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L-VAGUE CUT SET ON L-VAGUE SEMIRINGS OF L-SEMIRING

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Abstract

In this paper we introduce L-vague cut-set,L- vague semiring of a L-Semiring and studied their properties.These concepts are used in the development of some important results and theorems about L-vague cut-set and L-vague semiring of a L-semiring.Also some of their important properties have been investigated.

Keywords: Vague set,L-vague cut-set,L-vague Semiring, L-seiring.

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Introduction

The concept proposed by Zadeh.L.A.[16] defining a fuzzy subset A of a given universe X characterizing the membership of an element x of X belonging to A by means of a membership function μ_A defined from X into [0, 1] has revolutionized the theory of Mathematical modeling .Decision making etc.,in handling the imprecise real life situations mathematically.Now ,several branches of fuzzy mathematics like fuzzy algebra ,fuzzy topology ,fuzzy control theory ,fuzzy measure theory etc.,have emerged. But in the decision making ,the fuzzy theory takes care of membership of an element x only,that is the evidence against x belonging to A .It is felt by several decision makers and researchers that in proper decision making,the evidence belongs to A and evidence not belongs to A are both necessary .and how much X belongs to A or how much x does not belongs to A are necessary.
Several generalizations of Zadeh's fuzzy set theory have been proposed,such as L-fuzzy sets [5]. Interval valued fuzzy sets ,Intuitionistic fuzzy sets by Atanassov.K.T [1] ,Vague sets [4] are mathematically equivalent.Any such set A of a given Universe X can be charactrierized by means of a pair of function (t_A, f_A) where t_A and f_A are functions from X in to [0 1] such that $0 \leq t_A(x) + f_A(x) \leq 1$ for all x in X.The set t_A is called the truth function and the set f_A is called false function or non membership function and $t_A(x)$ gives the evidence of how much x $\in$ A $f_A(x)$ gives the evidence of how much x does not $\in$ A.These concepts are being applied in several areas like decision making, fuzy control, knowledge discovery and fault diagonsis etc.It is believed the vague sets (or equivalently instuitionistic fuzzy sets) will more useful in decision making, and other areas of Mathematical modeling.Through Atanassov's instuitionistic fuzzy sets,Gau and Buehrer and some other areas of Mathematical modeling.Since then the theory fuzzy sets developed extensively and embraced almost all subjects like engineering science and Technology.But the membership functionμ_A gives only a approximation belong to A .To avid this and obtain a better estimation and analysis of data decision making.

Gau.W.L and Bueher D.J. [4] have initiated the study of vague sets with the hope that they form a better tool to understand,interpret and solve real life problems which are in general vague,than the theory of vague sets do.Ranjit Biswas[10] initiated the study of vague groups and Ramakrishna.N [9], [11],[13],Ramakrishana.N,T. Eswarlal.T[16],[9],[11] are grate extended the study of vague algebra.The study L-vague sets of a vague set X,as a pair of function (t_μ, f_μ) where $t_\mu : X \to L$,$f_\mu : X \to L$ are L-mapings such that $t_\mu \leq 1_L - f_\mu$ and $f_\mu \leq 1_L - t_\mu$. The function $t_\mu : X \to L$ define the the degrees of membership function and the function $f_\mu : X \to L$ degrees of non membership function of the element $x \in X$ to $\mu \subset X$ respectively .The L-vague sets t_μ and f_μ should satisffy the conditions $t_\mu(x) \leq 1_L - f_\mu(x)$ and $f_\mu(x) \leq 1_L - t_\mu(x)$. for ever x in X (where $1_L - t_\mu(x)$ and (where $1_L - f_\mu(x)$ are elements of Brouwerian latice L,defined by means of the formula $(1_L - t_\mu(x)) = inf\{\alpha \in L :$

$1_L = t_\mu(x) \vee \alpha\}$ $for\ ever\ x\ in\ X)$ and $(1_L - f_\mu(x)) = inf\{\alpha \in L : 1_L = f_\mu(x) \vee \alpha\}$ $for\ ever\ x\ in\ X)$. In this paper we introduced L-vague cut-set and L-vague semiring of L-Semiring respectively.The objective of this paper is to contribute further,to the study of vague algebra by introducing concepts of L-vague cut-set and L-vague semiring of L-Semiring also homomorphism of L-vague semiring of a L-Semiring.

