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Application of Weighted Fuzzy Exponential J-Divergence Measure in Engiography

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Abstract

The idea of measure of fuzziness in image processing was developed by Pal and Bezdek in 1994. Thresholding is the popular technique for image processing/segmentation. In local thresholding image is partitioned into several sub regions and for each sub-region a threshold is determined. Techniques for thresholding can be classified as bi-level thresholding and multi-thresholding. In bi-level thresholding, image is partitioned in to two parts or regions, one is object and other is background. In the present communication, an exponential weighted fuzzy divergence and the weighted fuzzy exponential J-divergence and presented the general technique of grey level thresholding. Further, the concept of fuzziness and membership function for the J-divergence exploiting Gamma distribution to be utilized in grey level thresholding for image processing/engiography are presented. Lastly the application of weighted fuzzy exponential J-Divergence in image processing / engiography and its methodology / algorithm is established through the experimental results and the table of weighted fuzzy exponential J-divergence with grey level.

Keywords: Weighted Fuzzy Exponential Divergence (WFED), Weighted Fuzzy Exponential J-Divergence (WFEJ-D), Gamma Distribution Grey Level, Thresholding Technique.

1. INTRODUCTION

Zadeh proposed fuzzy set theory in 1965 which has gained much importance in various fields such as, fuzzy logic social science, Cybernetics, Engineering, Management, Medicine, Environmental Modeling, Multi Media, Signal Processing, Image Processing etc.

The idea of measure of fuzziness in image processing was developed by Pal and Bezdek in 1994. The weighted entropy allows subjective quantification of the information on content associated with each event in (X, P). In the same spirit, a similar concept for any discrete fuzzy set A ε $P_n(X)$, the "weighted fuzziness" was defined as :

$$H_{WFE}(A;W) = \sum_{i=1}^{n} w_i g(\mu_i) \tag{1.1}$$

where $$W = \left\{ w_i \geq 0, \forall i \mid \sum_{i=1}^{n} w_i = 1 \right\}$$

and g can have either multiplicative or additive forms.

The S-function was defined by letting L = {1, 2, …., α} be the set of grey level and $\mu_\beta : \alpha \to [0, 1]$ be

$$\mu_\beta(l) = S(l;a,b,c) = \begin{cases} 0, & l \leq a \\ 2\left\{\dfrac{l-a}{c-a}\right\}^2, & a \leq l \leq b \\ 1-2\left\{\dfrac{l-c}{c-a}\right\}^2, & b \leq l \leq a \\ 1, & l \geq c \end{cases} \tag{1.2}$$

where $b = \frac{a+c}{2} . \mu_\beta(l)$ can be interpreted on the membership of the grey level l in the fuzzy set WHITE(BRIGHT). We note that $\mu_\beta(l) = 0.5$ as l increases above b membership in BRIGHT increases, conversely, as l decreases below b membership decreases. The b point is called crossover point/point of inflexion. The point of inflexion minimized the entropy of the fuzzy image, being the best threshold. Hence thresholding, a popular tool for image segmentation is being used for extracting the objects from the picture. In this technique segments are considered as different fuzzy subsets of the image.

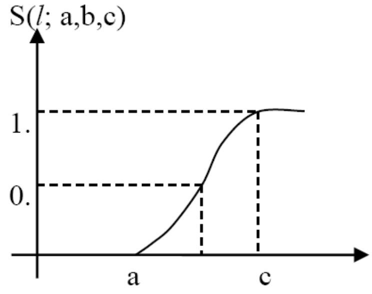

Fig.1: Graph of S

To minimize the class of separability Otsu (1979) used the thresholding technique, while information theoretic measures are used to threshold an image by Kapur et al (1985), Sahoo et al (1988), Sahoo and Wang (1989), Brink and Pendcock (1996) to threshold an image. Concept of spectral fuzzy sets is introduced by Pal and Dasgupta (1992) to find the membership value and then image segmentation.

According to Huang and Wang (1995) membership value is assigned by considering the reciprocal of the absolute difference of pixel and mean of the region containing that pixel. To select the best threshold neural network is used by Rama et al (2000) by using fuzzy measures viz. linear and quadratic indexes of fuzziness, logarithmic and exponential entropy. For image thresholding fuzzy homogeneity vectors and fuzzy co-occurrence matrix are used by Cheng and Chen (1997).

Membership function of the pixels of an image is determined by gamma distribution and then technique of minimization of fuzzy divergence defined by Chaira and Ray (2003) is used. In this technique divergence between the pixels in an ideally threshold image and the actual threshold image is considered. Gupta and Kumari (2014) examined the bounds and inequalities between weighted fuzzy mean difference-divergence measures.

