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Heat Transfer in Viscoelastic Boundary Layer Flow over a Stretching Sheet with Non-uniform Heat Source/Sink

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INTRODUCTION

The field of boundary layer flow problem over a stretching sheet has many industrial applications such as polymer sheet or filament extrusion from a dye or long thread between feed roll or wind-up roll, glass fiber and paper production, drawing of plastic films, liquid films in condensation process. Due to its high applicability in industrial phenomena, it has attracted the attentions of many researchers and one of the pioneering studies has been performed by Sakiadis [1]. It was extended by Crane [2] assuming that the velocity of moving strip is proportional to the distance from the slit. These types of flows usually occur in the drawing of plastic films and artificial fibers. The heat and mass transfer over a stretching sheet with or without suction/blowing by considering various situations was studied by many researchers (Gupta and Gupta [3], Fox et al. [4], Dutta et al. [5], Chen and Char [6] etc. All the above studies restrict their analyses to Newtonian flows.

In reality most liquids are non-Newtonian in nature, which are abundantly used in many industrial applications. Non-Newtonian fluids have gained considerable importance, because the power required in stretching a sheet and the heat transfer rate for a viscoelastic fluid is found to be less in Non – Newtonian fluids when compared to Newtonian fluids. In addition, the inadequacy of the classical Navier–Stokes theory to describe rheological complex fluids such as polymer solution, blood, paints, certain oils and greases, has led to the development of several theories of non-Newtonian fluids [7-10].

A series of studies on heat transfer effects on viscoelastic fluid have been made by many authors under different physical situations [11-14]. Khan and Sanjayanand [15] have derived a similarity solution of a viscoelastic boundary layer flow and heat transfer over an exponential stretching surface.

The effects of heat generation/absorption become important in view of various physical problems [16, 17]. Even, the mixed convection boundary layer flow of a Newtonian, electrically conducting fluid over an inclined continuously stretching sheet with power–law temperature variation in the presence of magnetic field, internal heat generation/absorption and wall suction/injection is analyzed by Abo-Eldahab and El Aziz [18].

Cortell [19] studied flow and heat transfer of a viscoelastic fluid over a stretching surface considering both constant sheet temperature and prescribed sheet temperature. Abel *et al*. [20] carried out a study of MHD viscoelastic boundary layer flow and heat transfer over a stretching surface in the presence of non-uniform heat source considering prescribed surface temperature and prescribed surface heat flux and solutions are obtained analytically in terms of Kummer's function.

In this paper, we use numerical method, Quasilinearisation technique to extend the work of [19,20] and found flow and heat transfer characteristics of viscoelastic boundary layer flow over a stretching sheet with non-uniform heat source. The numerical method, Quasilinearization technique is directly applicable to computer aided solution of non-linear two-point boundary value problems in many engineering problems. Results are in good agreement with available literature.

MATHEMATICAL FORMULATION

A steady two dimensional laminar flow of an incompressible, visco-elastic liquid (Walters' liquid B model) over a stretching sheet with non-uniform heat source is considered. The co-ordinate system is chosen such that x-axis is along the stretching sheet and y-axis normal to the sheet. The flow is generated as a consequence of linear stretching of the boundary sheet, caused by simultaneous application of two equal and opposite forces along the x-axis, while keeping the origin fixed. Let the temperature of the ambient fluid be T_∞. The continuous stretching sheet is assumed to have a linear velocity of the form $u(x) = bx$, where b (>0) is the linear stretching constant, x is the distance from the slit. Governing boundary layer equations for fluid flow are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - k_0\left\{u\frac{\partial^3 u}{\partial x \partial y^2} + v\frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x \partial y}\right\} \tag{2}$$

where u and v are the velocity components respectively along the x and y directions, υ is the kinematic viscosity, k_0 is the co-efficient of elasticity.

The boundary conditions for the velocity field are:

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$$u = u_w = bx,\ v = 0 \quad at\ y = 0, b > 0$$
$$u \to 0, \frac{\partial u}{\partial y} \to 0\ as\ y \to \infty \tag{3}$$

The condition $\partial u / \partial y \to 0\ as\ y \to \infty$ is the augmented condition, since the flow is in an unbounded domain. In this case, the flow is caused solely by the stretching of the sheet, since the free stream velocity is zero.

