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Common Fixed Point Theorems in Fuzzy Metric Space

Swati Agnihotri[1], K. K. Dubey[2], V. K. Gupta[3]

[1,2]Department of Applied & Humanities, Invertis University, Bareilly (U.P.) India.
[3]P.G. Department of Mathematics, Govt. Madhav Science P.G. College, Ujjain (M.P.) India.

Abstract

In this paper we use the concept of fuzzy metric space introduced by Kramosil and Michalek and modified by George and Veeramani with the help of t-norm. The purpose of this paper is to establish a common fixed point theorem for seven self-mappings in fuzzy metric space using weak compatibility and generalizing the result of Govery A. and Singh M.[10] and also we cite an example in support of our result.

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Keywords: Common fixed point, fuzzy metric space, compatible mappings and weakly compatible mappings.

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh (1965) [1] as a new way to represent vagueness in everyday life. The special feature of fuzzy set is that it assign partial membership for elements in its domain, while in ordinary set theory particular element has either full membership or no membership, intermediate situation is not considered. A large number of renowned mathematicians worked with fuzzy sets in different branches of Mathematics, Fuzzy Metric Space is one of them. This paper uses the concept of fuzzy metric space introduced by Kramosil and Michalek [2] and modified by George and Veeramani [3] with the help of t-norm. Grabiec [4] obtained the fuzzy version of Banach contraction principle, which is a milestone in developing fixed point theory in fuzzy metric space. Jungck [5] proposed the concept of compatibility. The concept of compatibility in fuzzy metric space was proposed by Mishra et al.[6]. Later on, Jungck [7] generalized the concept of compatibility by introducing the concept of weak compatibility. Singh and Chauhan [8] and Cho [9] provided fixed point theorems in fuzzy metric space for four self-maps using the concept of compatibility where two mappings needed to be continuous. In 2017 Govery A. and Singh M.[10] proved a common fixed point theorem for six self-mappings in fuzzy metric space using the concept of compatibility and weak compatibility where one map is needed to be continuous. In this paper, a theorem has been proved on common fixed point theorems for seven self-mappings in fuzzy metrics space, using weakly compatibility without continuity and using another inequality and generalizing the result of Govery A. and Singh M.[10] and some previous results.

2. PRELIMINARIES

Definition 2.1. [1] Let X be any set. A fuzzy set A in X is a function with domain in X and values in $[0,1]$.

Definition 2.2. [11] A binary operation $*: [0,1]\times [0,1]\rightarrow [0,1]$ is called a continuous t-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative,

(ii) $*$ is continuous,

(iii) $a*1 = a,$ for all $a\in [0,1]$,

(iv) $a*b \leq c*d$, whenever $a \leq c$ and $b\leq d$ for all $a, b, c, d\in[0,1]$.

Examples of t-norm are

$a*b = min \{a, b\}$ (minimum t-norm),

$a*b = ab$ (product t-norm).

Definition 2.3. [3] The 3-tuple $(X,M,*)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times(0, \infty)$ satisfying the following conditions

(FM-1) $M(x, y, t) > 0$,

(FM-2) $M(x, y, t) = 1$ if and only if $x = y$,

(FM-3) $M(x, y, t) = M(y, x, t)$,

(FM-4) $M(x, y, t)M (y, z, s) \leq M(x, z, t + s)$,

(FM-5) $M(x, y, .): (0, \infty)\rightarrow[0, 1]$ *is continuous*,

for all $x, y, z\in X$ and $t, s > 0$.

Let (X,d) be a metric space and let $a*b = a b$ or $a*b = min\{a, b\}$ for all $a, b\in[0, 1]$.

Let $M\left(x,y,t\right)=\dfrac{t}{t+d\left(x,\ y\right)}$; for all $x,\ y\in X$ and $t\ >0$.

Then $(X,M,*)$ is a fuzzy metric space, and this fuzzy metric *M* induced by d is called the standard fuzzy metric [3].

Definition 2.4. [4]A sequence $\left\{x_n\right\}$ in a fuzzy metric space $(X,M,*)$ is said to be convergent to a point $x\in X$, if $\lim_{n\to\infty} M(x_n,x,\ t)=1$ for all $t>0$.

Further, the sequence $\left\{x_n\right\}$ is said to be Cauchy if

$\lim_{n\to\infty} M(x_n,x_{n+p},\ t)=1$, for all $t>0$ and $p>0$.

The space $(X,M,*)$ is said to be complete if every Cauchy sequence in X is convergent in X.

