISTRICTS WITH REGARD TO THEIR MATHEMATICS SUBJECT USING NORMAL-EXPONENTIAL STOCHASTIC PRODUCTION FRONTIER MODEL

Translog Normal Exponential Stochastic Production Frontier Model

Estimation of Frontier Production Function Using Translog Normal Exponential Stochastic Production Frontier Model

The results of maximum likelihood estimates (MLE) of the translog normal exponential stochastic production frontier model are presented in Table 1. The coefficients of Learning disability and teaching related factors were of positive sign for MLE estimates showing efficient allocation of those input resources. However, the negative coefficient values of student-teacher ratio, socio-economic status, syllabus, school facilities and extra tuition classes showed an inefficient allocation of those inputs.

Estimation of Technical Efficiency using Translog Normal Exponential Stochastic Production Frontier Model

The parameters were estimated using the Method of Maximum Likelihood Estimation and the results were depicted in the following section

Table 1: Maximum Likelihood Estimate of Average Performance in Mathematics and Science Using Translog Normal Exponential Stochastic Production Frontier Model

Variables	Parameters	Coefficients	
		X	XII
Constant	β_0	296.345	298.254
ln STR	β_1	-40.231	-42.658
ln SES	β_2	-8.923	-9.541
ln SF	β_3	-5.856	-6.223
ln LD	β_4	1.005	1.128
ln SYL	β_5	-0.398	-0.429
ln TF	β_6	5.012	6.137
ln ETC	β_7	-20.089	-22.122
ln STR x ln STR	β_{11}	2.798	3.389
ln SES x ln SES	β_{22}	-0.715	-0.825
ln SF x ln SF	β_{33}	-0.451	-0.571
ln LD x ln LD	β_{44}	-0.742**	-0.812**
ln SYL x ln SYL	β_{55}	-0.069	-0.071
ln TF x ln TF	β_{66}	-0.156	-0.168

Variables	Parameters	Coefficients	
		X	XII
ln ETC x ln ETC	β_{77}	-0.201	-0.268
ln STR x ln SES	β_{12}	0.623	0.716
ln STR x ln SF	β_{13}	0.119	0.138
ln STR x ln LD	β_{14}	-0.538	-0.659
ln STR x ln SYL	β_{15}	0.062	0.089
ln STR x ln TF	β_{16}	0.059	0.067
ln STR x ln ETC	β_{17}	2.852	3.015
ln SES x ln SF	β_{23}	0.697	0.725
ln SES x ln LD	β_{24}	0.624	0.756
ln SES x ln SYL	β_{25}	0.078	0.082
ln SES x ln TF	β_{26}	-0.635	-0.715
ln SES x ln ETC	β_{27}	0.746	0.836
ln SF x ln LD	β_{34}	0.208	0.319
ln SF x ln SYL	β_{35}	0.141	0.173
ln SF x ln TF	β_{36}	0.245*	0.361**
ln SF x ln ETC	β_{37}	-0.085	-0.096
ln LD x ln SYL	β_{45}	0.211	0.337
ln LD x ln TF	β_{46}	0.192	0.210
ln LD x ln ETC	β_{47}	0.245	0.314
ln SYL x ln TF	β_{56}	-0.027	-0.035
ln SYL x ln ETC	β_{57}	-0.314	-0.411
ln TF x ln ETC	β_{67}	-0.459**	-0.562**
$\lambda = \frac{\sigma_u}{\sigma_v}$	0.9015		0.9125
$\sigma = \sqrt{\sigma_u{}^2 + \sigma_v{}^2}$	0.0514		0.0605
Log-likelihood	307.1895		308.1996
	Estimated Variances of the underlying variables		
v	0.00145		0.00158
u	0.00096		0.00099
ε	0.00241		0.00257
$\gamma = \frac{Var(u)}{Var(\varepsilon)}$	0.52000		0.61500

*Significant at 5% level

(Table 1 continued..)

