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An Integrated Vendor-buyer Production Inventory System for Deteriorating Items with Time Varying Demand and Shortages for Buyer

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Abstract:
In this paper, we formulate and solve a production-inventory problem for a vendor-buyer integrated system. We consider constant deterioration rate of items, increasing time-varying demand rate and shortages of items with complete backlogging. We provide a solution procedure and an algorithm for finding optimal scheduling of delivery plans and optimal total cost for both models with and without shortages. We compare the results of the models with and without shortage using a numerical example. We also provide the optimal solution for constant demand. Sensitivity analysis is carried out to study the effects of changes in the values of different parameters in the model.

Keywords: inventory system, time-varying demand, constant deterioration, vendor-buyer, multiple deliveries.

1. INTRODUCTION

The inventory problems deal with the optimal replenishment/production plans of items which are procured and stored for meeting future demands. The demand rate for an item is an important factor since the inventory position changes in response to customers demand. Deterioration of items in the form of direct spoilage or gradual physical decay in course of time is a realistic phenomena. In real life situations, the stock level of an item in an inventory is continuously depleting due to the combined effect of its demand rate and deterioration rate. We need to maintain the inventory to satisfy the customers' future demand. As the demand increases with time, maintaining the inventory in huge amount is quite challenging because of two extreme situations - 'over stock' and 'running out of stock'. Some of the reasons for these two situations are:

(i) rapid change in the taste of customers

(ii) physical decay of the stocked items over time

(iii) requirement of large storage space

(iv) higher holding cost to maintain high inventory level

Under all these circumstances, we need to think about shortage of items. When there is a shortage, some of the customers will seek alternatives, while others will wait for the next replenishment. There are three different situations arising when shortages are allowed - complete backlogging of shortages, complete lost sale and partial backlogging of shortages. In the competitive market, complete lost sale or partial backlogging of shortages may lead to loss of goodwill which yields less profit. Hence for the smooth running of the business, one must allow backlogging of unsatisfied demand. Several researchers have developed models on Economic Order Quantity (EOQ) with different assumptions on demand and deterioration rate, with or without shortages, partial backlogging [[1], [2] and [3]] or complete backlogging. Balkhi and Benkherouf [4] developed a production lot size inventory model for deteriorating items where demand and production rate are varying with time in an arbitrary way. Shortages are allowed and completely backlogged. Benkherouf and Mahmoud [5] have presented an inventory model for deteriorating items with complete backlogging of shortages. They have considered time - varying demand rate and constant deterioration rate of items and presented a dynamic programming solution to their model. Wee and Law [6] developed a production-inventory model for deteriorating item where the rate of deterioration at any time follows two parameter Weibull distribution. Rate of demand is a decreasing linear function of the selling price and shortages are completely back ordered. Giri and Chaudhuri [7] developed a production – inventory model over an infinite planning horizon. Here the demand rate varies linearly with time, unit production cost varies as a function of the production rate, shortages are fully back ordered and the machine production rate is considered as a decision variable. Wee and Law [8] presented a deterministic inventory model for an item where demand rate is dependent on price, deterioration rate is varying with time and there is complete backlogging of unsatisfied demand. Wu [9] presented an EOQ inventory model in which inventory is depleted by both time varying demand rate and Weibull deterioration rate. Inventory cycle is started with shortages and ended without shortages and it is shown that there is a considerable decrease in both the order quantity and the system cost. Hariga [10] and Manna and Chaudhuri [11] showed through numerical illustrative example, that the inventory model with shortages and complete backlogging is considered to be better economically.

In today's global markets, because of increased competition, individual and independent entities can not compete as solely and can not survive for long run in business. It is better to work together as a supply chain to manage inventories more efficiently.

Due to rising costs, shrinking resources, shortened product life cycle, increasing competitive pressure and quicker response, much attention has been paid to the collaboration between the members of the supply chain. In a supply chain inventory system, buyer purchases products from vendor. The buyer may be a regular purchaser of goods from vendor based on long term agreements. The new global market requires long term close cooperation between vendor and buyer for their successful business. Trust between them would develop over time due to various factors, such as, quality of items, adherence to delivery schedule, permissible delay in payment etc.