Lemma 2.5. [4] Let $(X,M,*)$ be a fuzzy metric space. Then M is non-decreasing for all $x,\ y\in X$.

Lemma 2.6. [12] Let $(X,M,*)$ be a fuzzy metric space. Then M is a continuous function on $X^2\times\left(0,\ \infty\right)$.

Throughout in this paper $(X,M,*)$ will denote the fuzzy metric space with the following condition:

(FM-6) $\lim_{n\to\infty} M\left(x\ ,y\ ,t\right)=1$ for all $x,\ y\in X$ and $t\ >0$.

Lemma 2.7. [6] If there exists $k\in\left(0,\ 1\right)$ such that

$M\left(x,y,kt\right)\geq M\left(x,y,t\right)$

for all $x,\ y\in X$ and $t\ >0$, then $x=y$.

Definition 2.8. [8] Let f and g be self-mappings on a fuzzy metric space $(X,M,*)$. The pair $\left(f,g\right)$ is said to compatible if

$\lim_{n\to\infty} M\left(fgx_n,gfx_n,t\right)=1$, for all $t\ >0$,

Whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} fx_n=\lim_{n\to\infty} gx_n=z$, for some $z\in X$.

Definition 2.9. [13] A pair (A,B) of self-maps of a fuzzy metric space $(X,M,*)$ is said to be semi-compatible if $\lim_{n\to\infty} ABx_n=Bx$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n=\lim_{n\to\infty} Bx_n=\ x$.

It follows that if (A,B) is semi-compatible and $Ax=Bx$ then $ABx\ =\ BAx$ that means every semi-compatible pair of self-maps is weak compatible but the converse is not true in general.

Definition 2.10. [14] Let f and g be self-mappings on a fuzzy metric space $(X,M,*)$. Then the mappings are said to be weakly compatible if they commute at their coincidence points.

$fx=gx$ implies $fgx=gfx$.

It is known that a pair of $\left(f,g\right)$ compatible maps is weakly compatible but converse is not true in general.

Govery. A. and Singh M.[10] proved the following result:

Theorem 2.11. Let $(X,M,*)$ be a complete fuzzy metric space and let A,B,S,T,P and Q be mappings from *X* into itself such that the following conditions are satisfied:

(2.11.1) $P\left(X\right)\subset ST\left(X\right),Q\left(X\right)\subset AB\left(X\right)$;

(2.11.2) $AB=BA,ST=TS,PB=BP,QT=TQ$;

(2.11.3) Either *AB* or *P* is continuous;

(2.11.4) $\left(P,AB\right)$ is compatible and (Q,ST) is weakly compatible;

(2.11.5) There exists $q\in(0,1)$ such that for every $x,y\in X$ and $t>0$

$$M\left(Px,Qy,qt\right)\geq M\left(ABx,STy,t\right)*M\left(Px,ABx,t\right)*M\left(Qy,STy,t\right)*M\left(Px,STy,t\right)$$

Then A,B,S,T,P and Q have a unique common fixed point in X.

3. MAIN RESULT

Our result generalizes the results of Govery A. and Singh M.[10] as we are using the concept of weak compatibility for both pairs which is lighter condition than compatible and semi-compatible and continuity is not require for existence of fixed point . We are proving the result for seven self-maps in a fuzzy metric space using another lighter inequality.

Theorem 3.1 Let $(X,M,*)$ be a complete fuzzy metric space and let A,B,R,S,T,P and Q be mappings from *X* into itself such that the following conditions are satisfied:

(3.1.1) $P\left(X\right)\subset STR\left(X\right),Q\left(X\right)\subset ABR\left(X\right)$;

(3.1.2) $AB=BA,ST=TS,PB=BP,QT=TQ,$

$PR=RP,TR=RT,BR=RB$;

(3.1.3) (P,ABR) and (Q,STR) are weakly compatible;

(3.1.4) There exists $q\in\left(0,1\right)$ such that for every $x,y\in X$ and $t>0$

$$M\left(Px,Qy,qt\right)\geq \min\left\{\begin{matrix} M\left(ABRx,STRy,t\right),M\left(Px,ABRx,t\right),M\left(Qy,STRy,t\right), \\ M\left(Px,STRy,t\right) \end{matrix}\right\}$$

Then A,B,R,S,T,P and Q have a unique common fixed point in X .

Proof: Let x_0 be an arbitrary point in X .

