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Numerical Illustration for Availability Function of Two Unit Non Identical System using M.L. Estimation.

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Abstract:

This paper presents the method of Maximum Likelihood Estimation to evaluate the estimate of availability [Av(t)] function of a two unit non-identical system under the influence of Lethal and Non-lethal Common Cause Shock (CCS) failures. The estimates of the above measures are given for series system. The numerical evidences are given to justify the use of maximum likelihood estimation in the present case.

Keywords: M L estimation, System Availability, CCS failures, Monte-Carlo simulation

INTRODUCTION

The system reliability evaluation usually assumes that components are subject to self-failures only. But components are often further subject to Common Cause Shock (CCS) failures. Failure to consider CCS failures results in exaggerating system reliability and availability. These CCS failures are of two types namely, lethal and non-lethal Common Cause Shock (LCCS & NCCS) failures. Hence many methods of system reliability evaluation with CCS failures have been developed [1-7]. Billinton and Allan [2] discussed the role of CCS failures. Atwood [1], Meachum and Atwood [4] used the BFR model for CCS failures to the data associated with nuclear power plants and given in the nuclear regulatory commission reports. The quantification and estimation of CCS failure rates were discussed by them. Chari et. al [3] discussed the concept of CCS failures in evaluation of reliability and availability measures. Reddy Y R [5] developed the reliability and availability functions for 2-unit system in the presence of lethal and non-lethal common cause shock failures. Sagar G Y [6] and Verma et.al [7] discussed the role of CCS failures as well as human errors in evaluating system reliability. This paper attempts the estimation of availability and frequency of failure functions for two component non-identical system with LCCS and NCCS failures by M L estimation approach in the case of series systems.

Assumptions:

(i) The system has two statistically -independent and non-identical units.

(ii) The system is affected by lethal as well as non-lethal CCS failures in addition to individual failures.

(iii) The components fail individually at the rate λ_i and failure probability is 'p_1'and also fail simultaneously when LCCS failures hit the system at a rate 'ω'.

(iv) The components fail due to NCCS failures, which is occurring at the rate of 'β' and failure probability is 'p_2'.

(v) The individual failures, LCCS and NCCS failures occur independently with each other and follow exponential distribution.

(vi) The failed components are serviced singly and service time follows exponential distribution.

NOTATIONS.

$\lambda_{i1,}\lambda_{i2}$: failure rates of 1st and 2nd components respectively

λ_{12} : rate of common mode failures.

ω : rate of LCCS failure

β : rate of NCCS failure

μ_0, μ_1: Service rates of the 1st and 2nd components respectively

$Av_{LNS}(t)$: time dependent system availability for the series configuration.

$\widehat{Av_{LNS}}(t)$:M L estimate of time dependent system availability for series configuration.

$\bar{x}$: sample mean of individual failures

$\bar{y}$:sample mean of NCCS failures

$\bar{\omega}$: sample mean of LCCS failures

$\bar{z}$: sample mean of service time of the components

$\hat{\bar{x}}$: sample estimate of individual failure rate

$\hat{\bar{y}}$: sample estimate of NCCS failure rate

$\hat{\bar{w}}$:sample estimate of LCCS failure rate

$\hat{\bar{z}}$:sample estimate of service time of the components

N : number of simulated samples

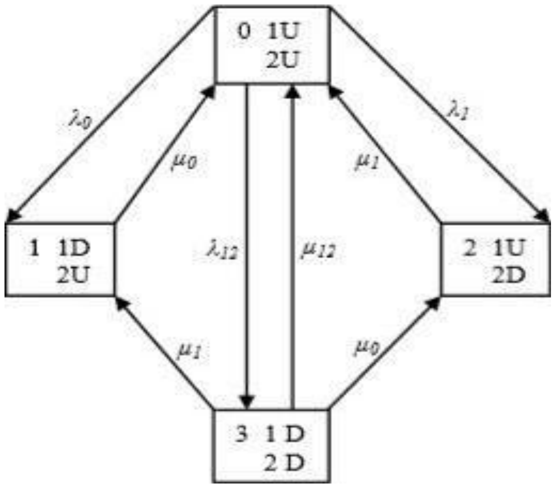
MSE : mean square error.