Good relationship between the vendor and buyer may allow for the sharing of information, forecasts and knowledge between them. Close cooperation between them can result in more cost effective production and distribution as well as a faster response to customer's demand which creates a beneficial environment for them. Team work of vendor and buyer leads to considerable success in business. Hence coordination between vendor and buyer is an important way to gain competitive advantage in supply chain management as it lowers supply chain cost.

Decision regarding the shipment schedule and the size of shipment must be taken jointly by both vendor and buyer. The integration between them would result in reduction of wastage. The vendor-buyer integrated inventory model aims to determine the number of shipments from vendor to buyer, the shipment schedule and the size of shipment quantity in each shipment which minimizes the integrated total cost of the inventory system.

The integration approach has been studied by many researchers with different assumptions on the demand rate and the deterioration rate. [Yang and Wee ([12], [13]), Wee et al. [14], Yang and Pan [15], Ouyang et al. [16], Yang et al. [17], Lo et al. [18], Zanoni and Zavanella [19], Chung and Wee [20], Shah et al. [21], Aarya and Kumar [22]]. These studies have thrown light on the need and importance of vendor-buyer integrated inventory system. Also these studies have shown that if the number of deliveries is decided in cooperation of both the vendor and the buyer, then the overall integrated cost can be minimized. But only little work has been done on vendor-buyer system with shortages [Singh and Chandramouli [23] and Singh and Singh [24]]. But these two papers consist several missing information and there are some inconsistencies in the numerical example used for illustration of the models.
This paper is organized as follows: Assumptions and notations are given in section 2. Vendor's and buyer's inventory systems and total costs are derived in section 3. Integrated total cost is also given in section 3. In section 4, an algorithm to obtain the optimal solution is provided. Section 5 considers numerical example and its results. Section 6 gives sensitivity analysis and observation based on sensitivity analysis. Summary and scope for future work are provided respectively in sections 7 and 8.

2. ASSUMPTIONS AND NOTATIONS

The proposed model is developed using the following assumptions and notations

2.1 Assumptions

The following assumptions are made in developing the vendor-buyer inventory model.

1. A single-vendor single-buyer inventory system with single item is considered.
2. Both demand and production rates are functions of time, denoted by
$D(t) = a + bt$ and $P(t) = \alpha(a + bt)$ respectively where $\alpha > 1$.
3. Shortages are allowed and completely backlogged.
4. Initial and final inventories are zero.
5. Shortages are not allowed in the last replenishment cycle.
6. The items deteriorate at a constant rate θ $(0 < \theta < 1)$.
7. The deteriorated units can neither be repaired nor replaced during the cycle time.
8. Lead time is assumed to be zero.
9. The deterioration occurs only when the item is effectively in stock.
10. T is the time length of each cycle/replenishment cycle.
11. n is the number of deliveries per cycle time.
12. $t_i = (i-1)T/n$ is the time of the i^{th} replenishment, $i = 1, 2, 3, ..., n$.
13. s_i is the time at which the inventory level in the i^{th} replenishment cycle drops to zero, $i = 1, 2, 3, ..., n$ with $s_n = T$.
14. Multiple deliveries per order are considered.

2.2 Notations

The following notations are used throughout the paper

T_1	The length of production time (a decision variable)
T_2	The length of non-production time (a decision variable), $(T_1 + T_2 = T)$
$I_1(t)$	Inventory level for vendor when t is between 0 and T_1
$I_2(t)$	Inventory level for vendor when t is between 0 and T_2
$I_3(t)$	Inventory level for buyer when t is between t_i and s_i
$I_4(t)$	Inventory level for buyer when t is between s_i and t_{i+1}
c_1	Ordering cost per order for buyer
c_2	Holding cost per unit per unit time for buyer
c_3	Deteriorating cost per unit per unit time for buyer
c_4	Shortage cost per unit per unit time for buyer
c_5	Vendor's set up cost per cycle time
c_6	Holding cost per unit per unit time for vendor
c_7	Deteriorating cost per unit per unit time for vendor
TCV	Total cost per unit time for vendor
TCB	Total cost per unit time for buyer
TC	Integrated total cost

3. MATHEMATICAL MODEL AND SOLUTION

Main objective of developing this model is to obtain the optimal stock of items, production-shipment schedule in terms of the size and frequency of shipments transferred between both parties and optimal total cost of the system, when shortages are allowed.