According to (3.1.1), there exists some points $x_1,x_2\in X$ such that

$Px_0 = STRx_1 = y_0$ and $Qx_1 = ABRx_2 = y_1$.

We can construct sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$y_{2n} = Px_{2n} = STRx_{2n+1}$ and $y_{2n+1} = Qx_{2n+1} = ABRx_{2n+2}$, for $n = 0, 1, 2,\ldots\ldots$

Now, we first show that $\{y_n\}$ is a Cauchy sequence in X .

Using condition (3.1.4)

$$M\left(y_{2n+1},\ y_{2n},qt\right)=M\left(y_{2n},\ y_{2n+1},qt\right)=M\left(Px_{2n},Qx_{2n+1},qt\right)$$

$$M\left(Px_{2n},Qx_{2n+1},qt\right)\geq \min\left\{\begin{matrix} M\left(ABRx_{2n},STRx_{2n+1},t\right),M\left(Px_{2n},ABRx_{2n},t\right), \\ M\left(Qx_{2n+1},STRx_{2n+1},t\right),M\left(Px_{2n},STRx_{2n+1},t\right) \end{matrix}\right\}$$

$$M\left(y_{2n},y_{2n+1},qt\right)\geq \min\left\{\begin{matrix} M\left(y_{2n-1},y_{2n},t\right),M\left(y_{2n},y_{2n-1},t\right),M\left(y_{2n+1},y_{2n},t\right), \\ M\left(y_{2n},y_{2n},t\right) \end{matrix}\right\}$$

$$M\left(y_{2n},\ y_{2n+1},qt\right)\geq \min\left\{M\left(y_{2n-1},y_{2n},t\right),M\left(y_{2n+1},y_{2n},t\right),1\right\}$$

$$M\left(y_{2n},y_{2n+1},qt\right)\geq M\left(y_{2n-1},y_{2n},t\right).$$

Similarly, $M\left(y_{2n+1},y_{2n+2},qt\right)\geq M\left(y_{2n},\ y_{2n+1},t\right).$

Therefore for all n and $t>0$,

$$M\left(y_n,y_{n+1},qt\right)\geq M\left(y_n,y_{n-1},t\right)$$

$$M\left(y_n,y_{n+1},t\right)\geq M\left(y_n,y_{n-1},\frac{t}{q}\right)\geq M\left(y_{n-1},y_{n-2},\frac{t}{q^2}\right)\geq ..\geq M\left(y_1,y_0,\frac{t}{q^n}\right).$$

On taking $n\to\infty$, we get

$$\lim_{n\to\infty} M\left(y_{n+1},y_{n},\ t\right)=1,\quad \forall t>0.$$

Now for any integer p we have

$$M\left(y_n,y_{n+p},t\right)\geq M\left(y_n,y_{n+1},\frac{t}{p}\right)*M\left(y_{n+1},y_{n+2},\frac{t}{p}\right)*\ldots *M\left(y_{n+p-1},y_{n+p},\frac{t}{p}\right).$$

Therefore $\lim_{n\to\infty} M\left(y_n,y_{n+p},t\right)\geq 1*1*1*\ldots\ldots\ldots *1=1.$

Above result show that $\{y_n\}$ is Cauchy sequence in X which is complete.

Therefore $\{y_n\}$ sequence converges to 'z'.

Hence the sub sequences $\{Px_{2n}\}$, $\{STRx_{2n+1}\}$, $\{Qx_{2n+1}\}$ and $\{ABRx_{2n+2}\}$ also converge to z.

$$\lim_{n\to\infty} Px_{2n} = \lim_{n\to\infty} Qx_{2n+1} = \lim_{n\to\infty} STRx_{2n+1} = \lim_{n\to\infty} ABRx_{2n+2} = z.$$

Case (I) Since $P(X) \subset STR(X)$ and $\lim_{n\to\infty} Px_{2n} = z$.

Then there exist $u \in X$ such that $STRu = z$. ......(i)

Putting $x = x_{2n}$ and $y = u$ in condition [3.1.4]

$$M(Px_{2n,} Qu, qt) \geq \min \begin{Bmatrix} M(ABRx_{2n}, STRu, t), M(Px_{2n}, ABRx_{2n}, t), M(Qu, STRu, t), \\ M(Px_{2n}, STRu, t) \end{Bmatrix}$$

$$M(Px_{2n,} Qu, qt) \geq \min \begin{Bmatrix} M(ABRx_{2n}, z, t), M(Px_{2n}, ABRx_{2n}, t), M(Qu, z, t), \\ M(Px_{2n}, z, t) \end{Bmatrix}.$$

Let $n \to \infty$ and using above result we get

$$M(z, Qu, qt) \geq \min\{M(z,z,t), M(z,z,t), M(Qu,z,t), M(z,z,t)\}$$

$$M(z\ , Qu, qt) \geq \min\{1, M(Qu,z,t)\}$$

$$M(z, Qu, qt) \geq M(Qu, z, t).$$

By lemma (2.7) $Qu = z$.