1. Preliminaries

In this section we briefly present the necessary material on latices, Boolean latices, Brouwerian latices and illustriate with examples.

Definition 1.1 [15] A poset $(L, \leq)$ is called a lattice if $sup\{x, y\}$ also denoted by $(x \vee y)$ and $inf\{x, y\}$ also denoted by $(x \wedge y)$ exists for every pair of elements $x, y\ in\ L$.

Definition 1.2 [15] A lattice $(L, \leq)$ in which every subset of L has g.l.b and l.u.b in it is called a complete lattice.

Definition 1.3 [15] A lattice L is said to be distributive if it satisfies $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ and $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ $for\ all\ x, y, z\ in\ L$.

Definition 1.4 [15] A lattice L is said to be bounded if L has least element and greatest element.usually least element of L is denoted by 0_L and greatest element is denoted by 1_L

Definition1.5[15] An L-vague set μ in X is a pair $\mu = (t_\mu, f_\mu)$ where $t_\mu : X \to L$ $f_\mu : X \to L$ are mappings such that $t_\mu \leq 1_L - f_\mu$ and $f_\mu \leq 1_L - t_\mu$ The mapping $t_\mu : X \to L$ is defined the degree of membership function and $f_\mu : X \to L$ defined by the degree of non membership function of the element $x \in X$ to $\mu \subset X$ respectively .The function t_μ, f_μ satisfy the condition $t_\mu \leq 1_L - f_\mu$ and $f_\mu \leq 1_L - t_\mu$ ie $t_\mu(x) \leq 1_L - f_\mu(x)$ and $f_\mu(x) \leq 1_L - t_\mu(x)$ for x in X,where $1_L - t_\mu(x)$ and $1_L - f_\mu(x)$ are elements in Brouwerian lattice L.

we recall here that in ordinary vague sets and Boolean vague sets if (t_A, f_A) is a vague set or Boolean vague set then the condition $t_A \leq 1 - f_A$ or $t_A \leq f_A^1$ implies $f_A \leq 1 - t_A$, $f_A \leq t_A^1$.But in our L-vague sets it is not the case,and so in the definition itself we have in included that $t_A \leq 1_L - f_A^1$ and $f_A \leq 1_L - t_A^1$ which is not un natural as this implies that truth is contained in the complement of falsity and falsity is contained in the complement of truth.

Definition 1.6 [15] Let $(G, *)$ be a group.An L-vague set $A = (t_A, f_A)$ of G is said to be L-vague group of G if it satisfies the following conditions

$(1)V_A(xy) \geq \vee\{V_A(x), V_A(y)\}$ for all x,y$\in G$ $(2)V_A(x^{-1}) \geq \{V_A(x)\}$ ie

$(1)t_A(xy) \geq \wedge\{t_A(x), t_A(y)\}$

$(2)f_A(xy) \leq \vee\{f_A(x), f_A(y)\}$

$(3)t_A(x^{-1}) \geq t_A(x)$ $(4)f_A(x^{-1}) \leq f_A(x)$ for all $x, y \in G$

Definition 1.7 [8]Let A be a Vague group of a group G then the set $\mathbb{N}(A)$

$= \{a \in G | V_A(axa^{-1}) = V_A(x) for\ all\ a, \in G\}$ is called vague normalizer of A.

Definition 1.8 [8] Let A be a vague group of a group G Then the Set $\mathbb{C}(A)$

$= \{a \in G | V_A([a, x]) = V_A(e) for\ all\ x \in G\}$ is called vague centralizer of A,where [a,x]=$a^{-1}x^{-1}ax$.

Defination 1.9[2]Let A be a vague group of G,and $GV_A = \{x \in G : V_A(x) = V_A(e)\}$.then the order of A is defined as the order of the crisp sub group GV_A and it is denoted by O(A).

2.L-Vague Cut-set on L-Vague Semirings of L-semiring

In this section we define the following

Definition 2.1 Let S be a L-semiring.An L-vague set $A = (t_A, f_A)$ is said to be L-vague semiring of L-semiring S if it satiesfies the following properties.