Vendor's inventory system:
Vendor's inventory system can be explained with the help of Figure 1. Vendor's inventory system starts with zero level. The production starts at time $t = 0$. The rate of production increases with time. Since the production rate is greater than the demand rate, inventory level increases. During the interval $[0,\ T_1]$, inventory level depends on production, demand and deterioration of items. The production stops at time $t = T_1$ and inventory reaches its maximum level. During the interval $[0,\ T_2]$, there is no production and inventory level decreases because of combined effects of demand and deterioration. Inventory level reaches zero at the end of the cycle.

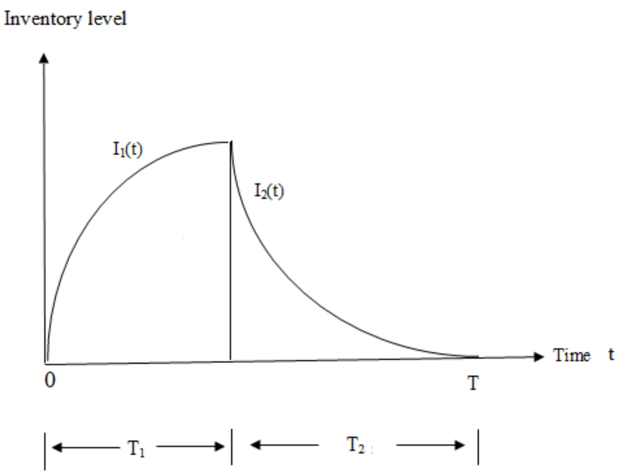

Figure 1: Vendor's inventory system

The vendor's inventory system at any time t during the interval $[0,\ T_1]$ is represented by the following differential equation:

$$\begin{aligned}\frac{dI_1(t)}{dt} &+ \theta I_1(t) = P(t) - D(t) \\ &= (a+bt)(\alpha-1),\ \ \alpha > 1,\ \ 0 \le t \le T_1 \quad (1)\end{aligned}$$

with initial condition $I_1(0) = 0$.
The vendor's inventory system at any time t during the interval $[0,\ T_2]$ is represented by the following differential equation:

$$\begin{aligned}\frac{dI_2(t)}{dt} + \theta I_2(t) &= -D(t) \\ &= -(a+bt),\quad 0 \le t \le T_2 \qquad (2)\end{aligned}$$

with boundary condition $I_2(T_2) = 0$.

The solutions of these differential equations are

$$I_1(t) = \int_0^t (a+bu)(\alpha-1)e^{\theta(u-t)}\,du,\ \ 0 \le t \le T_1 \quad (3)$$

$$I_2(t) = \int_t^{T_2} (a+bu)e^{\theta(u-t)}\,du,\ \ 0 \le t \le T_2 \quad (4)$$

Buyer's inventory system:
The buyer's inventory system can be explained with the help of Figure 2. The buyer's inventory system starts with a replenishment and ends without replenishment. We have considered an equal length of replenishment cycle. There are n replenishments during the cycle time T from vendor to buyer. Each replenishment is carried out at time $t = t_i,\ \ i = 1, 2, 3, ..., n$ where $t_i = (i-1)T/n$. Because of demand and deterioration of items, inventory level decreases and it reaches zero at time $t = s_i,\ \ i = 1, 2, 3, ..., n$. Due to demand, shortage of items starts to occur and accumulates during the interval $[s_i,\ t_{i+1}]$, where $i = 1, 2, 3, ..., n-1$. At the end of the last cycle, all accumulated shortages will be completely fulfilled, final inventory level becomes zero. Shortage is not allowed in the last cycle.

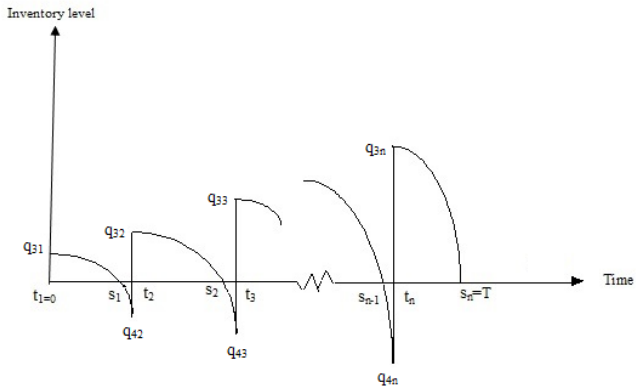

Figure 2: Buyer's inventory system

The buyer's inventory system at any time t during the interval $[t_i,\ s_i]$ can be described by the following differential equation:

