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Wavelet Based Adaptive Sliding Mode Control for Delayed Uncertain Non Linear Systems

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Abstract

This paper addresses the issue of adaptive sliding mode controller design for uncertain delayed systems. Wavelet networks, which have been proved an effective and efficient approximators as compare to classical neural network, are used for approximation of uncertain systems dynamics and the inherent robustness property of the sliding mode control is utilized to make the system performance insensitive to the approximation errors inserted by wavelet networks. Adaptation laws for wavelet networks are derived in the sense of Lyapunov function assuring the systems stability.

Keywords: Sliding mode control, wavelet network, nonlinear systems, delayed systems.

I. INTRODUCTION

Sliding mode control (SMC) has been proved to be very effective controlling strategy for nonlinear systems with uncertain dynamics and subjected to disturbances. The sliding mode control utilizes a variable structure scheme that drives state trajectories toward a specific hyper plane and thereafter maintains the trajectories sliding on hyper plane until the origin of the state space is reached [1]. Robustness, can be considered as one of the distinguished property of sliding mode control, this property arises due to sliding hyper plane and switched control settings by the consideration of modeling uncertainties and disturbances. Chattering phenomenon is a major issue of concern associated with sliding mode control. Chattering often results in excitation of high frequency unmodelled dynamics which may result in system instability. One commonly used technique to attenuate the chattering is the insertion of a boundary layer in the vicinity of sliding surface. This technique results tradeoff between tracking precision and the input quality of chattering[1-5].

Wavelet networks plays an important role in the controller designing for nonlinear systems with uncertain dynamics due to their universal approximation capability, rapid learning rate and fast conversions, they are used to approximate a wide range of nonlinear function with arbitrary accuracy[6-9]. Architecture of feed forward wavelet network can be viewed as a network composed of single layer of translated and dilated versions of mother wavelet function. Development of wavelet networks, mathematical properties and approximation capabilities of wavelet networks are explored in [8,9]. Application of wavelet network in controller design has been cited in[10-14].

Time delay is often encountered in practical systems, presence of time delay results in degradation of control performance and potential instability of the systems. Several control schemes for delayed nonlinear systems have been cited in the literature. Most of the schemes are based on Lypenov Krasovskii functional [15,16].

This paper presents an adaptive sliding mode control schemes for time delayed uncertain systems. The proposed controller integrates the sliding mode control strategy with wavelet neural networks to insure the effective tracking performance of state variables. The stability of the system is insured by constructing an appropriate Lyapunov Krasovskii functional in the control design.

The organization of the paper is as under. In section 2 fundamentals of wavelet network are discussed. System discription and adaptive sliding mode control schemes along with stability analysis are presented in Section 3. Simulation results are illustrated in Section 4, whereas Section 5 concludes the paper .

II. BASICS OF WAVELET NETWORK

Wavelet network is an efficient architecture meant for function approximation. The wavelet network is realized as an architecture composed of some translated and dileted versions of a wavelet function. The feed forward architecture of wavelet neural network can be modified by inserting the recurrences. These recurrent architectures are suitable for the approximation of nonlinearities which are the function of delayed states[17,18].

Figure 1. shows Feed-Forward WNN with self feedback wavelon layer as a modified version of SRWNN. Self feedback wavelon layer enables the wavelet network (SRWNN) to store past information, high degree approximation of dynamic nonlinearities and more convenient for adaptive control as compared with conventional.

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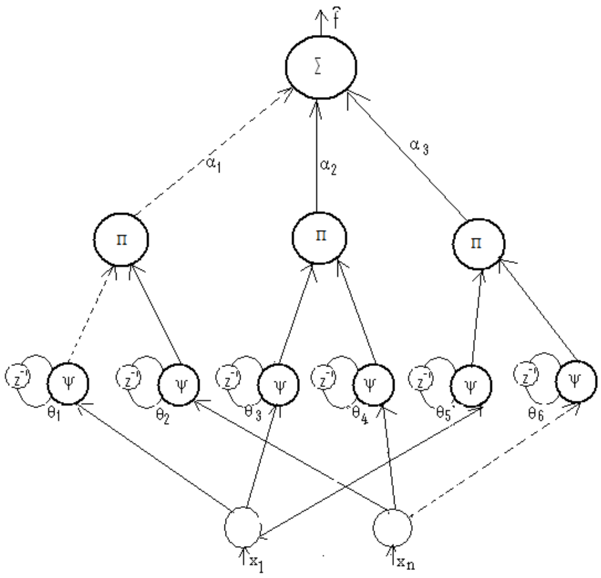