Model. The above assumptions can formulate a Markov model to derive the availability for series in presents of LCCS and NCCS failures are defined as

$$\lambda_0 = \lambda_{i1} + \beta\, p_1\, (1 - p_2);$$

$$\lambda_1 = \lambda_{i2} + \beta\, p_2\, (1 - p_1);$$

$$\lambda_{12} = \beta p_1 p_2 + \omega$$



U - Up State
L- Down State

Time dependent system availability function for series configuration.

The time dependent system availability expression for series configuration in the presence of LCCS and NCCS failure model is

Series Configuration

Thus, the time-dependent system availability expression for series configuration in the presence of LCCS and NCCS failure model is

$$Av_{LNS}(t) = Q_1 exp(\gamma_1 t) - Q_2 exp(\gamma_2 t) + Q_3 exp(\gamma_3 t) - A_3/\gamma_1\gamma_2\gamma_3$$

Where

$Q_1 = (\gamma_1^3 + \gamma_1^2 A_1 + \gamma_1 A_2 + A_3)/\gamma_1(\gamma_1 - \gamma_2)\,(\gamma_1 - \gamma_3)$

$Q_2 = (\gamma_2^3 + \gamma_2^2 A_1 + \gamma_2 A_2 + A_3)/\gamma_2(\gamma_1 - \gamma_2)\,(\gamma_2 - \gamma_3)$

$Q_3 = (\gamma_3^2 + \gamma_3^2 A_1 + \gamma_3 A_2 + A_3)/\gamma_3(\gamma_1 - \gamma_3)\,(\gamma_2 - \gamma_3)$

$\gamma_1 = -\gamma \sin(\alpha) - B_1/3$

$\gamma_2 = \gamma \sin(\pi/3 + \alpha) - B_1/3$

$\gamma_3 = \gamma \sin(-\pi/3 + \alpha) - B_1/3$

$\gamma = (2/3)(B_1^2 - 3B_2)^{1/2}$

$\alpha = \sin^{-1}(-4q/\gamma^3)/3$

$q = B_3 - (B_1 B_2)/3 + 2B_1^3/27$

$A_1 = 2(\mu_0 + \mu_1) + \mu_2$

$A_2 = \mu_0\mu_1 + [\mu_0 + \mu_1 + \mu_{12}](\mu_0 + \mu_1)$

$A_3 = \mu_0\mu_1(\mu_0 + \mu_1 + \mu_{12})$

$B_1 = \lambda_0 + \lambda_1 + \lambda_{12} + 2(\mu_0 + \mu_1) + \mu_{12}$

$B_2 = \lambda_0(\mu_0 + 2\mu_1 + \mu_{12}) + \lambda_1(2\mu_0 + \mu_1 + \mu_{12}) + 2\lambda_{12}(\mu_0 + \mu_1) + \mu_0\mu_1 + (\mu_0 + \mu_1 + \mu_{12})(\mu_0 + \mu_1)$

$B_3 = \lambda_0\mu_1(\mu_0 + \mu_1 + \mu_{12}) + \lambda_1 + \lambda_{12}\mu_0(\mu_0 + \mu_1 + \mu_{12}) + \lambda_{12}\mu_0(\mu_0 + \mu_1) + \lambda_{12}\mu_1^2 + \mu_0\mu_1(\mu_0 + \mu_1 + \mu_{12})$

$\lambda_0 = \lambda_{i1} + \beta p_1(1 - p_2)$

$\lambda_1 = \lambda_{i2} + \beta p_2(1 - p_1)$

$\lambda_{12} = \beta p_1 p_2 + \omega$

γ_1, γ_2 and γ_3 are always negative. $\forall\ \lambda_{i1}, \lambda_{i2} \geq 0, p_1 p_2 \in (0, 1)$

Estimation of the time dependent system availability of series configuration

The maximum likelihood estimate of time-dependent availability function for series system in the presence of LCCS and NCCS failure model is