$$\begin{aligned}\frac{dI_3(t)}{dt} + \theta I_3(t) &= -D(t) \\ &= -(a+bt),\quad t_i \le t \le s_i \qquad (5)\end{aligned}$$

with boundary condition $I_3(s_i) = 0$ where $i = 1, 2, 3, ..., n$. The buyer's inventory system at any time t during the interval $[s_i,\ t_{i+1}]$ can be described by the following differential equation:

$$\begin{aligned}\frac{dI_4(t)}{dt} &= D(t) \\ &= (a+bt),\quad s_i \le t \le t_{i+1} \qquad (6)\end{aligned}$$

with boundary condition $I_4(s_i) = 0$ where $i = 1, 2, 3, ..., n-1$. The solutions of these differential equations are

$$I_3(t) = \int_t^{s_i} (a+bu)e^{\theta(u-t)}\,du,\quad t_i \le t \le s_i \qquad (7)$$

$$I_4(t) = \int_{s_i}^{t} (a+bu)\,du,\quad s_i \le t \le t_{i+1} \qquad (8)$$

Inventory carried during the i^{th} cycle is

$$I_i = \int_{t_i}^{s_i} I_3(t)\,dt$$
$$= \int_{t_i}^{s_i} \left(\int_{t}^{s_i} (a+bt)e^{\theta(t-t_i)}\,du\right) dt,\ i = 1,2,3,...,n \tag{9}$$

After changing the order of integration, I_i can be rewritten as

$$I_i = \theta^{-1}\int_{t_i}^{s_i} (e^{\theta(t-t_i)} - 1)(a+bt)\,dt, \quad i = 1,2,3,...,n \tag{10}$$

Similarly, the amount of shortage during the i^{th} replenishment cycle is

$$S_i = \int_{s_i}^{t_{i+1}} (t_{i+1} - t)(a+bt)\,dt, \quad i = 1,2,3,...,n-1 \tag{11}$$

Total amount deteriorated during the i^{th} replenishment cycle is

$$\begin{aligned} D_i &= \int_{t_i}^{s_i} \theta I_3(t)\,dt \\ &= \theta I_i, \quad i = 1,2,3,...,n \end{aligned} \tag{12}$$

From Figure 2, $I_3(t_i) = q_{3i},\ i = 1,2,3,...n$ and $I_4(t_i) = q_{4i},\ i = 2,3,...,n$. Hence from (7) and (8)

$$q_{3i} = \frac{1}{\theta^2}(b - a\theta + (e^{\theta(s_i - t_i)})(-b + a\theta + b\theta s_i) - b\theta t_i) \tag{13}$$

$$q_{4i} = a(-s_i + t_{i+1}) + b(\frac{-s_i^2}{2} + \frac{t_{i+1}^2}{2}) \tag{14}$$

Buyer's total cost

The buyer's total cost per unit time is a sum of ordering cost, holding cost, shortage cost and deteriorating cost

$$TCB = \frac{1}{T}\left(nc_1 + c_2\sum_{i=1}^{n} I_i + c_3\sum_{i=1}^{n} D_i + c_4\sum_{i=1}^{n-1} S_i\right)$$
$$= \frac{1}{T}\left(nc_1 + (c_2 + \theta c_3)\theta^{-1}\sum_{i=1}^{n}\int_{t_i}^{s_i}(e^{\theta(t-t_i)} - 1)(a+bt)\,dt \qquad + c_4\sum_{i=1}^{n}\int_{s_i}^{t_{i+1}}(t_{i+1} - t)(a+bt)\,dt\right) \tag{15}$$

Vendor's total cost

Inventory carried by the vendor during the cycle time $[0, T]$ is given by

$$I[0,T] = \int_0^{T_1} I_1(t)\,dt + \int_0^{T_2} I_2(t)\,dt \tag{16}$$

The total amount of deteriorated units for vendor is given by

$$\begin{aligned} D &= \int_0^{T_1} (P(t) - D(t))\,dt - \int_0^{T_2} D(t)\,dt \\ &= \int_0^{T_1} (a+bt)(\alpha - 1)\,dt - \int_0^{T_2} (a+bt)\,dt \end{aligned} \tag{17}$$

Therefore the vendor's total cost per unit time is the algebraic sum of set up cost, holding cost and deteriorating cost.

