

# Application of Fractional Capacitor and Fractional Inductor in Automatic Voltage Regulator

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## Abstract

The aim of this paper is to compare the performance of an automatic voltage regulator (AVR) by controlling it with a fractional model which has been realized using fractional capacitor and fractional inductor. In this work a fractional model is designed for the automatic voltage regulator and its performance is analysed and compared with the response of AVR controlled by integer order PID controller using MATLAB/Simulink. For the tuning of PID controller and to find the values of parameters of the fractional model, Ziegler-Nichols method is used. It has been found that by using the fractional capacitor and inductor the performance of the voltage regulator has improved.

**Keywords:** Automatic voltage regulator, Fractional order system, Fractional capacitor, Fractional inductor, PID controller.

## 1. INTRODUCTION

In most of the industries the Automatic Voltage Regulator (AVR) is widely used in order to obtain the stability and good regulation of different electrical apparatus. An electronic circuit which provides a stable or constant DC voltage automatically is known as automatic voltage regulator which is independent of temperature, load current and AC line voltage variation [1]. The voltage regulator can be designed in various ways; either it can be a simple feed forward design, or it may include a negative feedback or it uses passive/active elements i.e. electronic components or uses electromechanical mechanism. It is used to regulate one/more AC/DC voltages, depending up on its design [2].

AVR works on the principle of 'Detection of errors'. If the output voltage is low, then the regulation element is commanded to produce a higher output voltage and vice versa[1].

In the recent years there has been a considerable progress in the field of fractional order circuits and systems. The subject of fractional order circuits was approached by theorists back in 1960. The concept of fractional capacitor came in to existence by [3] in 1964 and thus a new era of fractional order system began.

A dynamical system which is modelled by a fractional differential equation which contains derivatives of non-integer order is known as a fractional order system [4]. Fractional order systems are used to study the behaviour of dynamical

systems in biology, physics, viscoelasticity and electrochemistry [5].

Fractional-calculus deals with derivatives and integrals of an arbitrary order, for example they can be of order 0.5 or 1.3 or of order .

In [6] a fractional order filter has been designed using the fractional order capacitors and fractional order inductors and realized that it offers additional degrees of design freedom for better flexibility of filter design.

In [7] FC and FI was implemented in band pass filter and the result was found better than integer order filter. The paper [8] represents the use of a conventional PID controller with a simple gain adjustor in automatic voltage regulator to get good performance.

Here, in this paper, an AVR system is controlled both by PID controller and a fractional order model and the performance is compared by using Simulink.

The paper is organized as follows. In section II, background of fractional order system has been discussed. In section III, details of fractional order model has been presented. The AVR system has been discussed in section IV. In section V simulation results are presented and in section VI, the conclusions are drawn.

## 2. THEORITICAL BACKGROUND OF FRACTIONAL CAPACITOR AND FRACTIONAL INDUCTOR

The transfer function of a fractional order circuits comprises of real powers of 's' in numerator and denominator. This is in contrary to conventional circuits, where all the powers of 's' in the transfer function are integers. Traditional continuous type analog circuits are of integer order, since their transfer functions contain integer powers of 's'. But, if one or more conventional capacitors and/or inductors in the circuits are replaced by fractional capacitor (FC) or fractional inductors (FI), then the circuit's transfer function will contain real powers of 's'. Such systems are also governed by linear differential equations, whose orders are non-integer.

From a hardware point of view, a fractional-order electric circuit is one which typically involves one or more fractional capacitors. Recently, there had been some attempts to manufacture solid state fractional capacitors and inductors in[9. 10]. Most recently, these fabricated fractional capacitors and inductors has been used in the fractional order filter circuits, fractional order PLL and fractional order differentiator circuits and was found that their performances

are very close to that of ideal fractional order circuits [11, 12, 13, 14, 15]. Here FC and FI whose theoretical concepts are given below, has been used to realize a model for controlling the response of an AVR system.