$\hat{A}v_{LNS}(t) = Q_1^{'} exp(D_1 t) - Q_2^{'} exp(D_2 t) + Q_3^{'} exp(D_3 t) - A_3^{'}/D_1 D_2 D_3$

where

$Q_1^{'} = (D_1^3 + D_1^2 A_1^{'} + D_1 A_2^{'} + A_3^{'})/D_1(D_1 - D_2)\,(D_1 - D_3)$

$Q_2^{'} = (D_2^3 + D_2^2 A_1^{'} + D_2 A_2^{'} + A_3^{'})/D_2(D_1 - D_2)\,(D_2 - D_3)$

$Q_3^{'} = (D_3^3 + D_3^2 A_1^{'} + D_3 A_2^{'} + A_3^{'})/D_3(D_1 - D_3)\,(D_2 - D_3)$

$D_1 = -D \sin(\alpha^{'}) - B_1^{'}/3$

$D_2 = D \sin(\pi/3 + \alpha^{'}) - B_1^{'}/3$

$D_3 = D \sin(-\pi/3 + \alpha^{'}) - B_1^{'}/3$

$D = (2/3)((B_1^{'})^2 - 3B_2^{'})^{1/2}$

$\alpha^{'} = (\sin^{-1}(-4q^{'}/D^3))/3$

$q^{'} = B_3^{'} - (B_1^{'} B_2^{'})/3 + 2(B_1^{'})^3/27$

$A_1^{'} = 2\,(\hat{\hat{z}}_1 + \hat{\hat{z}}_2) + \overline{Z}_3$

$A_2^{'} = \hat{\hat{z}}_1 \hat{\hat{z}}_2 + (\hat{\hat{z}}_1 + \hat{\hat{z}}_2 + \hat{\hat{z}}_3)\,(\hat{\hat{z}}_1 + \hat{\hat{z}}_2)$

$A_3^{'} = \hat{\hat{z}}_1 \hat{\hat{z}}_2 + (\hat{\hat{z}}_1 + \hat{\hat{z}}_2 + \hat{\hat{z}}_3)$

$B_1^{'} = \lambda_0 + \lambda_1 + \lambda_{12} + 2\,(\hat{\hat{z}}_1 + \hat{\hat{z}}_2) + \hat{\hat{z}}_3$

$B_2^{'} = \lambda_0\,(\hat{\hat{z}}_1 + 2\,\hat{\hat{z}}_2 + \hat{\hat{z}}_3) + \lambda_1\,(2\,\hat{\hat{z}}_1 + \hat{\hat{z}}_2 + \hat{\hat{z}}_3) + 2\lambda_{12}\,(\hat{\hat{z}}_1 + \hat{\hat{z}}_2) + \hat{\hat{z}}_1 \hat{\hat{z}}_2 + (\hat{\hat{z}}_1 + \hat{\hat{z}}_2 + \hat{\hat{z}}_3)\,(\hat{\hat{z}}_1 + \hat{\hat{z}}_2)$

$B_3^{'} = \lambda_0\,\hat{\hat{z}}_1\,(\hat{\hat{z}}_1 + \hat{\hat{z}}_2 + \hat{\hat{z}}_3) + \lambda_1\,\hat{\hat{z}}_1\,(\hat{\hat{z}}_1 + \hat{\hat{z}}_2 + \hat{\hat{z}}_3) + \lambda_{12}\,\hat{\hat{z}}_1\,(\hat{\hat{z}}_1 + \hat{\hat{z}}_2) + \lambda_{12}\,\hat{\hat{z}}_2 \hat{\hat{z}}_2 + \hat{\hat{z}}_1 \hat{\hat{z}}_2\,(\hat{\hat{z}}_1 + \hat{\hat{z}}_2 + \hat{\hat{z}}_3)$

here we have to read λ_0, λ_1, λ_{12} as

$\lambda_0 = (\hat{\hat{x}}_1 + \hat{\hat{y}} p_1 (1 - p_2))$

$\lambda_1 = (\hat{\hat{x}}_2 + \hat{\hat{y}} p_2 (1 - p_1))$

$\lambda_{12} = (\hat{\hat{y}} p_1 p_2 + \hat{\hat{w}})$

Where, $\hat{\hat{x}}_1$, $\hat{\hat{x}}_2$, $\hat{\hat{y}}$, $\hat{\hat{w}}$, $\hat{\hat{z}}_1$, $\hat{\hat{z}}_2$ & $\hat{\hat{z}}_3$ are the maximum likelihood estimates of individual non-identical

are the maximum likelihood estimates of individual non-identical failure rates of first and second components *(λ_{i1} & λ_{i2}),* NCCS failure rate 'β' and LCCS failure rate 'ω' and repair rates of the components μ_0, μ_1, μ_{12} respectively .