$$\begin{aligned} TCV &= \frac{1}{T}\left(c_5 + c_6 I[0,T] + c_7 D\right) \\ &= \frac{1}{T}\Bigg[c_5 + c_6\left(\int_0^{T_1} I_1(t)\,dt + \int_0^{T_2} I_2(t)\,dt\right) \\ &\qquad + c_7\left(\int_0^{T_1}(a+bt)(\alpha - 1)\,dt - \int_0^{T_2}(a+bt)\,dt\right)\Bigg] \end{aligned} \tag{18}$$

The integrated total cost for the vendor and the buyer is

$$TC = TCB + TCV \tag{19}$$

Our optimization problem is

Minimize TC

Subject to constraint

$$\int_0^{T_1} P(t)\,dt \geq \sum_{i=1}^{n} q_{3i} + \sum_{i=2}^{n} q_{4i} \tag{20}$$

That is the total production must be greater than or equal to the total items delivered from vendor to buyer.

Total cost of the integrated system is a function of $n, s_i, (i = 1, 2, 3, ..., n-1)$ and T_1. Our problem is to find the values of $n, s_i, (i = 1, 2, 3, ..., n-1)$ and T_1 which minimizes the total cost of the integrated system.

For $n = 1$, t_1=0 and s_1=T, the total cost of the integrated system reduces to

$$\begin{aligned} TC(1) &= \frac{1}{T}\Bigg\{c_1 + \frac{(c_2+\theta c_3)}{2\theta^3}[2(1-e^{\theta T})(b-a\theta) + \theta(2(be^{\theta T} - a\theta)T - b\theta T^2)] + c_5 \\ &\quad + c_6(\alpha - 1)\Bigg[\frac{e^{-\theta T_1}(b-a\theta)}{\theta^2} - \frac{1}{2}T_1(2a + bT_1) + \frac{-b + a\theta + b\theta T_1}{\theta^2} \\ &\quad - \frac{1}{2\theta^3}(2(-1+e^{\theta T_2})(b-a\theta) + \theta T_2(-2be^{\theta T_2} + 2a\theta + b\theta T_2))\Bigg] \\ &\quad + c_7\left[-aT_1 + a\alpha T_1 - \frac{b{T_1}^2}{2} + \frac{b\alpha {T_1}^2}{2} - aT_2 - \frac{b{T_2}^2}{2}\right]\Bigg\} \end{aligned} \tag{21}$$

4. SOLUTION PROCEDURE

To obtain the optimum solution, we have considered the solution procedure given in Hariga [10]
One of the assumptions considered to develop the model for buyer is that the replenishments should be made at equal intervals of time throughout the cycle. Since n is an integer, for a given value of n, t_i values are obtained by

$$t_i = (i-1)T/n, \quad i = 1, 2, 3, ..., n \text{ and } t_1 = 0 \tag{22}$$

The necessary conditions for the integrated total cost to be minimum are

$$\frac{\partial TC}{\partial s_i} = 0, \quad i = 1, 2, 3, ..., n-1 \text{ and } s_n = T \tag{23}$$

and

$$\frac{\partial TC}{\partial T_1} = 0 \tag{24}$$

Equation (23) gives

$$\frac{1}{\theta}\left[(e^{\theta(s_i - t_i)} - 1)(c_2 + \theta c_3) - c_4\theta(t_{i+1} - s_i)\right] = 0, \quad i = 1, 2, 3, ..., n-1 \tag{25}$$

Let K $(0 < K < 1)$ be the fraction of the i^{th} replenishment such that s_i can be obtained by

$$s_i = t_i + K(t_{i+1} - t_i), \quad i = 1, 2, 3, ..., n-1 \text{ and } s_n = T$$

But, because of equal replenishment, we have

$$t_{i+1} - t_i = T/n, \quad i = 1, 2, 3, ..., n-1$$

Therefore

$$s_i = t_i + KT/n, \quad i = 1, 2, 3, ..., n$$

Hence equation (25) becomes

$$(c_2 + \theta c_3)\theta^{-1}(e^{\theta KT/n} - 1) - c_4(1-K)T/n = 0 \tag{26}$$

It is observed from equation (26), that the value of K does not depend on the demand function parameters a and b. Now, for the given value of n, find K using equation (26) by any one-dimensional search technique. Hence obtain $s_i = t_i + KT/n$. Using s_i and t_i $(i = 1, 2, 3, ..., n)$, find T_1 from equation (24). Substitute the values of n, t_i, s_i and T_1 in equation (19) to get the total cost of the integrated system.