1.$t_A(x + y) \geq \wedge\{t_A(x), t_A(y)\}$ and $f_A(x + y) \leq \vee\{t_A(x), t_A(y)\}$

2.$t_A(xy) \geq \wedge\{t_A(x), t_A(y)\}$ and $f_A(xy) \leq \vee\{t_A(x), t_A(y)\}$

3.$t_A(x \wedge y) \geq \wedge\{t_A(x), t_A(y)\}$ and 4.$f_A(x \vee y) \leq \vee\{t_A(x), t_A(y)\}$

Definition 2.2 Let A be an L-vague semiring of L-semiring S.Then the true $\alpha-$cut set $t_{A\alpha}$can be defined by $t_{A\alpha} = \{x \in S : t_A(x) \geq \alpha\}$.

Definition 2.3 Let A be an L-vague semiring of L-semiring S.Then the false $\alpha-$cut set $f_{A\alpha}$ can be defined by $f_{A\alpha} = \{x \in S : f_A(x) \leq \beta\}$.

Definition 2.4 Let A be an L-vague semiring of L-semiring S.Then the L-vague cut set $A_{(\alpha\beta)}$ can be defined by

$A_{(\alpha,\beta)}(x) = \{x \in S : t_A(x) \geq \alpha\ and\ f_A(x) \leq \beta\}$. for all $\alpha, \beta \in [0, 1]$.

Theorem 2.5 Let A L-vague semiring of a L-semiring S.Then true cut set $t_{A\alpha}$ is L-vague semiring of a L-semiring S for $\alpha \in [0,\ 1]$.

Proof Let x ,y be any in true α cut set $t_{A\alpha}$.

$\Rightarrow t_A(x) \geq \alpha$ and $t_A(y) \geq \alpha$.

1.$t_A(x + y) \geq \vee\{t_A(x), t_A(y)\} = \vee\{\alpha, \alpha\} = \alpha$

$\Rightarrow t_A(x + y) \geq \alpha$ for all x,y in $t_{A\alpha}$.

2.$f_A(x + y) \leq \wedge\{f_A(x), f_A(y)\} = \vee\{\alpha, \alpha\} = \alpha$

$\Rightarrow f_A(x + y) \leq \alpha$ for all x,y in $t_{A\alpha}$.

3.$t_A(x \wedge y) \geq \vee\{t_A(x), t_A(y)\} = \vee\{\alpha, \alpha\} = \alpha$

$\Rightarrow t_A(x \wedge y) \geq \alpha$ for all x,y in $t_{A\alpha}$.

4.$f_A(x \vee y) \leq \wedge\{f_A(x), f_A(y)\} = \vee\{\alpha, \alpha\} = \alpha$

$\Rightarrow f_A(x \vee y) \leq \alpha$ for all x,y in $t_{A\alpha}$.

5.$t_A(xy) \geq \vee\{t_A(x), t_A(y)\} = \vee\{\alpha, \alpha\} = \alpha$

$\Rightarrow t_A(xy) \geq \alpha$ for all x,y in $t_{A\alpha}$.

6.$f_A(xy) \leq \wedge\{f_A(x), f_A(y)\} = \vee\{\alpha, \alpha\} = \alpha$

$\Rightarrow f_A(xy) \leq \alpha$ for all x,y in $t_{A\alpha}$.

Hence $t_{A\alpha}$ is L-vague semiring of a L-semiring S for $\alpha \in [0, 1]$.

Theorem 2.6 Let A L-vague semiring of a L-semiring S.Then two true cut sets $t_{A\alpha_1}, t_{A\alpha_2}$ and α_1, α_1 are in $[0,\ 1]$ with $\alpha_1 < \alpha_2$ of A are equal if and only if thre is no x in S suchthat $\alpha_2 > t_A(x) > \alpha_1$.

Proof Assume that $t_{A\alpha_1} = t_{A\alpha_2}$
suppose there exists x in S such that $\alpha_2 > t_A(x) > \alpha_1$.
Then $t_{A\alpha_1} \subseteq t_{A\alpha_2}$
$\Rightarrow x \in t_{A\alpha_2}$ but not in $t_{A\alpha_1}$
$\Rightarrow$ which is a contradiction to $t_{A\alpha_1} = t_{A\alpha_2}$
Therefore there is no x in S such that $\alpha_2 > t_A(x) > \alpha_1$.
Conversely suppose that if there is no x in S such that $\alpha_2 > t_A(x) > \alpha_1$.
Then $t_{A\alpha_1} = t_{A\alpha_2}$. Hence the thorem follows.