Figure 1. Self Recurrent Wavelet Neural Network

Output of an 'n' dimensional SRWNN with 'm' wavelet nodes can be expressed as

$$f(x(t), x(t-\tau)) = \sum_{i=1}^{m} \alpha_i(t)\varphi_i\begin{pmatrix}\theta_i(t), \bar{\varphi}_i(t-\tau), \\ x(t), w_i(t), c_i(t)\end{pmatrix} \quad (1)$$

where φ_i is the i^{th} wavelet node given by

$$\varphi_i(x, \theta_i, \bar{\varphi}_i, w_i, c_i) = \prod_{j=1}^{n} \varphi_{ij}(x_j, \theta_{ij}, \bar{\varphi}_{ij}, w_{ij}, c_{ij}) \quad (2)$$

where φ_{ij} is the i^{th} wavelet node with j^{th} wavelon. $x(t) \in R^n$ shows input vector, with where τ is the point delay encountered in state vector.

$\bar{\varphi}_i = [\bar{\varphi}_{i1}, \bar{\varphi}_{i2}, \ldots, \bar{\varphi}_{in}]$ represents the feedback argument for the i^{th} wavelet node. The Network's previous information stored in the vector $\bar{\varphi}_i = [\bar{\varphi}_{i1}, \bar{\varphi}_{i2}, \ldots, \bar{\varphi}_{in}]$. $\theta_i = [\theta_{i1}, \theta_{i2}, \ldots, \theta_{in}]$ shows weighted vector , $w_i = [w_{i1}, w_{i2}, \ldots, w_{in}]$ and $c_i = [c_{i1}, c_{i2}, \ldots, c_{in}]$ represents are dilate and translate vectors for feedback input. This input vectors applied on i^{th} wavelet node which represents as follows-

$$[x_1 + \bar{\varphi}_{i1}\theta_{i1}, x_2 + \bar{\varphi}_{i2}\theta_{i2}, \ldots, x_n + \bar{\varphi}_{in}\theta_{in}]^T.$$

In matrix form (1) can be rewritten as

$$f = \alpha^T \varphi(x, \theta, \bar{\varphi}, w, c) \quad (3)$$

where

$w = [w_1, w_2, \ldots, w_m]^T \in R^{mn}$ and $c = [c_1, c_2, \ldots, c_m]^T \in R^{mn}$ are dilation and translation parameters respectively ; $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_m]^T \in R^m$ and $\theta = [\theta_1, \theta_2, \ldots, \theta_m]^T \in R^{mn}$ are the output and feedback weights respectively. $\bar{\varphi} = [\bar{\varphi}_1, \bar{\varphi}_2, \ldots, \bar{\varphi}_m]^T \in R^{mn}$ shows feedback, input vector of SRWNN.

With ideal wavelet approximator, let f^* be the optimal function approximation

$$f = f^* + \Delta = \alpha^{*T}\varphi^* + \Delta \quad (4)$$

where $\varphi^* = \varphi(x, \theta^*, \bar{\varphi}, w^*, c^*)$ and the optimal parameter vectors are α^*, w^*, c^*, θ^* of α, w, c, θ respectively, and the approximation error denoted by Δ with assumption $|\Delta| \le \Delta^*$,where Δ^* is a positive constant [13].

The optimal parameter vectors for paramount approximation are difficult to determine. The definition of estimation functions is as follows-

$$\hat{f} = \hat{\alpha}^T \hat{\varphi} \quad (5)$$

where $\hat{\varphi} = \varphi(x, \hat{w}, \hat{c}, \hat{\theta}, \bar{\varphi})$ and $\hat{\alpha}, \hat{w}, \hat{c}, \hat{\theta}$ are the estimates of $\alpha^*, w^*, c^*, \theta^*$ respectively. Defining the estimation error as

$$\tilde{f} = f - \hat{f} = f^* - \hat{f} + \Delta = \tilde{\alpha}^T\hat{\varphi} + \hat{\alpha}^T\tilde{\varphi} + \tilde{\alpha}^T\tilde{\varphi} + \Delta \quad (6)$$

where $\tilde{\alpha} = \alpha^* - \hat{\alpha}$, $\tilde{\varphi} = \varphi^* - \hat{\varphi}$

With the appropriate number of nodes, the estimation error $\tilde{f}$ can be reduced to arbitrarily small value on the compact set so that the bound $\|\tilde{f}\| \le \tilde{f}_m$ holds for all $x \in \Re$.