The values of n, s_i, t_i $(i = 1, 2, 3, ..., n)$ and T_1 for which TC is minimum becomes the optimal values of n, s_i, t_i $(i = 1, 2, 3, ..., n)$ and T_1.

We can summarize the solution procedure in an algorithmic form as follows:

1. Start with $n = 1$
2. Find TC from equation (21) and denote it as $TC(1)$
3. Set $n = n + 1$
4. Find t_i=$(i-1)T/n$ for $i = 1, 2, 3, ..., n$ where t_1=0
5. Compute $K(n)$ from equation (26)
6. Find s_i=$t_i + KT/n$ for $i = 1, 2, 3, ..., n-1$ and $s_n = T$
7. Compute T_1 from equation(24)
8. Calculate total cost of the system from equation (19) and denote it as $TC(n)$
9. If $TC(n)$<$TC(n-1)$, go to step 3
10. If $TC(n) \geq TC(n-1)$, stop. Set $n^* = n-1$, t_i^*=t_i, s_i^*=s_i and T_1^*=T_1, TC^*=$TC(n-1)$

Particular case:
If shortages are not allowed, then $s_i = t_{i+1}$. But $t_{i+1} - t_i$= T/n because of equal replenishment. Hence total cost is obtained by replacing $t_{i+1} - t_i$=T/n. Here total cost is a function of n and T_1 only. Hence follow the relevant steps in the solution procedure to obtain the optimal solution.

5. NUMERICAL EXAMPLE

To substantiate mathematical modeling and the analytical solution, we have considered the following numerical values for parameters.
c_1=100, c_2=1.1, c_3=1.8, c_4=2.5, c_5=600, c_6=0.9, c_7=2.0, a=20, b=2, θ=0.01, T=11, α=1.2.
Since the integrated total cost of the system is a function of more number of variables, we have developed a programme to obtain the optimal values using MATHEMATICA 11.1.
We obtain the optimal solution as follows:
n^*=4, K^*=0.6134, T_1^*=9.6580, T_2^*=1.3420, TC^*=153.3790 [Table 1].

t_1^*=0, s_1^*=1.6868, t_2^*=2.75, s_2^*=4.4368, t_3^*=5.5, s_3^*=7.1868, t_4^*=8.25, s_4^*=11.

For particular case:

If shortages are not allowed, then $s_i = t_{i+1}$. Hence, optimal solution is:

n^*=5, T_1^*=9.6822, T_2^*=1.3178, TC^*=166.149 [Table 2].

t_1^*=0, t_2^*=2.2, t_3^*=4.4, t_4^*=6.6, t_5^*=8.8, t_6^*=11.

Table 2: Optimal solution with infinite shortage cost

n	T_1	T_2	TC
1	10.0584	0.9416	295.0540
2	9.8142	1.1858	203.8550
3	9.7396	1.2604	176.1370
4	9.7035	1.2965	167.4900
5	9.6822	1.3178	166.1490*
6	9.6682	1.3318	168.3710

∗Optimal solution from the integrated perspective.

Observations from numerical result
It is observed from Table 1 and Table 2 that the model with complete backlogging of shortages gives minimum total cost compared to the model without shortages. So, one can allow shortages with complete backlogging for buyer instead of without shortages to minimize the total cost of the system.

6. SENSITIVITY ANALYSIS AND OBSERVATIONS

To study the effect of changes in parameters on optimal solution, we have carried out sensitivity analysis. To analyse the result we have considered $PICD$ (Percentage of Integrated total Cost Difference) which is defined as:

$$PICD = \frac{(TC - TC^*)}{TC^*}$$

The results of the sensitivity analysis executed by changing the value of each of the parameters by -20%, -10%, 10% and 20% and keeping all other parameters unchanged are tabulated in the Tables 3 to 11.

Table 1: Optimal solution with finite shortage cost

n	K	T_1	T_2	TCB	TCV	TC
1	1	10.0584	0.9416	232.1120	62.9421	295.0540
2	0.6114	9.7549	1.2451	103.4500	81.6896	185.1390
3	0.6127	9.6860	1.3140	78.2805	81.1600	159.4410
4	0.6134	9.6580	1.3420	72.4312	80.9476	153.3790*
5	0.6138	9.6432	1.3568	73.2585	80.8368	154.0950

∗Optimal solution from the integrated perspective.