In this communication, the fuzzy exponential J-divergence is generalized by taking the concept of weightage. Also we emphasize the method of thresholding for the proposed measure.

2. WEIGHTED FUZZY EXPONENTIAL DIVERGENCE AND J-DIVERGENCE

In goal oriented problems, the importance of the event has been the criteria for human beings in real life problems. Reliability is another aspect in real life problems. A lot of literature is available in weighted fuzzy indices viz. entropy, divergence and inaccuracy etc. So we consider the following information scheme for the purpose of weighted exponential fuzzy divergence to be used here :

$$\begin{bmatrix} E \\ \mu_A \\ \mu_B \\ W \end{bmatrix} = \begin{bmatrix} E_1 & E_2 \ldots\ldots\ldots E_n \\ \mu_A(E_1) & \mu_A(E_2)\ldots\mu_A(E_n) \\ \mu_B(E_1) & \mu_B(E_2)\ldots\mu_B(E_n) \\ w_1 & w_2 \ldots\ldots\ldots w_n \end{bmatrix} \tag{2.1}$$

and define the weighted fuzzy divergence corresponding to Bandari and Pal [1993] fuzzy divergence :

$$D_F(A,B;W) = \sum_{i=1}^{n} w_i \begin{bmatrix} \mu_A(E_i)\log\dfrac{\mu_A(E_i)}{\mu_B(E_i)} + (1-\mu_A(E_i)) \\ \times \log\dfrac{(1-\mu_A(E_i))}{(1-\mu_B(E_i))} \end{bmatrix} \tag{2.2}$$

By using single vector in fuzzy exponential entropy fuzzy divergence is proposed by Fan and Xie (1999). This concept of divergence introduced by Fan and Xie was extended to an image is represented by matrix.

The exponential entropy for an image of size M×M having probabilities (p_1, p_2,…p_{L-1}) and L different grey level was defined as :

$$H(P) = \sum_{i=1}^{L-1} p_i e^{1-p_i} \tag{2.3}$$

The weighted fuzzy exponential entropy for the above image is defined as :

$$H(A;W) = \frac{1}{n\left(\sqrt{e}-1\right)} \sum_{i=0}^{M-1}\sum_{j=0}^{M-1} w_{ij}\left[\left(\mu_A f_{ij}\right)e^{1-\mu_A f_{ij}} + \left(1-\mu_A f_{ij}\right)e^{\mu_A f_{ij}} - 1\right] \tag{2.4}$$

where

$n = M^2$ and i, j = 0, 1, 2,……., M−1,

$\mu_A f_{ij}$ is the membership values of the pixels in the image and f_{ij} is the (i,j)th pixel of the image A.

The weighted fuzzy exponential divergence between image A and B is given as :

$$D_{FAW}(A,B;W) = \sum_{i=0}^{M-1}\sum_{j=0}^{M-1} w_{ij}\left[1-\left(1-\mu_A f_{ij}\right)e^{\mu_A f_{ij}} - e^{\mu_B f_{ij}} - \mu_A f_{ij} e^{\mu_B f_{ij}} - \mu_A f_{ij}\right] \tag{2.5}$$

In the same fashion, the divergence of B from A may be given as :

$$D_{FBW}(B,A;W) = \sum_{i=0}^{M-1}\sum_{j=0}^{M-1} w_{ij}\left[1-\left(1-\mu_B f_{ij}\right)e^{\mu_B f_{ij}} - e^{\mu_A f_{ij}} - \mu_B f_{ij} e^{\mu_A f_{ij}} - \mu_B f_{ij}\right] \tag{2.6}$$

Hence the Weighted Fuzzy Exponential J-Divergence is defined as :

$$\begin{aligned} J_{F(A\cup B)W} &= D_{FAW}(A,B;W) + D_{FBW}(B,A;W) \\ &= \sum_{i=0}^{M-1}\sum_{j=0}^{M-1} w_{ij} \begin{bmatrix} 2-\left(1-\mu_A f_{ij}+\mu_B f_{ij}\right)e^{\mu_A f_{ij}} - e^{\mu_B f_{ij}} \\ -\left(1-\mu_B f_{ij}+\mu_A f_{ij}\right)e^{\mu_B f_{ij}} - \mu_A f_{ij} \end{bmatrix} \end{aligned} \tag{2.7}$$

3. GREY LEVEL THRESHOLDING

In local thresholding image is partitioned into several sub regions and for each sub-region a threshold is determined. Techniques for thresholding can be classified as bi-level thresholding and multi-thresholding. In bi-level thresholding, image is partitioned in to two parts or regions, one is object and other is background. In multilevel thresholding, several characteristics such as (light intensity image, objects having different coefficient of reflection, objects with different depths in range image) of an image are taken into consideration, then several thresholds for image segmentation are needed. We handle that situation by considering a set of thresholds (t_1, t_2, …, t_i) having all pixels with i = 0, 1, .., k constituting the i^{th} region type (t_0 and t_{k+1} are taken as 0 and L − 1 respectively).