Theorem 2.7 Let A be a L-vague semiring of L-semiring S such that $t_{A\alpha}$ be true α cut-set of S. If $\alpha \in [0\ 1]$, then $t_{A\alpha}$ is a L-vague semiring of L-semiring S.
Proof Let A be a L-vague semiring of L-semiring S and $t_{A\alpha}$ be true α cut-set of S. $\Rightarrow t_A(x) \geq \alpha$ and $t_A(y) \geq \alpha$
$case : (i)$
If $\alpha_1 \geq \alpha_2$ then x, y in $t_{A\alpha_2}$
as $t_{A\alpha_2}$ is a L-vague semiring of L-semiring S, then $x + y, xy, x \vee y, x \wedge y$ in $t_{A\alpha_2}$.
Now, $(1) t_A(x + y) \geq \alpha_2 = \vee\{\alpha, \alpha\} = \vee\{t_A(x), t_A(y)\}$
$\Rightarrow t_A(x + y) \geq \vee\{t_A(x), t_A(y)\}$ for all x,y in S.
$(2) f_A(x + y) \leq \alpha_2 = \wedge\{\alpha, \alpha\} = \wedge\{t_A(x), t_A(y)\}$
$\Rightarrow t_A(x + y) \geq \wedge\{t_A(x), t_A(y)\}$ for all x,y in S.
$(3) t_A(xy) \geq \alpha_2 = \vee\{\alpha, \alpha\} = \vee\{t_A(x), t_A(y)\}$
$\Rightarrow t_A(xy) \geq \vee\{t_A(x), t_A(y)\}$ for all x,y in S.
$(4) f_A(xy) \leq \alpha_2 = \wedge\{\alpha, \alpha\} = \wedge\{f_A(x), f_A(y)\}$
$\Rightarrow f_A(xy) \geq \wedge\{f_A(x), f_A(y)\}$ for all x,y in S.
$(5) t_A(x \vee y) \geq \alpha_2 = \vee\{\alpha, \alpha\} = \vee\{t_A(x), t_A(y)\}$
$\Rightarrow t_A(x \vee y) \geq \vee\{t_A(x), t_A(y)\}$ for all x,y in S.
$(6) f_A(x \wedge y) \leq \alpha_2 = \wedge\{\alpha, \alpha\} = \wedge\{t_A(x), t_A(y)\}$
$\Rightarrow f_A(x \wedge y) \geq \wedge\{f_A(x), f_A(y)\}$ for all x,y in S.

$case : (ii)$
If $\alpha_1 \leq \alpha_2$ If $\alpha_1 \leq \alpha_2$ then x, y in $t_{A\alpha_2}$
as $t_{A\alpha_2}$ is a L-vague semiring of L-semiring S, then $x + y, xy, x \vee y, x \wedge y$ in $t_{A\alpha_2}$.
$(1) t_A(x + y) \geq \alpha_2 = \vee\{\alpha, \alpha\} = \vee\{t_A(x), t_A(y)\}$
$\Rightarrow t_A(x + y) \geq \vee\{t_A(x), t_A(y)\}$ for all x,y in S.
$(2) f_A(x + y) \leq \alpha_2 = \wedge\{\alpha, \alpha\} = \wedge\{t_A(x), t_A(y)\}$
$\Rightarrow t_A(x + y) \geq \wedge\{t_A(x), t_A(y)\}$ for all x,y in S.
$(3) t_A(xy) \geq \alpha_2 = \vee\{\alpha, \alpha\} = \vee\{t_A(x), t_A(y)\}$
$\Rightarrow t_A(xy) \geq \vee\{t_A(x), t_A(y)\}$ for all x,y in S.
$(4) f_A(xy) \leq \alpha_2 = \wedge\{\alpha, \alpha\} = \wedge\{f_A(x), f_A(y)\}$
$\Rightarrow f_A(xy) \geq \wedge\{f_A(x), f_A(y)\}$ for all x,y in S.
$(5) t_A(x \vee y) \geq \alpha_2 = \vee\{\alpha, \alpha\} = \vee\{t_A(x), t_A(y)\}$
$\Rightarrow t_A(x \vee y) \geq \vee\{t_A(x), t_A(y)\}$ for all x,y in S.
$(6) f_A(x \wedge y) \leq \alpha_2 = \wedge\{\alpha, \alpha\} = \wedge\{t_A(x), t_A(y)\}$
$\Rightarrow f_A(x \wedge y) \geq \wedge\{f_A(x), f_A(y)\}$ for all x,y in S.