As the wavelet network representation is nonlinear in parameter which respect to certain adjustable parameters, to facilitate the derivation of tuning laws a Taylor expansion based following partial linearization is carried out [13,14].

$$\tilde{\varphi} = A^T\tilde{w} + B^T\tilde{c} + C^T\tilde{\theta} + h \quad (7)$$

where $\tilde{w} = w^* - \hat{w}$, $\tilde{c} = c^* - \hat{c}$, $\tilde{\theta} = \theta^* - \hat{\theta}$ and h are the vectors of higher order terms and

$$A = \left[\frac{d\varphi_1}{dw}, \frac{d\varphi_2}{dw}, \ldots, \frac{d\varphi_m}{dw}\right]_{w=\hat{w}}$$

$$B = \left[\frac{d\varphi_1}{dc}, \frac{d\varphi_2}{dc}, \ldots, \frac{d\varphi_m}{dc}\right]_{c=\hat{c}}$$

$$C = \left[\frac{d\varphi_1}{d\theta}, \frac{d\varphi_2}{d\theta}, \ldots, \frac{d\varphi_m}{d\theta}\right]_{\theta=\hat{\theta}}$$

with

$$\frac{d\varphi_i}{dw} = \left[0, \ldots, 0, \frac{d\varphi_i}{dw_{1i}}, \frac{d\varphi_i}{dw_{2i}}, \ldots, \frac{d\varphi_i}{dw_{ni}}, 0 \ldots 0\right]^T$$

$$\frac{d\varphi_i}{dc} = \left[0, \ldots, 0, \frac{d\varphi_i}{dc_{1i}}, \frac{d\varphi_i}{dc_{2i}}, \ldots, \frac{d\varphi_i}{dc_{ni}}, 0 \ldots 0\right]^T$$

$$\frac{d\varphi_i}{d\theta} = \left[0, \ldots, 0, \frac{d\varphi_i}{d\theta_{1i}}, \frac{d\varphi_i}{d\theta_{2i}}, \ldots, \frac{d\varphi_i}{d\theta_{ni}}, 0 \ldots 0\right]^T$$

Substituting (7) into (6)

[illegible] (8)

the uncertain term expression are as follows-

[illegible]

Assumption of uncertainty ε satisfies the following Lipschitz condition

$$|\varepsilon| \le q_1 \|x(t)\| + q_2 \|x(t-\tau)\| + \zeta \quad (9)$$

where. $q_i \le 0, \zeta > 0$.

III. SYSTEM DISCRIPTION AND CONTROLLER DESIGN

To quantify the process of controller design, consider the following nonlinear system of the form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_n &= f(x(t), x(t-\tau)) + g(x(t))w(t) \\ y &= x_1 \end{aligned} \quad (10)$$

where $x = [x_1, x_2, ..., x_n]^T, y$ represents the state variable and output respectively where as u is the control effort applied to the system through an actuator with output $w(t)$. Function $g(.)$ is always bounded away from zero .It implies that $g(.)$ is strictly either positive or negative for all *x(t)*. $f(x(t), x(t-\tau)) : \Re^{2n} \to \Re$ is a smooth unknown, nonlinear function of present and delayed values of state variables, τ is the known time delay encountered in state variables. In this work the function $f(x(t), x(t-\tau))$ is approximated by a self recurrent wavelet network.

Objective is to design the control input *u* using sliding mode methodology so that the tracking error $(y(t) - y_d(t))$ converges to small neighborhood of origin. Here $y_d(t)$ is the desired trajectory, assumed to be smooth, continuous C^n and available for measurement.

Defining an error system by applying the change of coordinates for system (10) as

$$e = [e_1, e_2, ..., e_n]^T = x - y'_d = [x_1 - y_d, x_2 - \dot{y}_d, ..., x_n - y_d^{n-1}]^T$$

where

$$y'_d = [y_d, \dot{y}_d, ..., y_d^{n-1}]^T$$

so the resulting error dynamics becomes

$$\left.\begin{aligned} \dot{e}_i &= e_{i+1} 1 \le i \le n-1 \\ \dot{e}_n &= f(x) + g(x)u - y_d^n \end{aligned}\right\} \quad (11)$$

Defining a sliding hyper plane of the form

$$s = ke \quad (12)$$

where $k = [k_1, k_2, ..., k_n]$ are known positive terms.

Considering $f(x)$ to be the unknown dynamics of the system (10). As per the sliding mode theory the control scheme is supposed to drive the system variables such that the condition mentioned in (13) is satisfied and thereafter maintained until the origin is reached[1].

$$s = 0 \quad (13)$$

The necessary and sufficient condition to satisfy (13) is

$$s\dot{s} = -\lambda |s| \quad (14)$$

where $\lambda > 0$.