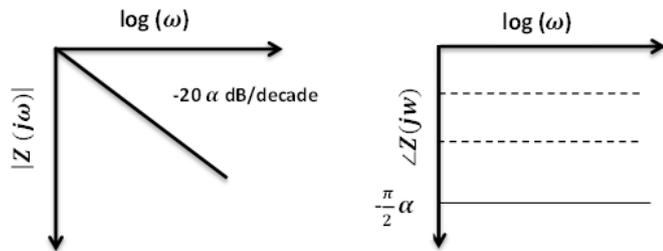
**Fractional Capacitor**

A fractional capacitor (FC) can be defined as a passive circuit element which gives phase angle between 0 to -90 degree and remains constant with frequency [11, 16]. The impedance of a fractional capacitor (FC) can be expressed as

$$Z = \frac{1}{C_F s^\alpha} \tag{1}$$

Where  $C_F$  is the fractional capacitance of the FC and  $\alpha(0 < \alpha \leq 1)$  is its order.  $\alpha$  (Known as fractional operator) is used to interpret the voltage-current relationship of a fractional capacitor. The phase difference is  $-\frac{\pi\alpha}{2}$  between the voltage across its two terminals and current entering these terminals.

The unit of fractional capacitance can be expressed as  $F/s^{1-\alpha}$  where 's' denotes time in second and F denotes farad [6]. The magnitude characteristic of an ideal FC is a straight line with slope  $-20\alpha$  dB/decade while the phase angle remains constant for all frequencies, as shown in figure 1.

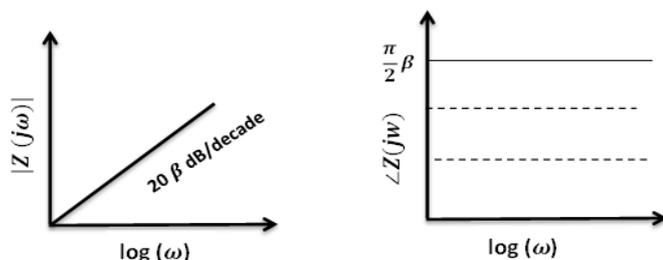


**Fig. 1:** Magnitude plot and Phase plot of fractional capacitor

**Fractional Inductor (FI):** Fractional inductor (FI) is defined as passive circuit element which gives constant phase angle between 0 to +90 degree and remains constant with frequency [6]. The impedance of a fractional inductor is expressed by

$$Z(S) = L_F s^\beta \tag{2}$$

Where  $L_F$  is called the fractional inductance and  $\beta(0 < \beta \leq 1)$  is its order. The magnitude characteristic of an ideal fractional inductor is a straight line with slope  $+20\alpha$  dB/decade while the phase angle remains constant for all frequencies as shown in figure 2.



**Fig. 2:** Magnitude plot and Phase plot of fractional inductor

Simulation of fractional-order systems is crucial for any application. There are many issues for the numerical simulations of fractional-order systems in time domain.

In the frequency domain, it is necessary to simulate the Laplace operator  $s^\alpha = (j\omega)^\alpha$

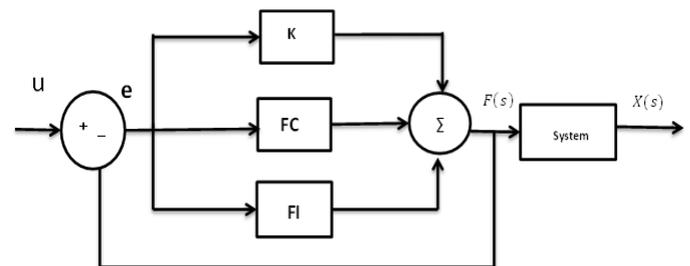
$$s^\alpha = (j\omega)^\alpha = \omega^\alpha \left( \cos \frac{\pi\alpha}{2} + j \sin \frac{\pi\alpha}{2} \right) \tag{3}$$

**3. FRACTIONAL MODEL**

A fractional model is realized by using fractional capacitor and fractional inductor. To use it in the voltage regulator a DC multiplier (K) is also used in order to achieve the best performance of the fractional model. It is used just before the AVR system.