Table 3: Sensitivity analysis when buyer's ordering cost c_1 and vendor's setup cost c_5 are changed

c_1	80	90	{100}	110	120
c_5	480	540	{600}	660	720
n	5	5	4	4	4
T_1	9.6433	9.6432	9.6580	9.6580	9.6580
TC	134.095	144.095	153.379*	162.470	171.560
$PICD$	-0.1257	-0.0605	0.0000	0.0593	0.1185

*: Optimal total cost; {}: Base column

Table 4: Sensitivity analysis when c_2 and c_6 are changed

c_2	0.88	0.99	{1.1}	1.21	1.32
c_6	0.72	0.81	{0.9}	0.99	1.08
n	4	4	4	4	4
T_1	9.6625	9.6601	9.6580	9.6561	9.6943
TC	143.521	148.502	153.379*	158.163	159.585
$PICD$	-0.0642	-0.0318	0.0000	0.0312	0.0404

*: Optimal total cost; {}: Base column

Table 5: Sensitivity analysis when c_3 and c_7 are changed

c_3	2	2.25	{2.5}	2.75	3.00
c_7	1.6	1.8	{2.0}	2.2	2.4
n	4	4	4	4	4
T_1	9.6581	9.6580	9.6580	9.6580	9.6579
TC	152.214	152.796	153.379*	153.961	154.543
$PICD$	-0.0076	-0.0038	0.0000	0.0038	0.0076

*: Optimal total cost; {}: Base column

Table 6: Sensitivity analysis when c_4 is changed

c_4	0.88	0.99	{1.1}	1.21	1.32
n	4	4	4	4	4
T_1	9.6536	9.6559	9.6580	9.65995	9.6618
TC	151.482	152.490	153.379*	154.168	154.874
$PICD$	-0.0124	-0.0058	0.0000	0.0051	0.0097

*: Optimal total cost; {}: Base column

Table 7: Sensitivity analysis when a is changed

a	16	18	{20}	22	24
n	4	4	4	4	5
T_1	9.7059	9.6807	9.6580	9.6375	9.6039
TC	145.585	149.485	153.379*	157.268	160.869
$PICD$	-0.0508	- 0.0254	0.0000	0.0254	0.0488

*: Optimal total cost; {}: Base column

Table 8: Sensitivity analysis when b is changed

b	1.6	1.8	{2.0}	2.2	2.4
n	4	4	4	4	4
T_1	9.6101	9.6353	9.6580	9.6785	9.6972
TC	148.658	151.021	153.379*	155.732	158.081
$PICD$	- 0.0308	- 0.0154	0.0000	0.0153	0.0306

*: Optimal total cost; {}: Base column

Table 9: Sensitivity analysis when θ is changed

θ	0.008	0.009	{0.01}	0.011	0.012
n	4	4	4	4	4
T_1	9.6465	9.6522	9.6580	9.6638	9.6695
TC	153.240	153.310	153.379*	153.448	153.518
$PICD$	- 0.0009	- 0.0004	0.0000	0.0004	0.0009

*: Optimal total cost; {}: Base column

Table 10: Sensitivity analysis when α is changed

α(>1)	0.96	1.08	{1.2}	1.32	1.44
n	-	4	4	4	4
T_1	-	10.4515	9.6580	8.9841	8.4037
TC	-	139.146	153.379*	164.558	173.634
$PICD$	-	-0.0927	0.0000	0.0729	0.1321

*: Optimal total cost; {}: Base column

Table 11: Sensitivity analysis when T is changed

T	8.8	9.9	{11}	12.1	13.2
n	3	4	4	5	5
T_1	7.6968	8.6665	9.6580	10.6350	11.6294
TC	160.402	155.579	153.379*	152.061	151.849
$PICD$	0.0458	0.0143	0.0000	-0.0086	-0.0099