To satisfy (14) the sliding mode control of the form is defined

$$u = \frac{1}{g(x)}(k_e e - \hat{f}(x) + y_d^n - s - \rho sgn(s)) \quad (15)$$

here , $\hat{f}(x)$ is the estimates of $f(x)$, and is defined in (5), $k_e = [0, k_1, ..., k_{n-1}]$, $\rho > 0$ and sgn(.) is signum nonlinearity defined as

$$\begin{cases} \text{sgn}(s) = 1, & \text{if s} < 0 \\ \text{sgn}(s) = -1, & \text{if s} > 0 \end{cases} \quad (16)$$

Tuning laws for the online adjustment of wavelet parameters are given as

$$\begin{aligned} &\dot{\hat{\alpha}} = -\dot{\tilde{\alpha}} = \gamma_1 s(\hat{\varphi} - A^T \hat{w} - B^T \hat{c}); \dot{\hat{w}} = -\dot{\tilde{w}} = \gamma_2 s A\hat{\alpha} \\ &\dot{\hat{c}} = -\dot{\tilde{c}} = \gamma_3 s B\hat{\alpha}; \dot{\hat{\theta}} = -\dot{\tilde{\theta}} = \gamma_4 s C\hat{\alpha} \end{aligned} \quad (17)$$

$\gamma_1, \gamma_2, ..., \gamma_4$ are the learning rates with positive constants.

In next subsection the proposed control law is examined.

IV. STABILITY ANALYSIS

To perform the convergence analysis of the closed loop systems with control term (15), consider a Lyapunov-Krasovskii functional of the form [21]

$$V = \frac{1}{2}s^2 + \frac{\tilde{\alpha}^T \tilde{\alpha}}{2\gamma_1} + \frac{\tilde{c}^T \tilde{c}}{2\gamma_2} + \frac{\tilde{w}^T \tilde{w}}{2\gamma_3} + \frac{\tilde{\theta}^T \tilde{\theta}}{2\gamma_4} + \int_{t-\tau}^{t} \xi(\sigma) d\sigma \quad (18)$$

Differentiating (18) along the trajectories of the system

$$\dot V = s\left(K_e e - f(x(t),x(t-\tau)) + g(x)u(t) - \overset{n}{y_d}\right) +$$
$$\frac{\tilde\alpha^T\dot{\tilde\alpha}}{\gamma_1}+\frac{\tilde c^T\dot{\tilde c}}{\gamma_2}+\frac{\tilde w^T\dot{\tilde w}}{\gamma_3}+\frac{\tilde\theta^T\dot{\tilde\theta}}{\gamma_4}+\xi(t)-\xi(t-\tau))$$

Substitution of control law u (15) in the above equation yields

$$\dot V = s\left(\tilde f(x) - s - \rho sgn(s)\right) +$$
$$\frac{\tilde\alpha^T\dot{\tilde\alpha}}{\gamma_1}+\frac{\tilde c^T\dot{\tilde c}}{\gamma_2}+\frac{\tilde w^T\dot{\tilde w}}{\gamma_3}+\frac{\tilde\theta^T\dot{\tilde\theta}}{\gamma_4}+\xi(t)-\xi(t-\tau))$$

Substituting adaptation laws (17) in above equation,

$$\begin{aligned}
&\le s\left(-s-\rho sgn(s)+\varepsilon_f\right)+\xi(t)-\xi(t-\tau))\\
&\le \left(-s^2-\rho|s|+|\varepsilon_f||s|\right)+\xi(t)-\xi(t-\tau))\\
&\le \left(-s^2-\rho|s|+(q_1\|x(t)\|+q_2\|x(t-\tau)\|+\zeta)|s|\right)+\xi(t)-\xi(t-\tau))
\end{aligned} \tag{19}$$

Selecting $\xi(t)$ as

$$\xi(t)=q_1\|x(t)\||s| \tag{20}$$

Substituting (20) into (19)

$$\le\left(-s^2-\rho|s|+\zeta|s|\right) \tag{21}$$

Selecting ρ such that the following inequality is satisfied

$$\rho>\zeta \tag{22}$$

with (22) , equation (21) reduces to

$$\begin{aligned}
&\dot V \le -(\rho-\zeta)|s|\\
&\dot V \le -c|s|
\end{aligned} \tag{23}$$

Thus the properly selecting ρ sufficient condition for driving the state variables to sliding hyper plane $s=0$ can be achieved and this in turn indicates the asymptotic convergence of error variables to origin.