**Significant at 1% level

The significant level of the parameter λ showed that there exists sufficient evidence to suggest the presence of technical inefficiency. The estimates of the error variances ($\sigma_u{}^2$, $\sigma_v{}^2$) were (0.00145, 0.00096) and (0.00158,0.00099) respectively at their X and XII standard level in case of the Mathematics subject .Therefore, it could be easily seen that the variance of the one-sided error, $\sigma_u{}^2$ is larger than the variance of the random error $\sigma_v{}^2$.Thus the value of λ=0.9015 and λ=0.9125 in case of Mathematics in their X and XII standards respectively of more than one clearly showed the dominant share of the estimated variance of the one-sided error term, u, over the estimated variance of the whole error term. This further implied that greater part of the residual variation in output was associated with the variation in technical inefficiency rather than with 'measurement error', which was associated with incontrollable factors related to the production process.

Moreover, both λ and σ variables of Coimbatore and Tirupur districts of Tamil Nadu entered the output of all students positively and significantly. The estimate of γ, which is the ratio of the variance of student-specific performance of technical efficiency of the total variance of output was 0.52000 and 0.61500 respectively at their X and XII standards indicating that the difference between the observed and frontier output was primarily due to the factors which were 52% and 62% at their X and XII standards respectively were under the control of the schools.

The level of technical efficiency for each of the 900 sample students was calculated using Translog Exponential Stochastic Frontier Production Model by estimating the one-sided error component u_i. The (max, min) estimated technical efficiency was (99.89 %, 89.98%) at their X standard level and was (99.96%, 89.97%) at their XII standard level using Translog Normal Exponential Stochastic Production Frontier Model. The mean level of Technical Efficiency was 94.94% and 94.97% at their X standard level and XII standard level which implied that the sample students realized 94.94% and 94.97% of their technical abilities at their X and XII standard levels with regard to their Mathematics subject and the mean level of Technical Efficiency was 94.84% and 94.89% at their X standard level and XII standard level in case of Science subject which implied that the sample students realized 94.84% and 94.89%and of their technical abilities at their X and XII standard levels with regard to their Science subject. A firm is considered technically inefficient even if the firm registered a technical efficiency of 82%. By this standard, 100% of the students were considered technically efficient in the sample under study using Translog Normal Exponential Stochastic Production Frontier Model as no student has reported the technical efficiency score of less than 88 %.

However, for better indication of the distribution of individual efficiencies, a frequency distribution of predicted technical efficiencies within ranges of five using Translog Normal Exponential Stochastic Production Frontier Model is depicted in Table 2. This indicated less variations in the level of technical efficiency across students.

Table 2 Frequency Distribution of Student Specific Technical Efficiency Estimates Using Translog Normal Exponential Stochastic Production Frontier Model.

Efficiency Score(%)	X-Standard		XII-Standard	
	Number of Students	Percentage	Number of Students	Percentage
Below 85	-	-	-	-
85-90	**10**	**1.11**	**8**	**0.89**
90-95	**80**	**8.89**	**77**	**8.56**
95-100	**810**	**90**	**815**	**90.56**

The highest number of students (810,815) at their X and XII standard levels respectively were found in the technical efficiency class of 95-100%. However the model range lies between 94.94% and 94.97% at their X standard level and XII standard level. No student has reported a technical efficiency score of less than 85 percent.

To test whether the model Translog Normal Exponential Stochastic Frontier Model predicted technical efficiency accurately, correlation coefficient between observed efficiency and technical efficiency has been calculated and presented in the following section.

Chi-square Test for Goodness of Fit of Translog normal Exponential Stochastic Production Frontier Model-

The Chi-square value was obtained as 1.3632 and 1.4592 at their X and XII standard levels respectively.

Correlation Analysis for Translog Normal Exponential Stoochastic Production Frontier Model

The strength of relationship between the observed efficiency and technical efficiency using Translog Normal Exponential Stochastic Production Frontier model was given by the correlation coefficient ,'r', 0.584,0.579 at their X and XII standard levels respectively.