*: Optimal total cost; {}: Base column

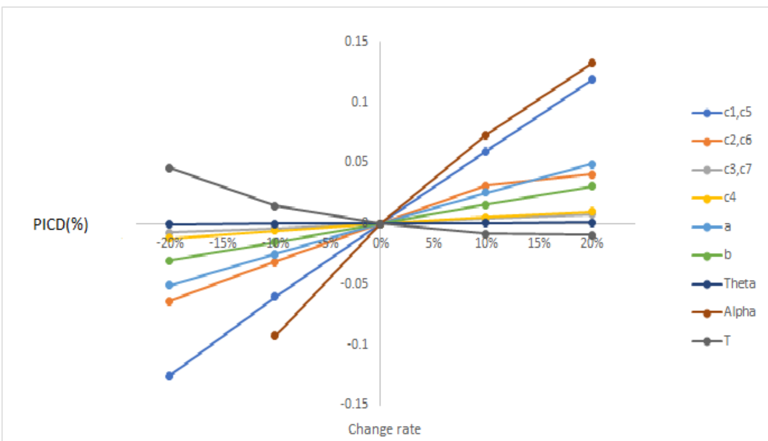

Figure 3: PICD vs. change rate of various parameters

Observations:
The observations drawn from the sensitivity analysis are as follows (Table 3 to Table 11 and Figure 3):

- Production time T_1 is less sensitive to all the cost parameters - buyer's ordering cost and vendor's set-up cost $(c_1,\ c_5)$, buyer's and vendor's holding costs $(c_2,\ c_6)$, buyer's and vendor's deterioration costs $(c_3,\ c_7)$, buyer's shortage cost c_4, the demand parameters a, b and the deterioration rate θ. It is more sensitive to the parameters cycle length T and production rate α. As the production rate α increases, the production period T_1 decreases. As the cycle length T increases, the production period T_1 increases.

- The optimal number of deliveries n is sensitive to the parameters - buyer's ordering cost and vendor's set-up cost $(c_1,\ c_5)$, constant demand rate a, and the cycle length T. As buyer's ordering cost and vendor's set-up cost$(c_1,\ c_5)$ increases, n decreases. As constant demand rate a and the cycle length T increase, n increases. It is insensitive to all the remaining parameters - buyer's and vendor's holding costs $(c_2,\ c_6)$, buyer's and vendor's deterioration costs $(c_3,\ c_7)$, buyer's shortage cost c_4, b, deterioration rate θ and vendor's production rate α.

- Total cost is sensitive to the parameters - buyer's ordering cost and vendor's set-up cost $(c_1,\ c_5)$, buyer's and vendor's holding costs $(c_2,\ c_6)$, constant demand rate a, b, vendor's production rate α and cycle length T. As buyer's ordering cost and vendor's set-up cost $(c_1,\ c_5)$, buyer's and vendor's holding costs $(c_2,\ c_6)$, constant demand rate a, b, vendor's production rate α increase, total cost increases. As T increases, total cost decreases. Total cost is less sensitive to buyer's and vendor's deterioration costs $(c_3,\ c_7)$, buyer's shortage cost c_4 and deterioration rate θ.

7. SUMMARY

Complete success of a supply-chain inventory system depends on good coordination between vendor and buyer. Because of several conditions, buyer can not stock sufficient goods. So, he may allow shortages. But complete lost or partial backlogging may result in loss of goodwill. With the support of vendor he can consider complete backlogging of shortages. We developed a vendor-buyer integrated inventory system for deteriorating items with increasing time-varying demand rate and complete backlogging of shortages. We have showed numerically that the model with complete backlogging of shortages gives minimum total cost compared to the model without shortages. For numerical result, we have used Mathematica 11.1.

8. SCOPE FOR FUTURE WORK

The review of research works related to vendor-buyer inventory system focuses only on a few selected areas and business scenarios. We have identified some of the possible extensions/variations to the inventory systems, that could be pursued in future.

(i) In vendor-buyer production-inventory system, one may explore the possibility of imposing capacity constraints on vendor's production and inventory levels.

(ii) Demand for some items and deterioration are uncertain in nature and firms do not have any control on them. In cases where the demand rate and deterioration rate are unknown, they are assumed to be a random variable with a known probability distribution. Attempts could be made to develop and analyse the model under probabilistic setting.

(iii) In some of the practical situations like (a) when demand is stock-dependent and the capacity of warehouse is limited and (b) customer's unwillingness to wait for backlogged items during a shortage period; maintaining inventory in single warehouse is not profitable. In such cases, retailers have to consider two warehouses, one owned by themselves and the other one on rent, for the smooth running of business. Hence the study can be extended to two ware house situation.

(iv) Several possible models with multi-vendor, multi-buyer and multi-item scenarios can be studied.

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