SIMULATION AND VALIDITY.

The proposed estimates of availability time-dependent by maximum likelihood estimation approach do not find analytical form of density and it is not possible to attempt or develop analytical verification of properties of proposed M L estimates. Hence in this paper empirical approach is considered and Monte Carlo simulation procedure is used for validity of results.

For a range of specified values of the rates of individual (λ_{i1}, λ_{i2}), LCCS failures (ω), NCCS failures (β) and service rates

(μ_0, μ_1, μ_{12}) and for the samples of sizes n = 5 (5) 30 are simulated using computer package in this paper and the sample estimates are computed for N = 10000 (20000) 90000 and mean square error (MSE)of the estimates for Av_{LNS} *has* obtained and given in numerical illustration. For large samples M L estimates are undisputedly better since they are CAN estimators. However it is observed that for a sample size as low as (n=5) M L estimate is still reasonably good and gives more accurate.

Numerical Validity:

Time dependent availability – Series Configuration with

$$\lambda_{i1} = 0.03, \lambda_{i2} = 0.04, \beta = 0.4, \ \ \omega = .001, \mu_0 = 3, \mu_1 = 2, \mu_{12} = 0.1, p_1 = 0.05, p_2 = 0.05, t = 1:$$

Sample Size (n= 5)			
N	$Av_{LNS}(t)$	$\widehat{Av_{LNS}}(t)$	M S E
10000	0.9594	0.9499	0.00058
30000	0.9593	0.9499	0.00058
50000	0.9593	0.9501	0.00056
70000	0.9593	0.9502	0.00056
90000	0.9593	0.9501	0.00056
Sample Size (n = 10)			
10000	0.9593	0.9551	0.00018
30000	0.9593	0.9549	0.00018
50000	0.9593	0.9548	0.00018
70000	0.9593	0.9549	0.00018
90000	0.9593	0.9549	0.00018
Sample Size (n = 15)			
10000	0.9593	0.9561	0.00010
30000	0.9593	0.9563	0.00010
50000	0.9593	09563	0.00010
70000	0.9593	0.9563	0.00010
90000	0.9593	0.9592	0.00010
Sample Size (n = 20)			
10000	0.9593	0.9569	0.000075
30000	0.9593	0.9569	0.000073
50000	0.9593	0.9568	0.000074
70000	0.9593	0.9563	0.000074
90000	0.9593	0.9568	0.000074
Sample size(n=25)			
10000	0.9593	0.9573	0.000057
30000	0.9593	0.9572	0.000057
50000	0.9593	0.9572	0.000057
70000	0.9593	0.9572	0.000057
90000	0.9593	0.9572	0.000057
Sample Size(n = 30)			
10000	0.9593	0.9574	0.000048
30000	0.9593	0.9575	0.000046
50000	0.9593	0.9575	0.000046
70000	0.9593	0.9575	0.000047
90000	0.9593	0.9575	0.000047

CONCLUSIONS

This paper attempts to evaluate the estimates of system availability function [Av_{LNS} (t)] of two-unit non-identical system under the influence of LCCS and NCCS failures along with individual failures for both series and parallel systems. The M L method proposedhere is giving almost accurate estimation in the case of sample size 20 and above (n ≥ 20) which is verified by simulation processes in the absence of analytical approach. Thus empirical evidence was developed which indicate that mean squared error is found very small and satisfactory for the estimation process. Therefore, the M L estimation approach to evaluate the estimates of the reliability measures like availability (time-dependent, steady-state) for various configurations with samples of size as low as n = 5 is reasonably appropriate as the MSE is very low. This shows that M L estimation approach is satisfactory and is quite useful in estimating reliability indices.

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