$Qu = STu = z$ ....(ii)

'u' is coincident point of X such that $Qu = STRu = z$ and (Q, STR) is weakly compatible mappings.

$QSTRu = STRQu$.

$Qu = z \to STRQu = STRz$ and $STRu = z \to QSTRu = Qz$

$Qz = STRz$.

Now Putting $x = x_{2n}$ and $y = z$ in condition [3.1.4]

$$M(Px_{2n}, Qz, t) \geq \min \begin{Bmatrix} M(ABRx_{2n}, STRz, t), M(Px_{2n}, ABRx_{2n}, t) M(Qz, STRz, t), \\ M(Px_{2n}, STRz, t) \end{Bmatrix}$$

$$M(Px_{2n}, Qz, qt) \geq \min \begin{Bmatrix} M(ABRx_{2n}, Qz, t), M(Px_{2n}, ABRx_{2n}, t), M(Qz, Qz, t), \\ M(Px_{2n}, Qz, t) \end{Bmatrix}$$

Let $n \to \infty$ and using above result we get

$$M(z, Qz, qt) \geq \min\{M(z,Qz,t), M(z,z,t), M(Qz,Qz,t), M(z,Qz,t)\}$$

$$M(z, Qz, qt) \geq \min\{M(z,Qz,t), 1\}$$

$$M(z, Qz, qt) \geq M(z, Qz, t).$$

By lemma (2.7) we get $Qz = z$. Therefore $Qz = STRz = z$. ….(iii)

Case (II) Since $Q(X) \subset ABR(X)$ and $\lim_{n\to\infty} Qx_{2n+1} = z$.

Then there exist $v \in X$ such that $ABRv = z$. …… (iv)

Putting $x = v$ and $y = x_{2n+1}$ in condition [3.1.4]

$$M\left(Pv,Qx_{2n+1},qt\right) \geq \min\begin{Bmatrix} M\left(ABRv,STRx_{2n+1},t\right), M\left(Pv,ABRv,t\right), \\ M\left(Qx_{2n+1},STRx_{2n+1},t\right), M\left(Pv,STRx_{2n+1},t\right) \end{Bmatrix}$$

$$M\left(Pv\ ,Qx_{2n+1},qt\right) \geq \min\begin{Bmatrix} M\left(z,STRx_{2n+1},t\right), M\left(Pv,z,t\right), M\left(Qx_{2n+1},STRx_{2n+1},t\right), \\ M\left(Pv,STRx_{2n+1},t\right) \end{Bmatrix}$$

Let $n \to \infty$ and using above result we get

$$M\left(Pv,z,qt\right) \geq \min\ \left\{M\left(z,z,t\right), M\left(Pv,z,t\right), M\left(z,z,t\right), M\left(Pv,z,t\right)\right\}$$

$$M\left(Pv,z,qt\right) \geq \min\left\{M\left(Pv,z,t\right),1\right\}$$

$$M\left(Pv,z,qt\right) \geq M\left(Pv,z,t\right)$$

By lemma (2.7) we get $Pv = z$ therefore

$$Pv = ABRv = z \qquad \text{……. (v)}$$

'v'is coincident point of X such that $Pv = ABRv = z$ and (P, ABR) is weakly compatible mappings.

$$PABRv = ABRPv$$

$Pv = z \to ABRPv = ABRz$ and $ABRv = z \to PABRv = Pz$.

$Pz = ABRz$.