Thus we have the following result

Theorem: With the adaptive control term (10), and adaptation laws (17), in the presented closed loop architecture, the error variables of the uncertain plant (10) show asymptotic convergence to the origin.

Proof: As per the facts established on the basis of Lyapunov stability theory inequality (23) reflects that the signals of the closed loop systems comprising of designed controller and the system of the form(10) show uniform boundedness and the tracking error converges to origin.

V. SIMULATION RESULTS

This section presents a simulation study carried out to illustrate the efficacy of the proposed controller.

Consider the following nonlinear delayed system with actuator constraints

$$\begin{aligned}
&\dot x_1 = x_2\\
&\dot x_2 = x_3\\
&\dot x_3 = x_4\\
&\dot x_4 = 0.5x_1(t)x_2(t)x_3^2+0.5x_1x_2(t-\tau)+\frac{u}{x_1^2+x_2^2+1}\\
&y = x_1
\end{aligned} \tag{24}$$

System belongs to the class of nonlinear systems defined by (10), with $n=4$ and $\tau=1\sec$. Proposed controller is applied to this system with an objective to solve the tracking problem of system.

The desired trajectory is taken as $y_d=0.5\sin t+0.5\cos t$. Initial conditions are taken as $x(0)=[.75,0,0,0]^T$. Controller parameters are taken as $k_1=15, k_2=4, k_3=5, k_4=1, \rho=2$. The wavelet network is constructed by using Mexican hat wavelet as the mother wavelet. The dilation, translation and gain parameters of the wavelet network are tuned online using adaptation laws (17). As reflected by the figure 2, the system output is effectively approaching the desired trajectory within a small span of time, this indicates the efficiency of the wavelet network to approximates the unknown system dynamics. For simulation of parameter tuning laws initial conditions are set to zero.

Simulation results are shown in Figure 2 and 3. As observed from the Figure 2, system response tracks the desired trajectory rapidly and a steady state error converges to the close neighborhood of origin under the effect of control effort shown in Figure 3.

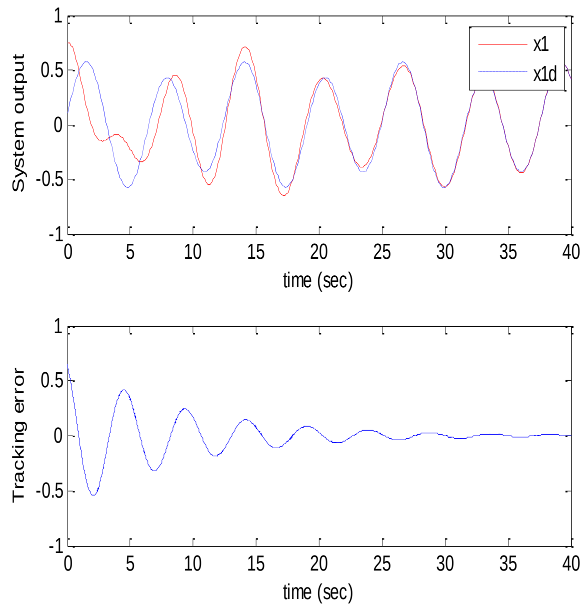

Figure 2. The system output, the reference signal, the tracking error

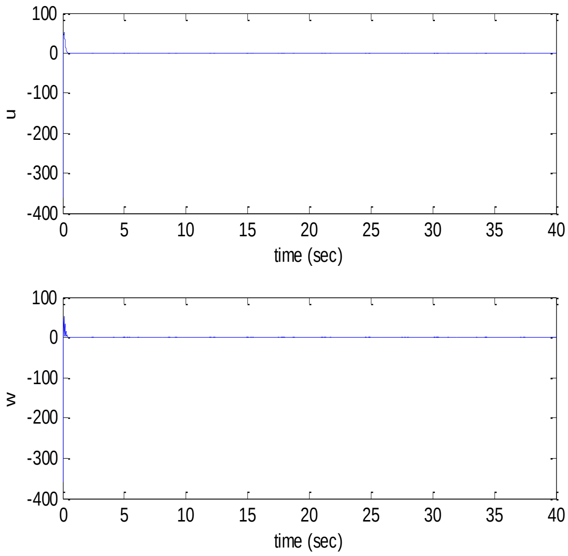

Figure 3. Control signal.

VI. CONCLUSION

In this paper the issue of controller design for a class of delayed nonlinear systems with uncertain dynamics is addressed. A closed loop system comprising of a sliding mode controller with wavelet approximator is realized. With the proposed controller the nonlinear system has shown a promising tracking performance which has been validated mathematically as well as through simulation.

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