Cobb-Douglas Normal Exponential Stochastic Production Frontier Model

Estimation of Frontier Production Using Cobb-Douglas Normal Exponential Stochastic Production Frontier Model

The Cobb-Douglas production function model considered for the study involved a total of seven independent variables. The results of the maximum likelihood(MLE) estimates of the Cobb-Douglas Normal Exponential Stochastic Frontier Production Model were presented in Table 3. The coefficient value of the input variable learning disability was of positive value with 1% significant level in all the cases considered showing its efficient allocation. Moreover, this positive and significant value indicated that there was a scope to increase the score in the subjects. The coefficient value of the variable extra tuition classes was -0.139 and -0.152 for the students at their X and XII standard levels respectively, which was also significant at 1% level. This gives an indication that many of the students were having extra tuition classes. Though the coefficient of the variables socio-economic status and syllabus were of positive values, they do not have much impact on output as they were significant neither at 1% nor at 5% levels.

Table 3: Maximum Likelihood Estimates of the Cobb-Douglas Normal Exponential Stochastic Production Frontier Model

Variables	Parameters	Coefficients	
		X	XII
Constant	$\boldsymbol{\beta_0}$	**7.736****	**8.254****
ln STR	$\boldsymbol{\beta_1}$	**-0.140**	**-0.165**
ln SES	$\boldsymbol{\beta_2}$	**0.049**	**0.052**
ln SF	$\boldsymbol{\beta_3}$	**0.015**	**0.019**
ln LD	$\boldsymbol{\beta_4}$	**0.238****	**0.312****
ln SYL	$\boldsymbol{\beta_5}$	**0.017**	**0.019**
ln TF	$\boldsymbol{\beta_6}$	**-0.028**	**-0.036**
ln ETC	$\boldsymbol{\beta_7}$	**-0.139****	**-0.152****
$\lambda = \frac{\sigma_u}{\sigma_v}$	1.2279		1.3158
$\sigma = \sqrt{\sigma_u{}^2 + \sigma_v{}^2}$	0.0567		0.0598
Log-likelihood	271.9871		280.5986
Estimated variances of the underlying variables			
v	0.00129		0.00136
u	0.00187		0.00193
ε	0.00316		0.00319
$\gamma = \frac{Var(u)}{Var(\varepsilon)}$	0.60158		0.61055

The estimates of the error variances ($\sigma_u{}^2$, $\sigma_v{}^2$) were (0.00129, 0.00187) and (0.00136,0.00193)at their X and XII standard levels respectively. The variance of one-sided error term, $\sigma_u{}^2$ was larger than that of the random error, $\sigma_v{}^2$. Thus the value of λ were 1.2279 and 1.3158 at their X and XII standard levels respectively. Hence, a greater part of the residual variation in output was associated with the variation in the technical inefficiency rather than the measurement error, which was associated with uncontrollable factors. The estimate of γ was 0.60158 and 0.61055 at their X and XII standard levels respectively indicating the fact that the difference between the observed and frontier output was primarily due to the factors, which were approximately 60% under the control of the sample schools in all the cases.

Estimation of Technical Efficiency Using Cobb-Douglas Normal Exponential Stochastic Production Frontier Model

The level of technical efficiency for each student was calculated and was given in

Table 4.. In this model, technical efficiency of sample students ranged between 82.45 % and 99.02% with an average of 95.79% with the judicious use of existing resources and technology.

Table 4: Frequency Distribution of Student Specific Technical Efficiency Estimates Using Cobb-Douglas Normal Exponential Stochastic Production Frontier Model

Efficiency Score(%)	X-Standard		XII-Standard	
	Number of students	Percentage	Number of Students	Percentage
Below 80	-	-	**-**	**-**
80-85	24	2.75	29	**3.22**
85-90	24	2.75	35	**3.89**
90-95	208	25.00	202	**22.44**
95-100	644	71.50	**634**	**70.44**

The frequency distribution of student-specific technical efficiency scores using Cobb-Douglas Normal Exponential Stochastic Production Frontier Model is depicted in Table 4 which indicated less variation in the level of technical efficiency across sample students. The highest number of students was found in the most efficient class 95-100 percent followed by 90-95 percent class and 85-90 percent class. No student operated in the efficiency score below 80% using Cobb-Douglas Stochastic Production Frontier Model.