Now putting $x = z$ and $y = x_{2n+1}$ in condition [3.1.4]

$$M(Pz,Qx_{2n+1},qt) \geq \min\begin{Bmatrix} M(ABRz,STRx_{2n+1},t), M(Pz,ABRz,t), M(Qx_{2n+1},STRx_{2n+1},t), \\ M(Pz,STRx_{2n+1},t) \end{Bmatrix}$$

$$M\left(Pz,Qx_{2n+1},qt\right) \geq \min\begin{Bmatrix} M(Pz,STRx_{2n+1},t), M(Pz,Pz,t), M(Qx_{2n+1},STRx_{2n+1},t), \\ M(Pz,STRx_{2n+1},t) \end{Bmatrix}$$

Let $n \to \infty$ and using above result we get

$$M\left(Pz,z,qt\right) \geq \min\left\{M\left(Pz,z,t\right), M\left(Pz,Pz,t\right), M\left(z,z,t\right), M\left(Pz,z,t\right)\right\}$$

$$M\left(Pz,z,qt\right) \geq \min\left\{M\left(Pz,z,t\right),1\right\}$$

$$M(Pz,z,qt) \geq M(Pz,z,t)$$

By lemma (2.7) we get $Pz = z$ therefore, $Pz = ABRz = z$. … (vi)

Again putting $x = Rz$ and $y = z$ in condition [3.1.4]

$$M\left(PRz,Qz,qt\right) \geq \min\begin{Bmatrix} M(ABRRz,STRz,t), M(PRz,ABRRz,t), M(Qz,STRz,t), \\ M(PRz,STRz,t) \end{Bmatrix}$$

$$M\left(RPz,Qz,qt\right)\geq \min\begin{Bmatrix} M(ARBRz,STRz,t),M(RPz,ARBRz,t),\\ M(Qz,STRz,t),M(RPz,STRz,t) \end{Bmatrix},\qquad \text{Since (3.1.2)}$$

$$M\left(Rz,z,qt\right)\geq \min\begin{Bmatrix} M(RABRz,z,t),M(Rz,RABRz,t),M(z,z,t),\\ M(Rz,z,t) \end{Bmatrix}\qquad \text{since (iii), (vi) and (3.1.2)}$$

$$M\left(Rz,z,qt\right)\geq \min\left\{M(Rz,z,t),M(Rz,Rz,t),M(z,z,t),M(Rz,z,t)\right\}$$

$$M\left(Rz,z,qt\right)\geq \min\left\{M(Rz,z,t),1\right\}$$

By lemma (2.7) we get $Rz=z$, therefore

$STRz=z\rightarrow STz=z$ and $ABRz=z\rightarrow ABz=z$

$$STz=ABz=z. \qquad \text{.... (vii)}$$

Again putting $x=Bz$ and $y=z$ in condition [3.1.4]

$$M\left(PBz,Qz,qt\right)\geq \min\begin{Bmatrix} M(ABRBz,STRz,t),M(PBz,ABRBz,t),M(Qz,STRz,t),\\ M(PBz,STRz,t) \end{Bmatrix}$$

$$M\left(BPz,Qz,qt\right)\geq \min\begin{Bmatrix} M(BARBz,STRz,t),M(BPz,BARBz,t),M(Qz,STRz,t),\\ M(BPz,STRz,t) \end{Bmatrix}\qquad \text{Since (3.1.2)}$$

$$M\left(Bz,z,qt\right)\geq \min\begin{Bmatrix} M(BARBz,z,t),M(Bz,BARBz,t),M(z,z,t),\\ M(Bz,z,t) \end{Bmatrix}\qquad \text{Since (iii) and (vi)}$$

$$M\left(Bz,z,qt\right)\geq \min\begin{Bmatrix} M(BABRz,z,t),M(Bz,BABRz,t),M(z,z,t),\\ M(Bz,z,t) \end{Bmatrix}$$

$$M\left(Bz,z,qt\right)\geq \min\begin{Bmatrix} M\left(Bz,z,t\right),M\left(Bz,Bz,t\right),M\left(z,z,t\right),\\ M\left(Bz,z,t\right) \end{Bmatrix},\qquad \text{since (vi)}$$

$$M\left(Bz,z,qt\right)\geq \min\left\{M\left(Bz,z,t\right),1\right\}$$

$$M\left(Bz,z,qt\right)\geq M\left(Bz,z,t\right)$$

By lemma (2.7) we get $Bz=z$, therefore $ABz=z\rightarrow Az=z$

$$Az=Bz=z. \qquad \text{...... .(viii)}$$

Putting $x=z$ and $y=Tz$ in condition [3.1.4]

$$M\left(Pz,QTz,qt\right)\geq \min\begin{Bmatrix} M\left(ABRz,STRTz,t\right),M\left(Pz,ABRz,t\right),M\left(QTz,STRTz,t\right),\\ M\left(Pz,STRTz,t\right) \end{Bmatrix}$$