$case : (iii)$
If $\alpha_1 = \alpha_2$ then the proof is trivial, in all the above cases $t_{A\alpha}$ is a L-vague semiring of L-semiring S.

Theorem 2.8 Let A be a L- vague semiring of a L-semiring S.If for any two true $\alpha-$sets $t_{A\alpha_1},\ t_{A\alpha_2}$ of A in S,then their intersection is also L- vague semiring of a L-semiring S.
Proof $\alpha_1, \alpha_2 \in [0,\ 1]$
$case : (i)$
$\alpha_1 < t_A(x) < \alpha_2$ then $t_{A\alpha_1} \subseteq t_{A\alpha_2}$
therefore $t_{A\alpha_1} \cap t_{A\alpha_2} = t_{A\alpha_1}$ but $t_{A\alpha_1}$ is a true $\alpha-$cut set of S.
$case : (ii)$
$\alpha_1 > t_A(x) > \alpha_2$ then $t_{A\alpha_1} \subseteq t_{A\alpha_2}$
therefore $t_{A\alpha_1} \cap t_{A\alpha_2} = t_{A\alpha_1}$ but $t_{A\alpha_2}$ is a true $\alpha-$cut set of S.
$case : (iii)$
$\alpha_1 = t_A(x) < \alpha_2$ then $t_{A\alpha_1} = t_{A\alpha_2}$.
In all the three cases their intersection is also L- vague semiring of a L-semiring S.

Theorem 2.9 Let A be a L- vague semiring of a L-semiring S.If for any two true $\alpha-$sets $t_{A\alpha_1}, t_{A\alpha_2}$ of A in S,then their union is also L-vague semiring of a L-semiring S.
Proof $\alpha_1, \alpha_2 \in [0, 1]$
$case : (i)$
$\alpha_1 < t_A(x) < \alpha_2$ then $t_{A\alpha_1} \subseteq t_{A\alpha_2}$
therefore $t_{A\alpha_1} \cup t_{A\alpha_2} = t_{A\alpha_1}$ but $t_{A\alpha_1}$ is a true $\alpha-$cut set of S.
$case : (ii)$
$\alpha_1 > t_A(x) > \alpha_2$ then $t_{A\alpha_1} \subseteq t_{A\alpha_2}$
therefore $t_{A\alpha_1} \cup t_{A\alpha_2} = t_{A\alpha_1}$ but $t_{A\alpha_2}$ is a true $\alpha-$cut set of S.
$case : (i)$
$\alpha_1 = t_A(x) < \alpha_2$ then $t_{A\alpha_1} = t_{A\alpha_2}$.
In all the three cases their union is also L- vague semiring of a L-semiring S.

3.Homomorphism of L-vague cut-set on L-Vague Semiring of a L-Semiring.
In this section we define the following
Definition 3.1 Let S_1 and S_2 be two L-semirings and $\phi : S_1 \rightarrow S_2$ be a maping.Let A be a L-vague set of S_1.Then the image of L-vague set A can be defined by $(V_{\phi(A)}(x) = (t_{\phi(A)}(x), f_{\phi(A)}(x))$
where $t_{\phi(A)}(x) = t_A(\phi(x))$ and $f_{\phi(A)}(x) = f_A(\phi(x))$.

Definition 3.2: Let S_1 and S_2 be two semirings and $\phi : S_1 \rightarrow S_2$ be a maping. Let A be a l-vague set of S_2. Then the inverse image of L-vague set A can be defined by $(V_{\phi^{-1}(A)}(x) = (t_{\phi^{-1}(A)}(x), f_{\phi^{-1}(A)}(x))$
where $t_{\phi^{-1}(A)}(x) = t_A(\phi^{-1}(x))$ and $f_{\phi^{-1}(A)}(x) = f_A(\phi^{-1}(x))$.

Theorem 3.3: The homomorphic image of a true cut-set $t_{A\alpha}$ of L-vague semiring of L-semiring S_1 is true cut-set $t_{A\alpha}$ of L-vague semiring of L-semiring S_2.