Defining new variables:

$$u = bxf_\eta(\eta),\ v = -\sqrt{b\upsilon}f(\eta),\ \eta = \sqrt{b/\upsilon}\,y \tag{4}$$

where $f_\eta(\eta)$ denotes differentiation with respect to η. Clearly u and v defined above satisfy the continuity equation (1), and the equation (2) reduces to

$$f_\eta^2 - ff_{\eta\eta} = f_{\eta\eta\eta} - k_1\left\{2f_\eta f_{\eta\eta\eta} - ff_{\eta\eta\eta\eta} - f_{\eta\eta}^2\right\} \tag{5}$$

where $k_1 = \frac{k_0 b}{\upsilon}$ is the viscoelastic parameter.

The boundary conditions (3) become

$$f(0) = 0,\ f_\eta(0) = 1 \tag{6a}$$

$$f_\eta(\eta) \to 0,\ f_{\eta\eta}(\eta) \to 0\ \ as\ \eta \to \infty \tag{6b}$$

HEAT TRANSFER ANALYSIS

The governing boundary layer equation with internal heat generation or absorption is given by

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + q''' \tag{7}$$

Where k is the thermal diffusivity and c_p is the specific heat of a fluid at constant pressure and q''' is the space and temperature dependent internal heat generation / absorption. This can be modeled in simplest terms as

$$q''' = \left(\frac{ku_w(x)}{x\upsilon}\right)\left[A^*(T_w - T_\infty)f'(\eta) + B^*(T - T_\infty)\right] \tag{8}$$

Here A* and B* are parameters of space and temperature dependent internal heat generation/ absorption. It is to be noted that A* > 0 and B* > 0 correspond to internal heat generation while A* < 0 and B* < 0 correspond to internal heat absorption.

Heat transfer analysis is considered for two types of heating processes, namely,

(i) Prescribed Surface Temperature (ii) Prescribed Heat Flux, which will be discussed below.

Prescribed Power Law Surface Temperature (PST case)

For this heating process, the prescribed surface temperature is assumed to be quadratic function of x and is given by

$$T = T_w[= T_\infty + A\left(\frac{x}{l}\right)^2]\ at\ y = 0$$
$$T \to T_\infty\ as\ y \to \infty \tag{9}$$

Here l is the characteristic length.

Define non-dimensional temperature as

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{10}$$

Using (4), equation (7) reduces to

$$\theta_{\eta\eta} + \Pr f\theta_\eta - \left(\Pr f_\eta - B^*\right)\theta + \left(A^* - \Pr\theta\right)f_\eta = 0 \tag{11}$$

where $\Pr = \frac{\mu C_p}{k}$, is the Prandtl number

with boundary conditions

$$\theta(0) = 1,\ \theta(\infty) \to 0 \tag{12}$$

Prescribed Power law surface Heat Flux (PHF Case)

Prescribed power law surface heat flux (PHF), where surface is subjected to a power law heat flux q_w on the wall surface is considered to be a quadratic power of x in the form

$$-k\frac{\partial T}{\partial y} = q_w = D\left(\frac{x}{l}\right)^2 \text{ at y = 0} \tag{13}$$

$T \to T_\infty$ as $y \to \infty$

where D is a constant, k is the thermal conductivity

Define a dimensionless, scaled temperature as

$$g(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{14}$$

where $T_w - T_\infty = \frac{D}{k}\left(\frac{x}{l}\right)^2\sqrt{\frac{\upsilon}{b}}$

Using (4), equation(7) reduces to

$$g_{\eta\eta} + \Pr fg_\eta - \left(\Pr f_\eta - B^*\right)g + \left(A^* - \Pr g\right)f_\eta = 0 \tag{15}$$

With corresponding boundary conditions as

$$g_\eta(\eta) = -1\ at\ \eta = 0$$
$$g(\eta) \to 0\ as\ \eta \to \infty \tag{16}$$

NUMERICAL SOLUTION OF THE PROBLEM

The flow equation (5) coupled with the energy equation (11) or (15) constitute a set of non-linear non-homogeneous differential equations, for which obtaining closed-form solution is difficult, hence the problem has been solved numerical approach Quasi-linearization technique .This method is also known as the generalized Newton-Raphson method, given by Bellman and Kalaba [21].This method is quadratically convergent, starting from the initial guess value and solution obtained is valid for a large range of parameters. Even when the required numbers of initial conditions are not given, this method converges at faster speed.