$$M\left(Pz,TQz,qt\right)\geq \min\begin{Bmatrix} M\left(ABRz,TSRTz,t\right),M\left(Pz,ABRz,t\right),M\left(TQz,TSRTz,t\right),\\ M\left(Pz,TSRTz,t\right) \end{Bmatrix}\qquad \text{since (3.1.2)}$$

$$M\left(Pz,TQz,qt\right)\geq \min\begin{Bmatrix} M\left(ABRz,TSTRz,t\right),M\left(Pz,ABRz,t\right),M\left(TQz,TSTRz,t\right),\\ M\left(Pz,TSTRz,t\right) \end{Bmatrix}\qquad \text{since (3.1.2)}$$

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$M(z,Tz,qt) \geq \min\{M(z,Tz,t), M(z,z,t), M(Tz,Tz,t), M(z,Tz,t)\}$ Since (iii) and (vi)

$M(z,Tz,qt) \geq \min\{M(z,Tz,t), 1\}$

$M(z,Tz,qt) \geq M(z,Tz,t)$

By lemma (2.7) we get $Tz = z$, therefore

$STz = z \to Sz = z$

$Sz = Tz = z$. … (ix)

Hence, $Pz = Qz = Rz = Sz = Tz = Az = Bz = z$.

Finally we get 'z' is a common fixed point of self-mappings P, Q, R, S, T, A and B.

Uniqueness:

Let 'w' is another common fixed point of self-mappings P, Q, R, S, T, A and B.

Such that,

$Pz = Qz = Rz = Sz = Tz = Az = Bz = z$.

$Pw = Qw = Rz = Sw = Tw = Aw = Bw = w$.

Putting $x = z$ and $y = w$ in (3.1.4)

$$M(Pz,Qw,qt) \geq \min\left\{\begin{array}{l} M(ABRz,STRw,t), M(Pz,ABRz,t), M(Qw,STRw,t), \\ M(Pz,STRw,t) \end{array}\right\}$$

$$M(z,w,qt) \geq \min\left\{\begin{array}{l} M(ABz,STw,t), M(z,ABz,t), M(w,STw,t), \\ M(z,STw,t) \end{array}\right\}$$

$$M(z,w,qt) \geq \min\{M(Az,Sw,t), M(z,Az,t), M(w,Sw,t), M(z,Sw,t)\}$$

$$M(z,w,qt) \geq \min\{M(z,w,t), M(z,z,t), M(w,w,t), M(z,w,t)\}$$

$$M(z,w,qt) \geq \min\{M(z,w,t), 1\}$$

$$M(z,w,qt) \geq M(z,w,t) \quad \Rightarrow z = w.$$

Remark 3.2: If we put $R = I$ in theorem (3.1) then condition (3.1.2) is satisfied trivially and we get the following theorem.

Corollary 3.3: Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S, T, P and Q be mappings from X into itself such that the following conditions are satisfied:

(3.3.1) $P(X) \subset \mathrm{ST}(\mathrm{X}), Q(X) \subset \mathrm{AB}(\mathrm{X})$;

(3.3.2) $AB = BA, ST = TS, PB = BP, QT = TQ$;

(3.3.3) (P, AB) and (Q, ST) are weakly compatible;

(3.3.4) There exists $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$$M(Px,Qy,qt) \geq \min\left\{\begin{array}{l} M(ABx,STy,t), M(Px,ABx,t), M(Qy,STy,t), \\ M(Px,STy,t) \end{array}\right\}$$

Then A, B, S, T, P and Q have a unique common fixed point in X.

Remark 3.4: If we put $B = T = I$ in corollary (3.3) then condition (3.3.2) is satisfied trivially and we get the following theorem.

Corollary 3.5 Let $(X, M, *)$ be a complete fuzzy metric space and let A, S, P and Q be mappings from X into itself such that the following conditions are satisfied:

(3.5.1) $P(X) \subset S(X), Q(X) \subset A(X)$;

(3.5.2) (P, A) and (Q, S) are weakly compatible;

(3.5.3) There exists $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$M(Px, Qy, qt) \geq \min\{M(Ax, Sy, t), M(Px, Ax, t), M(Qy, Sy, t), M(Px, Sy, t)\}$, Then A, S, P and Q have a unique common fixed point in X.

Example 3.6: Let $X = [-1,1]$ and (X, d) be a metric space where metric $d(x, y) = |x - y|$.