Proof: Let S_1, S_2 be any two L vague semirings and $\phi : S_1 \rightarrow S_2$ be a semiring homomorphism.
Then $\phi(x + y) = \phi(x) + \phi(y), \phi(xy) = \phi(x)\phi(y)$
$\phi(x \vee y) = \phi(x) \vee \phi(y), \phi(x \vee y) = \phi(x) \vee \phi(y)$

$\phi(x \wedge y) = \phi(x) \wedge \phi(y), \phi(x \wedge y) = \phi(x) \wedge \phi(y)$ for all x,y in S_1
true cut-set $t_{A\alpha}$ of L-vague semiring of L-semiring S_1
$\Rightarrow t_A(x) \geq \alpha \, and \, t_A(y) \geq \alpha$
$\Rightarrow t_A(x+y) \geq \alpha \, and \, t_A(xy) \geq \alpha$
(1) Let $x, y \in S_1$.
We have $t_A(\phi(x) + (\phi(y)) = \{t_A(\phi(x+y))\} \geq t_A(x+y)$
since ϕ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$
$\therefore \; t_A(\phi(x+y)) \geq \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$...(3.3.1)

(2) Let $x, y \in S_1$.
We have $f_A(\phi(x) + (\phi(y)) = \{f_A(\phi(x+y))\} \leq f_A(x+y)$
since ϕ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore \; f_A(\phi(x+y)) \leq \vee\{f_A(\phi(x)), f_A(\phi(y))\}$...(3.3.2)

(3) Let $x, y \in S_1$.
We have $t_A(\phi(x).(\phi(y)) = \{t_A(\phi(xy))\} \geq t_A(xy)$
since ϕ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$
$\therefore \; t_A(\phi(xy)) \geq \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$...(3.3.3)

(4) Let $x, y \in S_1$.
We have $f_A(\phi(x)\phi(y)) = \{f_A(\phi(xy))\} \leq f_A(xy)$
since ϕ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore \; f_A(\phi(xy)) \leq \vee\{f_A(\phi(x)), f_A(\phi(y))\}$...(3.3.4)

(5) Let $x, y \in S_1$.
We have $t_A(\phi(x) \vee (\phi(y)) = \{t_A(\phi(x \vee y))\} \geq t_A(x \vee y)$
since ϕ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$
$\therefore \; t_A(\phi(x \vee y)) \geq \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$...(3.3.5)

(6) Let $x, y \in S_1$.
We have $f_A(\phi(x) \wedge \phi(y)) = \{f_A(\phi(xy))\} \leq f_A(xy)$
since ϕ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore \; f_A(\phi(x \wedge y)) \leq \vee\{f_A(\phi(x)), f_A(\phi(y))\}$...(3.3.6)
from (3.3.1), (3.3.2), (3.3.3)(3.3.4), (3.3.5), (3.3.6)
Hence the theorem follows.

Theorem 3.4: The homomorphic pre image of a true cut-set $t_{A\alpha}$ of L-vague semiring of L-semiring S_1 is true cut-set $t_{A\alpha}$ of L-vague semiring of L-semiring S_2.
Proof: Let B be a L-vague semiring of S_2., Let $x, y \in S_1$.
Then Let S_1, S_2 be any two L vague semirings and $\phi^{-1} : S_2 \to S_1$ be a semiring homomorphism.
Then $\phi^{-1}(x+y) = \phi^{-1}(x) + \phi^{-1}(y), \phi(xy) = \phi^{-1}(x)\phi^{-1}(y)$
$\phi^{-1}(x \vee y) = \phi^{-1}(x) \vee \phi^{-1}(y), \phi^{-1}(x \vee y) = \phi^{-1}(x) \vee \phi^{-1}(y)$
$\phi^{-1}(x \wedge y) = \phi^{-1}(x) \wedge \phi^{-1}(y), \phi^{-1}(x \wedge y) = \phi^{-1}(x) \wedge \phi^{-1}(y)$
for all x,y in S_2
true cut-set $t_{A\alpha}$ of L-vague semiring of L-semiring S_1
$\Rightarrow t_A(x) \geq \alpha \, and \, t_A(y) \geq \alpha$
$\Rightarrow t_A(x+y) \geq \alpha \, and \, t_A(xy) \geq \alpha$
$\phi : S_1 \to S_2$ be a semiring homomorphism.
(1) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x) + (\phi^{-1}(y)) = \{t_A(\phi^{-1}(x+y))\} \geq t_A(x+y)$
since ϕ^{-1} is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore \; t_A(\phi^{-1}(x+y)) \geq \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$...(3.4.1)

(2) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x) + (\phi^{-1}(y)) = \{f_A(\phi^{-1}(x+y))\} \leq f_A(x+y)$
since ϕ^{-1} is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$
$\therefore \; f_A(\phi^{-1}(x+y)) \leq \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$...(3.4.2)