The use of fractional elements instead of integer order elements gives more flexibility because of the fractional operators  $\alpha$  and  $\beta$ . By varying the values of  $\alpha$  and  $\beta$  from 0.1 to 0.9 the output of the AVR system can be varied and better output can be achieved.

The DC multiplier, the fractional inductor and the fractional capacitor are connected in parallel and are added which operates as a controller for a system.



**Fig. 3:** Block diagram of Fractional Model used in voltage regulator

**4. AUTOMATIC VOLTAGE REGULATOR SYSTEM**

An automatic voltage regulator (AVR) is a device designed to regulate voltage automatically. The principle of working AVR system is 'detection of errors'. The constant voltage at the terminal point can get disturbed due to different factors like temperature, speed, load etc. AVR takes a fluctuating voltage level and changes it in to a constant voltage level. So, AVR is required to retain the magnitude of the terminal voltage within a specified limit.

The AVR system has four main parts- amplifier, exciter, generator and sensor (Figure 4)

The sensor's role is to sense the terminal voltage continuously which is then compared with a reference voltage in the comparator. The error voltage which is obtained from the comparator is used to control the field winding of the alternator by means of the exciter, after it gets amplified by amplifier.

In this paper, for the purpose of mathematical modelling of the system the transfer functions of the described components of the AVR system are assumed to be linear.

Amplifier- The transfer function of the amplifier is modelled by a gain and a time constant [17]

$$G_{\text{Amplifier}}(S) = \frac{K_A}{1+S\tau_A} \quad (4)$$

Where,  $K_A$ = Gain whose range is 10-40

And  $\tau_A$ = Time constant and its range is 0.02 to 0.1 sec

Exciter- The transfer function of exciter model can be expressed in terms of gain and time constant [17]

$$G_{\text{Exciter}}(S) = \frac{K_E}{1+S\tau_E} \quad (5)$$

Where,  $K_E$ = Gain whose range is 1-10

And  $\tau_E$ = Time constant and its range is 0.4 to 1 sec

Generator- The transfer function of generator model is modelled by a gain and a time constant [17]

$$G_{\text{Generator}}(S) = \frac{K_G}{1+S\tau_G} \quad (6)$$

Where,  $K_G$ = Gain whose range is 0.7-1. and  $\tau_G$ = Time constant and its range is 1 to 2 sec

Sensor- The transfer function of sensor model is given as [17]

$$G_{\text{Sensor}}(S) = \frac{K_S}{1+S\tau_S} \quad (7)$$

Where,  $K_S$ = Gain whose range is 0.9-1.1

And  $\tau_S$ = Time constant and its range is 0.001 to 0.06sec

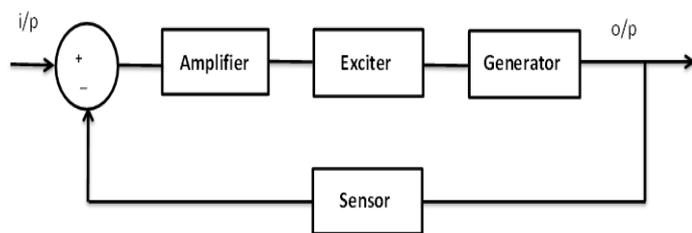


Fig. 4: Block diagram of Automatic Voltage Regulator System

The step response of the AVR system (Figure 5) conveys that it is a marginally stable system as oscillations in the output persists indefinitely.

As the system is marginally stable PID controller and the designed fractional model is used to get better response.

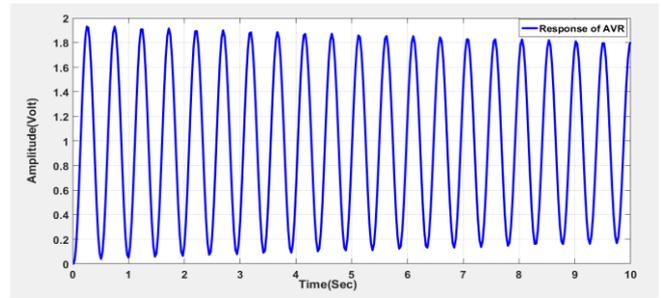


Fig. 5: Step response of AVR system

## 5. SIMULATION RESULTS

The AVR system is controlled with both PID controller and the designed fractional model and the results are compared. For the tuning of PID controller, Ziegler-Nichols method [18] is used and it is found that  $K_p = 1.1688$ ,  $K_i = 4.87$  and  $K_d = 0.070128$ .