Define $a * b = a.b$ where $a, b \in [0,1]$ and M fuzzy set on

$X^2 \times (0, \infty)$ such that $M(x, y, t) = \dfrac{|x-y|}{t + |x-y|}$

We take four functions $P, Q, S, T : [-1,1] \to [-1,1]$ define as follow:

$$P(x) = Q(x) = \begin{Bmatrix} \frac{1}{4} & ; & -1 \leq x < \frac{1}{5} \\ \frac{2}{5} - x & ; & \frac{1}{5} \leq x \leq 1 \end{Bmatrix},$$

$$S(x) = \begin{Bmatrix} 0 & ; -1 \leq x \leq 0 \\ \frac{1}{4} & ;\ 0 < x < \frac{1}{5} \\ \frac{1}{5} & ; \frac{1}{5} \leq x \leq 1 \end{Bmatrix} \quad \text{and} \quad T(x) = \begin{Bmatrix} \frac{1}{4} & ; -1 < x < \frac{1}{5} \\ \frac{1}{5} & ; \frac{1}{5} \leq x < 1 \end{Bmatrix}$$

Then $P(X) = Q(X) = \left\{\frac{1}{4}\right\} \cup \left[\frac{-3}{5}, \frac{1}{5}\right]$, $\quad S(X) = \left\{0, \frac{1}{4}, \frac{1}{5}\right\}$

and $T(X) = \left\{\frac{1}{4}, \frac{1}{5}\right\}$.

Therefore, $T(X) \subseteq P(X)$ and $S(X) \subseteq Q(X)$.

Consider a sequence $x_n = \frac{1}{5} + \frac{1}{n}$ then

$$\lim_{n\to\infty} T(x_n) = \lim_{n\to\infty} T\left(\frac{1}{5} + \frac{1}{n}\right) = \lim_{n\to\infty}\left(\frac{1}{5}\right) = \frac{1}{5},$$

$$\lim_{n\to\infty} Q(x_n) = \lim_{n\to\infty} Q\left(\frac{1}{5} + \frac{1}{n}\right) = \lim_{n\to\infty}\left[\frac{2}{5} - \left(\frac{1}{5} + \frac{1}{n}\right)\right] = \frac{1}{5},$$

Hence $\lim_{n\to\infty} T(x_n) = \lim_{n\to\infty} Q(x_n) = \frac{1}{5}$.

But $\lim_{n\to\infty} TQ(x_n) = \lim_{n\to\infty} TQ\left(\frac{1}{5} + \frac{1}{n}\right)$

$$=\lim_{n\to\infty} T\left[\frac{2}{5}-\left(\frac{1}{5}+\frac{1}{n}\right)\right]=\lim_{n\to\infty} T\left(\frac{1}{5}-\frac{1}{n}\right)=\frac{1}{4},$$

$$\lim_{n\to\infty} QT(x_n)=\lim_{n\to\infty} QT\left(\frac{1}{5}+\frac{1}{n}\right)=\lim_{n\to\infty} Q\left[\frac{1}{5}\right]=\lim_{n\to\infty}\left(\frac{2}{5}-\frac{1}{5}\right)=\frac{1}{5}$$

$$\lim_{n\to\infty} TQ(x_n)\neq\lim_{n\to\infty} QT(x_n).$$

Hence, (T,Q) is not compatible mapping but (T,Q) is weak compatible mapping as $T(x)=Q(x)\, when\, x=\frac{1}{5}$.

Then there exist a coincident point $x=\frac{1}{5}$ such that

$$TQ\left(\frac{1}{5}\right)=QT\left(\frac{1}{5}\right).$$

(T,Q) is commute at coincident points.

(T,Q) is weak compatible mapping.

Similarly (P,S) is also not compatible.

Again taking a sequence $\{x_n\}$ of X, such that $x_n=\frac{1}{5}+\frac{1}{n}$ then

$$\lim_{n\to\infty} P(x_n)=\lim_{n\to\infty} P\left(\frac{1}{5}+\frac{1}{n}\right)=\lim_{n\to\infty}\frac{2}{5}-\left(\frac{1}{5}+\frac{1}{n}\right)=\frac{1}{5}.$$

$$\lim_{n\to\infty} S(x_n)=\lim_{n\to\infty} S\left(\frac{1}{5}+\frac{1}{n}\right)=\lim_{n\to\infty}\frac{1}{5}=\frac{1}{5}.$$