When the regions are distinct, or in other words we can say that when the image is composed of regions with different grey level ranges different peaks are showed in the histogram of the image, each corresponding to one region and other is separated by a deep valley. For example if image has different object on a background the grey level histogram represents a deep valley which is called bimodal. In this case bottom of the valley is considered as threshold to separate object and background. When the histogram has a (or a set of) deep valley (S) selection of threshold (s) is not trivial. Hence detecting valleys for multiple thresholding is problematic. Different methods are available for this. In next section we present method of thresholding through weighted fuzzy exponential divergence.

4. FUZZINESS AND MEMBERSHIP FUNCTION

Let X = {f_{ij}, $\mu(f_{ij})$}, $\forall$ f_{ij} ε X} be an image of size M × M having L levels and f_{ij} be grey level of (i, j)th pixel in X. Let $\mu(f_{ij})$ denote the membership value of the (i, j)th pixel in X, where

$0 \le \mu(f_{ij}) \le 1$, with$\mu(f_{ij}) = 1$ full membership and $\mu(f_{ij}) = 0$, non-membership.

The membership function for a given threshold is derived by using gamma distribution. Value of absolute difference between the region containing pixel and grey level of that pixel is considered. It is observed that membership function is proportional to the exponential function of negative of the above said absolute difference. It is clear that this is inversely proportional to the membership value.

Let the count (f) denote the number of occurrences of the grey level f in the range. Given a certain threshold value t, which separates the object and the background, the average grey level of the background region is given by the relation :

$$\mu_0 = \frac{\sum_{f=0}^{t} f.\text{count}(f)}{\sum_{f=0}^{t} \text{count}(f)} \tag{4.1}$$

and the average grey level of the object region is given by

$$\mu_1 = \frac{\sum_{f=t+1}^{L-1} f.\text{count}(f)}{\sum_{f=t+1}^{L-1} \text{count}(f)} \tag{4.2}$$

4.1 The Gamma Distribution

The membership function of each pixel in the image depends on its affinity to the region to which it belongs. Gamma Distribution is used to find out the membership values of the pixels.

$$f(x) = \frac{\left(\frac{x-\nu}{\beta}\right)^{\gamma-1} \exp\left(\frac{-(x-\nu)}{\beta}\right)}{\Gamma(\gamma)} \tag{4.3}$$

Where γ is a shape parameter, ν is the location parameter and β is a scalar parameter and Γ is the Gamma function given by

$$\Gamma(\gamma) = \int_0^{\infty} u^{\gamma-1} e^{-u} du \tag{4.4}$$

Here we have two cases :

Case I : When $\nu = 0$, $\beta = 1$, the distribution takes the form

$$f(x) = \frac{x^{\gamma-1}\exp^{(-x)}}{\Gamma(\gamma)}, x \ge 0, \gamma > 0 \tag{4.5}$$

which is the Standard Gamma distribution.

Case II : When $\nu \neq 0$, $\beta = 1$ and $\gamma = 1$, the Gamma Distribution takes the form

$$f(x) = \exp^{(-(x-\nu))} \tag{4.6}$$

as $\Gamma 1 = 1$

Now, we have

$\mu(f_{ij}) = \exp(-c.|f_{ij} - \mu_0|)$, if $f_{ij} \le t$ for background

$= \exp(-c.|f_{ij} - \mu_1|)$, if $f_{ij} > t$ for object (4.7)

where t is any chosen threshold as stated above.

In the membership function, the constant 'c' is considered to ensure membership of the grey level feasible in the range [0, 1]. The value of 'c' can be chosen as

$$c = \frac{1}{(f_{max} - f_{min})} \tag{4.8}$$

Where f_{min} and f_{max} are the minimum and the maximum grey level in the image respectively. The absolute value of the distance between the mean of the region to which a pixel belongs and the grey level of that pixel is considered.

For tri-level thresholding, where there are three regions in the image, two thresholds values t_1 and t_2 are selected such that $0 \le t_1 < t_2 \le L-1$, where L is the maximum grey level of the image, the membership function in case of tri-level thresholding will take the following form

$\mu(f_{ij}) = \exp(-c_1.|f_{ij} - \mu_0|)$, if $f_{ij} \le t_1$

$= \exp(-c_1.|f_{ij} - \mu_1|)$, if $t_1 < f_{ij} \le t_2$

$= \exp(-c_1.|f_{ij} - \mu_2|)$, if $f_{ij} > t_2$ (4.9)

Where μ_0, μ_1 and μ_2 are average grey levels for the three regions separated by the thresholds t_1 and t_2 and the constant 'c_1' is considered in the same manner we choose 'c' in equation (4.8).