To solve the equations (5) coupled (11) **or** (15) by using Quasilinearization technique, first we convert them in to the following system of first order differential equations by substituting

$$\left(f, f', f'', f''', \theta, \theta'\right) = \left(x_1, x_2, x_3, x_4, x_5, x_6\right)$$

(OR)

$$\left(f, f', f'', f''', g, g'\right) = \left(x_1, x_2, x_3, x_4, x_5, x_6\right)$$

$$\frac{dx_1}{d\eta} = x_2$$
$$\frac{dx_2}{d\eta} = x_3$$
$$\frac{dx_3}{d\eta} = x_4 \qquad (17)$$
$$\frac{dx_4}{d\eta} = \frac{1}{k_1 x_1}\left(x_2^2 - x_1 x_3 - x_4 + 2k_1 x_2 x_4 - k_1 x_3^2\right)$$
$$\frac{dx_5}{d\eta} = x_6$$

$$\frac{dx_6}{d\eta} = -\Pr x_1 x_6 + \left(\Pr x_2 - B^*\right)x_5 - \left(A^* - \Pr x_5\right)x_2$$

Let $x_i^r, i = 1,2,\ldots,6$ be an approximate current solution and $x_i^{r+1}, i = 1,2,\ldots,6$ be an improved solution of equation (17). By taking Taylor's series expansion around the current solution and neglecting the second and higher order derivative terms, the coupled first order system (17) is linearized as:

$$\frac{dx_1^{r+1}}{d\eta} = x_2^{r+1}$$
$$\frac{dx_2^{r+1}}{d\eta} = x_3^{r+1}$$
$$\frac{dx_3^{r+1}}{d\eta} = x_4^{r+1}$$
$$\frac{dx_4^{r+1}}{d\eta} = -\frac{1}{k_1 (x_1^r)^2}\left(\left(x_2^r\right)^2 - x_4^r + 2k_1 x_2^r x_4^r - k_1\left(x_3^r\right)^2\right)x_1^{r+1} + \frac{1}{k_1 x_1^r}\left(2x_2^r + 2k_1 x_4^r\right)x_2^{r+1} + \frac{1}{k_1 x_1^r}\left(-x_1^r - 2k_1 x_3^r\right)x_3^{r+1}$$
$$+ \frac{1}{k_1 x_1^r}\left(-1 + 2k_1 x_2^r\right)x_4^{r+1} + \frac{1}{k_1 x_1^r}\left(-x_4^r\right) \qquad (18)$$
$$\frac{dx_5^{r+1}}{d\eta} = x_6^{r+1}$$
$$\frac{dx_6^{r+1}}{d\eta} = \left(-\Pr x_6^r\right)x_1^{r+1} + \left(2\Pr x_5^r - A^*\right)x_2^{r+1} + \left(2\Pr x_2^r - B^*\right)x_5^{r+1} + \left(-\Pr x_1^r\right)x_6^{r+1} + \left(\Pr x_1^r x_6^r - 2\Pr x_2^r x_5^r\right)$$

The boundary conditions reduce to

$x_1^{r+1}(0) = 0,\ x_2^{r+1}(0) = 1,\ x_5^{r+1}(0) = 1$ **(PST case)**

(or)

$x_1^{r+1}(0) = 0,\ x_2^{r+1}(0) = 1,\ x_6^{r+1}(0) = -1$ **(PHF case)**

$$x_2^{r+1} \to 0,\ x_3^{r+1} \to 0,\ x_5^{r+1} \to 0 \ as\ \eta \to \infty$$

The initial values are chosen as follows:

For the homogeneous solution:

$$x_i^{h_1}(\eta) = [0\ 0\ 1\ 0\ 0\ 0]$$
$$x_i^{h_2}(\eta) = [0\ 0\ 0\ 1\ 0\ 0] \qquad (19)$$
$$x_i^{h_3}(\eta) = [0\ 0\ 0\ 0\ 0\ 1]$$

(or)

$$x_i^{h_3}(\eta) = [0\ 0\ 0\ 0\ 1\ 0]$$

For particular solution:

$$x_i^{p}(\eta) = [0\ 1\ 0\ 0\ 1\ 0] \qquad (20)$$

(or)

$$x_i^{p}(\eta) = [0\ 1\ 0\ 0\ 0\ -1]$$

Since the differential equations are linear, the principle of superposition holds and the general solution may be written as,

$$x_i^{r+1}(\eta) = C_1 x_i^{h1}(\eta) + C_2 x_i^{h2}(\eta) + C_3 x_i^{h3}(\eta) + x_i^{p}(\eta), \quad (21)$$

where C_1, C_2, C_3 are the unknown constants and are determined by considering the boundary conditions as $\eta \to \infty$.This solution $\left(x_i^{r+1}, i = 1,2,... \quad ,6\right)$is then compared with solution at the previous step $\left(x_i^{r}, i = 1,2,... \quad ,6\right)$ and next iteration is performed if the convergence has not been achieved or greater accuracy is desired.