(3) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x).(\phi(y)) = \{t_A(\phi^{-1}(xy))\} \geq t_A(xy)$
since ϕ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore \; t_A(\phi^{-1}(xy)) \geq \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$...(3.4.3)

(4) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x)\phi(y)) = \{f_A(\phi^{-1}(xy))\} \leq f_A(xy)$
since ϕ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$
$\therefore \; f_A(\phi^{-1}(xy)) \leq \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$...(3.4.4)

(5) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x) \vee (\phi^{-1}(y)) = \{t_A(\phi^{-1}(x \vee y))\} \geq t_A(x \vee y)$
since ϕ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore \; t_A(\phi^{-1}(x \vee y)) \geq \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$...(3.4.5)

(6) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x) \wedge \phi^{-1}(y)) = \{f_A(\phi^{-1}(xy))\} \leq f_A(xy)$
since ϕ^{-1} is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore \; f_A(\phi^{-1}(x \wedge y)) \leq \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$...(3.4.6)
from (3.4.1), (3.4.2), (3.4.3)(3.4.4), (3.4.5), (3.4.6)
Hence the theorem follows.

Theorem 3.5: The homomorphic image of a $(\alpha\ \beta)$ cut-set $A_{(\alpha\beta)}$ of L-vague semiring of L-semiring S_1 is $(\alpha\ \beta)$ cut-set $A_{(\alpha\beta)}$ of L-vague semiring of L-semiring S_2.

Proof: Let S_1, S_2 be any two L vague semirings and $\phi : S_1 \to S_2$ be a semiring homomorphism.
Then $\phi(x + y) = \phi(x) + \phi(y), \phi(xy) = \phi(x)\phi(y)$
$\phi(x \vee y) = \phi(x) \vee \phi(y), \phi(x \vee y) = \phi(x) \vee \phi(y)$
$\phi(x\wedge y) = \phi(x)\wedge\phi(y), \phi(x\wedge y) = \phi(x)\wedge\phi(y)$ for all $x, yinS$
$(\alpha\ \beta)$ cut-set $t_{A(\alpha\beta)}$ of L-vague semiring of L-semiring S_1
$\Rightarrow t_A(x) \geq \alpha\ and\ t_A(y) \geq \alpha$
$\Rightarrow t_A(x + y) \geq \alpha\ and\ t_A(xy) \geq \alpha$
$\Rightarrow f_A(x) \leq \beta\ and\ f_A(y) \leq \beta$
$\Rightarrow f_A(x + y) \leq \beta\ and\ f_A(xy) \leq \beta$
(1) Let $x, y \in S_1$.
We have $t_A(\phi(x) + (\phi(y)) = \{t_A(\phi(x + y))\} \geq t_A(x + y)$
since ϕ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$
$\therefore\ t_A(\phi(x + y)) \geq \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$...(3.5.1)

(2) Let $x, y \in S_1$.
We have $f_A(\phi(x) + (\phi(y)) = \{f_A(\phi(x + y))\} \leq f_A(x + y)$
since ϕ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore\ f_A(\phi(x + y)) \leq \vee\{f_A(\phi(x)), f_A(\phi(y))\}$...(3.5.2)

(3) Let $x, y \in S_1$.
We have $t_A(\phi(x).(\phi(y)) = \{t_A(\phi(xy))\} \geq t_A(xy)$
since ϕ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$
$\therefore\ t_A(\phi(xy)) \geq \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$...(3.5.3)

(4) Let $x, y \in S_1$.
We have $f_A(\phi(x)\phi(y)) = \{f_A(\phi(xy))\} \leq f_A(xy)$
since ϕ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore\ f_A(\phi(xy)) \leq \vee\{f_A(\phi(x)), f_A(\phi(y))\}$...(3.5.4)

(5) Let $x, y \in S_1$.
We have $t_A(\phi(x) \vee (\phi(y)) = \{t_A(\phi(x \vee y))\} \geq t_A(x \vee y)$
since ϕ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$
$\therefore\ t_A(\phi(x \vee y)) \geq \wedge\{t_A(\phi(x)), t_A(\phi(y))\}$...(3.5.5)

(6) Let $x, y \in S_1$.
We have $f_A(\phi(x) \wedge \phi(y)) = \{f_A(\phi(xy))\} \leq f_A(xy)$
since ϕ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore\ f_A(\phi(x \wedge y)) \leq \vee\{f_A(\phi(x)), f_A(\phi(y))\}$...(3.5.6)
from (3.5.1), (3.5.2), (3.5.3)(3.5.4), (3.5.5), (3.5.6) we have the homomorphic image of a L-vague semiring of S_1 is a L-vague semiring of S_2.
Hence the theorem follows.