5. METHODOLOGY/ ALGORITHM

A searching methodology based on image histogram is considered for bilevel or multilevel thresholding. The region for searching is the region between two consecutive peaks. In multilevel thresholding the search regions are selected for each valley corresponding to each successive and preceding peaks of image histogram. We compare the membership values of the threshold image with ideally threshold image corresponding to each threshold value. Thus the equation (2.7) of the weighted exponential fuzzy divergence reduces to

$$J_{F(A\cup B)W}(A,B;W) = \sum_{i=0}^{M-1}\sum_{j=0}^{M-1} w_{ij} \begin{bmatrix} 2-\left(1-\mu_A f_{ij}+1\right)e^{\mu_A f_{ij}-1} \\ -\left(1-1+\mu_A f_{ij}\right)e^{1-\mu_A f_{ij}} \end{bmatrix} \tag{5.1}$$

As the membership values of each pixel in ideally threshold image (which is image B in this case) are taken as unity.

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Hence (5.1) may be presented after simplification as

$$J_{F(A\cup B)W}(A,B;W)=\sum_{i=0}^{M-1}\sum_{j=0}^{M-1} w_{ij}\begin{bmatrix}2-\left(2-\mu_A f_{ij}\right)e^{\mu_A f_{ij}-1}\\ -\mu_A f_{ij}.e^{1-\mu_A f_{ij}}\end{bmatrix} \quad (5.2)$$

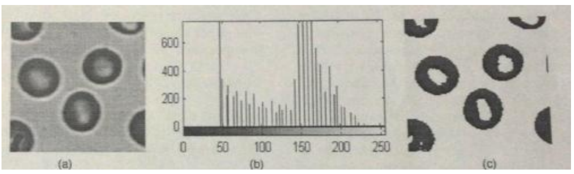
In an ideally threshold image, the image is segmented in such a way that the pixels which are in the object or in the background belong completely to the respective region. The J-Fuzzy exponential weighted divergence is found out by using the value of each pixel between ideally segmented image and our selected threshold image. For good thresholding, membership value of each pixel in the threshold image is expected to lie close to that of ideally threshold image. If a pixel lies in the object or background, it should belong to the corresponding region totally. Weighted fuzzy exponential J-divergence for each threshold is calculated. According to equation (5.2) and then Commulative divergence is computed for the whole image. Minimum divergence is found out and the optimum threshold is chosen by corresponding grey level of the minimum J-divergence we can find out a measure of maximum belongingness of each object/background pixel to their corresponding regions. The thresholded image leads almost towards the ideally threshold image after thresholding.

6. EXPERIMENTAL RESULTS

The method of thresholding explained above is tested on bimodal, in order to evaluate the effectiveness of the proposed method. Several images were tested, the images are in 'tif' format as follows

The 'blood' of size 128×128 (bimodal) Fig. 2(a) and (b) shows the 'Blood' image of size 128×128 and its bimodal histogram. Fig. 2(c) shows the threshold image, which is thresholded at grey level |3|.

Image	Divergence Value	Divergence value where Wi = 2, ∀ i	Grey Level
Blood	117.0447	234.894	107-113



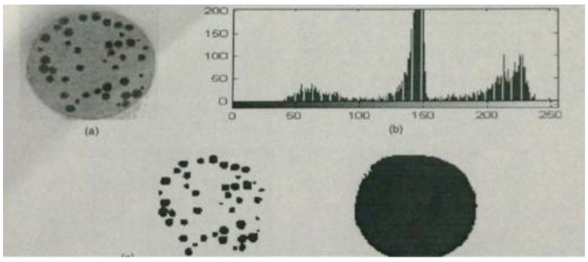


Fig.1(a) Input image 'Blood1' image, (b) its histogram, (c) thresholded image

7. CONCLUSION

In this communication the concept of weighted fuzzy divergence as well as J-divergence, due to the non-symmetry of the fuzzy divergence is presented to choose the best threshold for object extraction. The threshold obtained in this case provides optimal segmentation of the object from the background. The method is based on the minimization of the weighted fuzzy J-divergence of the image. To locate the deep valley of the histogram, we used the concept of fuzzy range. The concept non symmetric weighted fuzzy divergence as well as J-divergence is presented to choose the best threshold for extraction of object from background.

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