RESULTS AND DISCUSSION

The steady two dimensional laminar flow and heat transfer of an incompressible viscoelastic fluid over stretching sheet in presence of non-uniform heat source has been considered. Heat transfer analysis has been carried out for two cases (i) Prescribed Surface Temperature (ii) Prescribed Heat Flux. The effect of various parameters on flow and heat transfer is depicted in the following graphs.

Fig 1 shows that horizontal velocity profile f_η (η) decreases with increase in viscoelastic parameter. The effect of increasing values of viscoelastic parameter (k_1) is to shift the streamlines towards stretching boundary and thereby reduces thickness of the momentum boundary layer. Therefore the effect of viscoelastic parameter is seen to decrease the boundary layer velocity throughout the boundary layer but significantly near the stretching sheet.

Fig 2(a) and 2(b) show that increasing the viscoelastic parameter (k_1) is to increase the temperature profile in both PST and PHF cases respectively. Effect of viscoelastic parameter increases viscoelastic normal stress, therefore boundary layer thickness increases. Temperature remains unchanged at the wall in PST case with the change of physical parameters and it tends to zero in the free stream region in both the cases.

Fig 3(a) and 3(b) shows the effect of Prandtl number on temperature profiles in both PST and PHF cases respectively. Both the graphs demonstrate that the increase of Prandtl number results in the decrease of temperature distribution at a particular point. This is due to the fact that increasing of Prandtl number decreases thermal boundary layer thickness. Temperature distribution in both the situations asymptotically approaches to zero in the free stream region.

Fig 4(a) and 4(b) illustrates the effect of heat source/sink parameter B* on temperature profiles. When B*>0, energy is released, which results in increase in magnitude of temperature in both the cases. When B*<0, energy is absorbed, which results in decrease in temperature in both the cases. Fig 5(a) and 5(b) depicts the effect of space-dependent heat source/sink parameter A* in both cases. The explanation on the effect of A* on temperature is similar to that B*.

The heat transfer phenomenon is usually analyzed from the numerical values of two physical parameters, namely (i). Wall temperature gradient $\theta'_\eta(0)$ in the PST case, and (ii). Wall temperature $g(0)$ in the PHF case, which are recorded for various values of parameters in Table1. Analysis of this table reveals that the effect of increasing the values of k_1, A*, B* is to increase the wall temperature gradient in PST case and wall temperature in PHF case; the opposite trend is observed in case of Prandtl number (Pr).

CONCLUSIONS

In this paper an analysis has been carried out to study the viscoelastic boundary layer fluid flow past a stretching sheet in presence of non-uniform heat source. Effect of several parameters on velocity and temperature are shown graphically and discussed briefly. Some of the important findings of our analysis, obtained from graphical representations are listed below:

i. Horizontal velocity profile decreases with increase in viscoelastic parameter.
ii. Temperature profiles increase with increase in viscoelastic parameter in both PST and PHF cases. Hence viscoelastic liquids having low viscous dissipation must be chosen.
iii. Thermal boundary layer thickness decreases with increase in Prandtl number.
iv. The effect of space and temperature dependent heat source/sink parameters is to generate temperature for increasing positive values and absorb temperature for decreasing negative values. Hence space and temperature dependent heat sinks are better suited for cooling purposes.
v. The Power law Heat Flux boundary condition is better suited for effective cooling of the stretching sheet.

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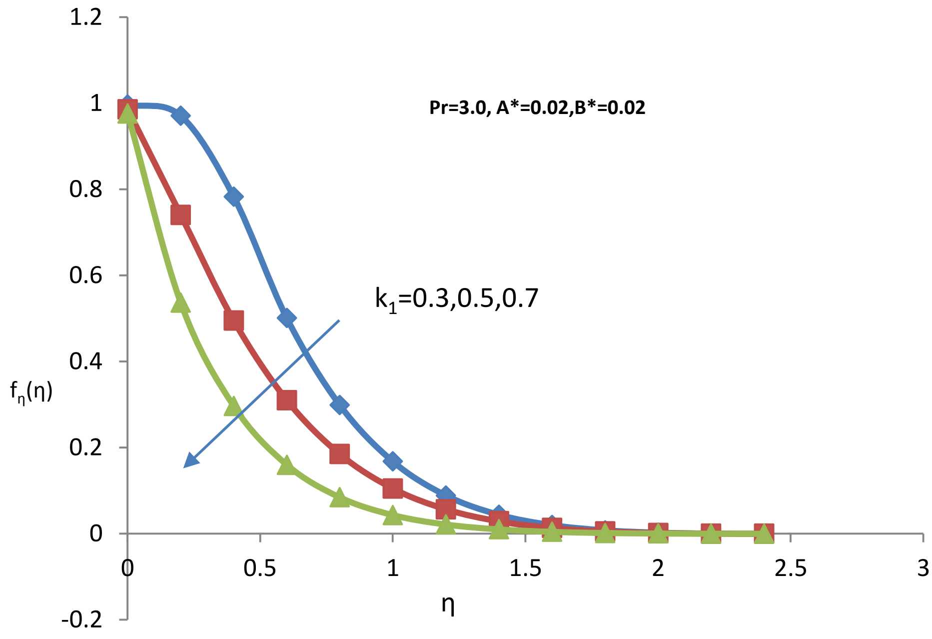