Theorem 3.6: The homomorphic pre image of a $(\alpha\ \beta)$ cut-set $A_{(\alpha\beta)}$ of L-vague semiring of L-semiring S_1 is $(\alpha\ \beta)$ cut-set $A_{(\alpha\beta)}$ of L-vague semiring of L-semiring S_2.

Proof: Let B be a L-vague semiring of S_2., Let $x, y \in S_1$.
Then Let S_1, S_2 be any two L vague semirings and $\phi^{-1} : S_2 \to S_1$ be a semiring homomorphism.
Then $\phi^{-1}(x + y) = \phi^{-1}(x) + \phi^{-1}(y), \phi(xy) = \phi^{-1}(x)\phi^{-1}(y)$
$\phi^{-1}(x\vee y) = \phi^{-1}(x)\vee\phi^{-1}(y), \phi^{-1}(x\vee y) = \phi^{-1}(x)\vee\phi^{-1}(y)$
$\phi^{-1}(x\wedge y) = \phi^{-1}(x)\wedge\phi^{-1}(y), \phi^{-1}(x\wedge y) = \phi^{-1}(x)\wedge\phi^{-1}(y)$
for all $x, yinS$
$(\alpha\ \beta)$ cut-set $t_{A(\alpha\beta)}$ of L-vague semiring of L-semiring S_1
$\Rightarrow t_A(x) \geq \alpha\ and\ t_A(y) \geq \alpha$
$\Rightarrow t_A(x + y) \geq \alpha\ and\ t_A(xy) \geq \alpha$
$\phi : S_1 \to S_2$ be a semiring homomorphism.
(1) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x) + (\phi^{-1}(y)) = \{t_A(\phi^{-1}(x + y))\} \geq t_A(x + y)$
since ϕ^{-1} is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore\ t_A(\phi^{-1}(x + y)) \geq \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$...(3.6.1)

(2) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x) + (\phi^{-1}(y)) = \{f_A(\phi^{-1}(x + y))\} \leq f_A(x + y)$
since ϕ^{-1} is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$
$\therefore\ f_A(\phi^{-1}(x + y)) \leq \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$...(3.6.2)

(3) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x).(\phi(y)) = \{t_A(\phi^{-1}(xy))\} \geq t_A(xy)$
since ϕ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$
$\therefore\ t_A(\phi^{-1}(xy)) \geq \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$...(3.6.3)

(4) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x)\phi(y)) = \{f_A(\phi^{-1}(xy))\} \leq f_A(xy)$
since ϕ is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$
$\therefore\ f_A(\phi^{-1}(xy)) \leq \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$...(3.6.4)

(5) Let $x, y \in S_1$.
We have $t_A(\phi^{-1}(x) \vee (\phi^{-1}(y)) = \{t_A(\phi^{-1}(x \vee y))\} \geq t_A(x \vee y)$
since ϕ is a homomorphisim
$\geq \wedge\{t_A(x), t_A(y)\}$
$= \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$

$\therefore\ t_A(\phi^{-1}(x \vee y)) \geq \wedge\{t_A(\phi^{-1}(x)), t_A(\phi^{-1}(y))\}$...(3.6.5)

(6) Let $x, y \in S_1$.
We have $f_A(\phi^{-1}(x) \wedge \phi^{-1}(y)) = \{f_A(\phi^{-1}(xy))\} \leq f_A(xy)$ since ϕ^{-1} is a homomorphisim
$\leq \vee\{f_A(x), f_A(y)\}$
$= \vee\{f_A(\phi(x)), f_A(\phi(y))\}$
$\therefore\ f_A(\phi^{-1}(x \wedge y)) \leq \vee\{f_A(\phi^{-1}(x)), f_A(\phi^{-1}(y))\}$...(3.6.6)
from $(3.6.1), (3.6.2), (3.6.3)(3.6.4), (3.6.5), (3.6.6)$
Hence the theorem follows.

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