$$\lim_{n\to\infty} P(x_n)=\lim_{n\to\infty} S(x_n)=\frac{1}{5}.$$

But $\lim_{n\to\infty} PS(x_n)=\lim_{n\to\infty} PS\left(\frac{1}{5}+\frac{1}{n}\right)=\lim_{n\to\infty} P\left(\frac{1}{5}\right)=\lim_{n\to\infty}\left(\frac{2}{5}-\frac{1}{5}\right)=\frac{1}{5}.$

$$\lim_{n\to\infty} SP(x_n)=\lim_{n\to\infty} SP\left(\frac{1}{5}+\frac{1}{n}\right)=\lim_{n\to\infty} S\left[\frac{2}{5}-\left(\frac{1}{5}+\frac{1}{n}\right)\right]=\lim_{n\to\infty} S\left(\frac{1}{5}-\frac{1}{n}\right)=\frac{1}{4}.$$

$$\lim_{n\to\infty} PS(x_n)\neq\lim_{n\to\infty} SP(x_n).$$

Hence, (P,S) is not compatible mapping but (P,S) is weak compatible mapping.

Because $P(x)=S(x)$ when $x=\frac{1}{5}$.

There exist a coincident point $x=\frac{1}{5}$ in X where commutativity is satisfied such that $PS\left(\frac{1}{5}\right)=SP\left(\frac{1}{5}\right)$.

There is a common unique fixed point $x=\frac{1}{5}$ in $X=[-1,1]$

such that, $P\left(\frac{1}{5}\right)=Q\left(\frac{1}{5}\right)=S\left(\frac{1}{5}\right)=T\left(\frac{1}{5}\right)=\frac{1}{5}$.

Remark: Example (3.4) shows that (Q,T) and (P,S) are not compatible mappings but there exist a unique common fixed point due to the weak compatibility. Every compatible mapping is weak compatible but converse is not true.

4. CONCLUSION

This paper is generalization of the result of Govery A. and Singh M.[10] in the sense of using the weak compatible mapping for both pairs which is lighter condition than compatible and continuity is not require for the existence of fixed point for seven self-mappings in complete fuzzy metric space. Corollary (3.5) is generalization of Cho [9] as using weak compatibility with another inequality and continuity in not require.

REFERENCES

[1] Zadeh, L.A., Fuzzy sets, *Inform.and control*, 8(1965), 338-353.

[2] Kramosil, I. & Michalek, J. Fuzzy metric and statistical metric spaces, *Kybernetica,* 11,(1975). 336-344.

[3] George, A. & Veeramani, P. On some results in Fuzzy metric spaces. *Fuzzy Sets and Systems*, 64(1994), 395-399.

[4] M. Grebiec, Fixed points in fuzzy metric spaces, *Fuzzy Sets and System* 27 (1988), 385-389.

[5] Jungck, G: Compatible mappings and common fixed points. *Int. J. Math. Math. Sci*. 9(4), (1986), 771-779.

[6] Mishra, S.N., Sharma, S.N. and Singh, S.L., Common fixed point of maps in fuzzy metric spaces, *Internat. J. Math. Sci.*17 (1994), 253-258.

[7] Jungck, G., Common fixed points for non continuous non self-maps on non-metric spaces. *Far East J. Math. Sci*.4, (1996), 199-215.

[8] Singh, B. & Chouhan, M. S. Common fixed points of compatible maps in fuzzy metric spaces. Fuzzy Sets and Systems, 115, (2000), 471-475.

[9] Cho S. H., 'On common fixed points in fuzzy metric spaces' *Int. Mathematical Forum*, 1, no.10, 2006 471-479.

[10] Govery A. K. and Singh M., 'Fixed point theorem in fuzzy metric space through weak compatibility *J. Ana. Num. Theor*. 5, No. 1 (2017), 1-4.

[11] Schweizer, B. and Sklar, A., *Statistical metric spaces*, Pacific J. Math. 10 (1960), 314 -334.

[12] Lopez, J., Rodrigues and Romaguera, S., *The Hausdorff fuzzy metric on compact sets*, Fuzzy Sets and Systems 147 (2), (2004),273-283.

[13] Singh, B. and Jain, S., Semi-compatibility and fixed point theorems in fuzzy metric space using implicit relation, *international journal of Mathematics and Mathematical sciences*, 16(2005), 2617-2629.

[14] Singh, B., Jain, S. and Jain, S., Generalized theorems on Fuzzy metric spaces, *Southeast Asian Bulletin of Mathematics*, 31(2007), 963-978.