Correlation Analysis for Cobb-Douglas Stochastic Production Frontier model.

The strength of relationship that exists between the observed efficiency and technical efficiency using Cobb-Douglas Normal Exponential Stochastic Production Frontier Model was given by correlation coefficient r-0.786,0.793 at their X and XII standard levels respectively.

Chi-square Test for Goodness of Fit of Cobb-Douglas Normal Exponential Stochastic Production Frontier model

The Chi-square value was obtained as 1.11712,1,9896 at their X and XII standard levels respectively.

Table 5: Comparison of Frequency Distribution of Student Specific Technical Efficiency Estimates

Efficiency Score (%)	Number of Students(Percentage)			
	Translog Exponential Production Function		Cobb-Douglas Exponential Production Function	
	X	XII	X	XII
55-60	-	-	-	-
60-65	-	-	-	-
65-70	-	-	-	-
70-75	-	-	-	-
75-80	-	-	-	-
80-85	-	-	24(2.75)	29(3.22)
85-90	10(1.11)	8(0.89)	24(2.75)	23(2.5)
90-95	80(8.89)	77(8.56)	208(25)	216(24)
95-100	810(90)	815(90.56)	644(71.5)	634(70.5)

Table 6: Summary Statistics of Efficiency Estimates

Statistic	Efficiency Score of Translog Exponential Model		Efficiency Score of Cobb-Douglas Exponential Model	
	X	XII	X	XII
Mean	93.42	93.18	90.74	90.65
Minimum	87.39	87.13	82.39	82.19
Maximum	99.45	99.23	99.09	99.10

Potential of Technical Efficiency Improvement in Efficiency of Schools

The present analysis focuses on the achievement of higher scores in Mathematics and Science subjects with the existing resources technology. Based on the technical efficiency of the most efficient student in each of the chosen models, the average potential to increase the score in the subjects was determined using the formula (Saha and Jain 2004)[9]

$$Potential\ for\ increasing\ the\ score\} =$$

$$\left[1 - \left(\frac{Mean\ Technical\ Efficiency\ of\ the\ system}{Maximum\ technical\ efficiency\ of\ the\ system}\right)\right] * 100$$

The average potential of increasing the score through technical efficiency improvement across various schools are presented in Table 7.

Table 7: Increasing Technical Efficiency Potential using Various Models

Model	Mean Technical Efficiency		Maximum Technical Efficiency		Mean Potential to Increase Technical Efficiency	
	X	XII	X	XII	X	XII
Translog Normal Exponential Stochastic Production Frontier Model	93.42	93.18	99.45	99.23	6.06	6.10
Cobb-Douglas Normal Exponential Stochastic Production Frontier Model	90.74	90.65	99.09	99.10	8.43	8.53

Table 8: Statistical Association of the Models Under Study

Model	Correlation Coefficient		Chi-square Value	
	X	XII	X	XII
Translog Normal Exponential Stochastic Production Frontier Model	0.584	0.579	1.3632	1.4592
Cobb-Douglas Normal Exponential Stochastic Production Frontier Model	0.786	0.793	1.1171	1.9896

CONCLUSION

The analysis done among 900 students in Coimbatore and Tirupur districts from three sectors of school regarding their views on Mathematics subject at their X and XII standard levels revealed the following results

(i) The mean technical efficiency obtained using Translog Normal Exponential Stochastic Frontier Production Model was high compared to the meam technical efficiency obtained using Cobb-Douglas Normal Exponential Stochastic Frontier Production Model.

(ii) On comparing the efficiency scores with respect tot heir views and scores at their X and XII standard levels shows that the scores of the technical efficiency was high at their X standard levels compared to their XII standard level.

(iii) Among the seven input factors considered for analysis, learning disability plays a major role in affecting the technical efficiency(i.e technical inefficiency factor).

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