The same method is used to calculate the values of the constant K and of FC and FI of fractional model.

The settling time ( $t_s$ ), rise time ( $t_r$ ) and percentage overshoot ( $M_p\%$ ) of the AVR system with PID controller and with fractional model by varying integral order and derivative order are recorded and compared.

The figure 6 represents the response of AVR system with PID controller. Now the simulation is done by varying  $\alpha$  and  $\beta$  values of the fractional model from 0.1 to 0.9 and the responses are compared with that of PID and then conclusion is drawn.

Figure 7 shows the simulation results of AVR system with fractional model with different values of  $\alpha$  and  $\beta$ .

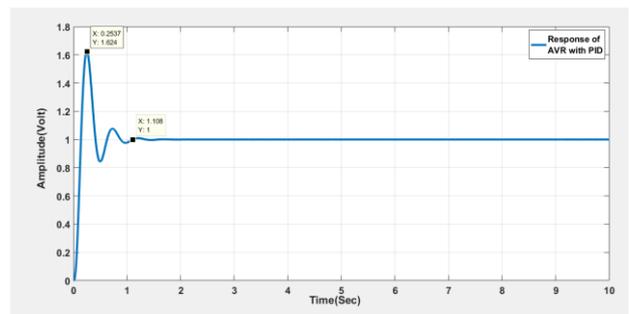


Fig. 6: Response of AVR system with PID controller

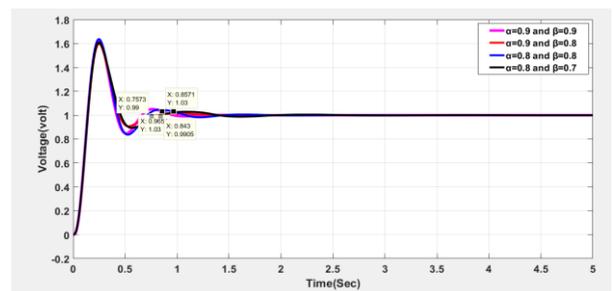


Fig. 7: Response of AVR system with fractional model by varying  $\alpha$  and  $\beta$

**Table I:** Settling time, rise time and percentage overshoot of AVR system for different values of  $\alpha$  and  $\beta$

AVR system with fractional model	SettlingTime( $t_s$ ) In sec	Rise Time( $t_r$ ) In sec	Percentage Overshoot ( $M_p$ %)
$\alpha=0.9$ and $\beta=0.9$	0.8571	0.086	63.3
$\alpha=0.9$ and $\beta=0.8$	0.7573	0.086	60.5
$\alpha=0.8$ and $\beta=0.8$	0.9654	0.086	64.3
$\alpha=0.8$ and $\beta=0.7$	0.843	0.086	61.4

**Table II:** Settling time, rise time and percentage overshoot of the AVR system

AVR System	SettlingTime( $t_s$ ) In sec	Rise Time( $t_r$ ) In sec	Percentage Overshoot ( $M_p$ %)
With PID Controller	1.108	0.076	62.4
With Fractional Model ( $\alpha=0.9, \beta=0.8$ )	0.7573	0.086	60.5

## 6. CONCLUSION

This paper presents the performance of an AVR system using both by PID controller and a fractional model through simulation results (MATLAB R2016b Simulink). The realization and performance analysis has been carried out by varying the values of  $\alpha$  and  $\beta$  of the fractional model from 0.1 to 0.9 and compared with that of conventional PID control. Also the parameter like settling time ( $t_s$ ), rise time ( $t_r$ ) and percentage overshoot ( $M_p$ %) are investigated and found that the fractional model has a better performance.

It is to be noted that from table II that for  $\alpha=0.9$  and  $\beta=0.8$  of the fractional model, the AVR system's response is much better response as compared to the system using PID because of its low settling time and percentage overshoot.

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