Figure 1. Plot of velocity ($f_\eta(\eta)$) vs η for different values of viscoelastic parameter

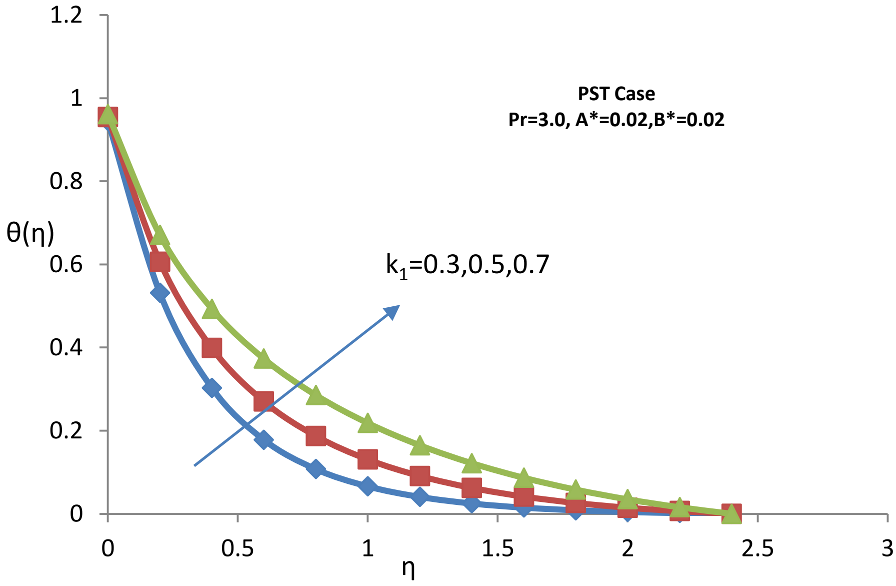

Figure 2(a). Effect of visco-elastic parameter (k1) on temperature distribution in PST case

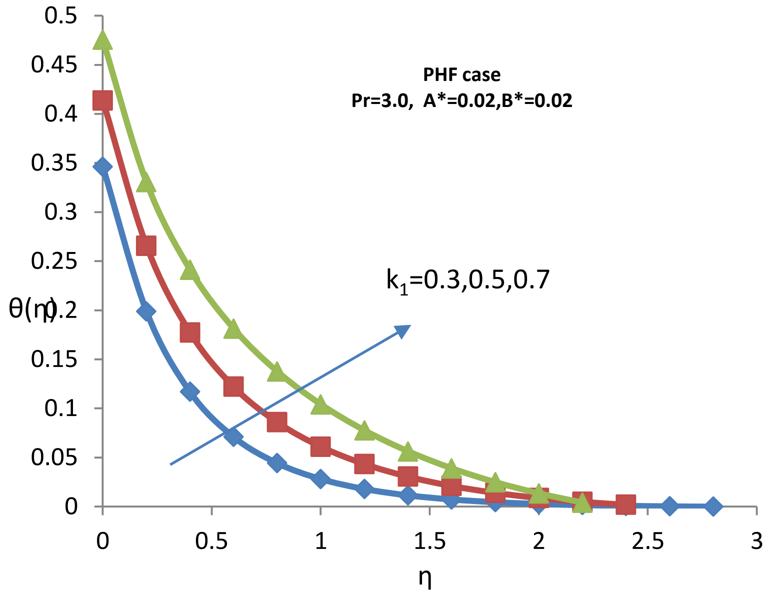

Figure 2(b). Effect of visco-elastic parameter (k1) on temperature distribution in PHF case

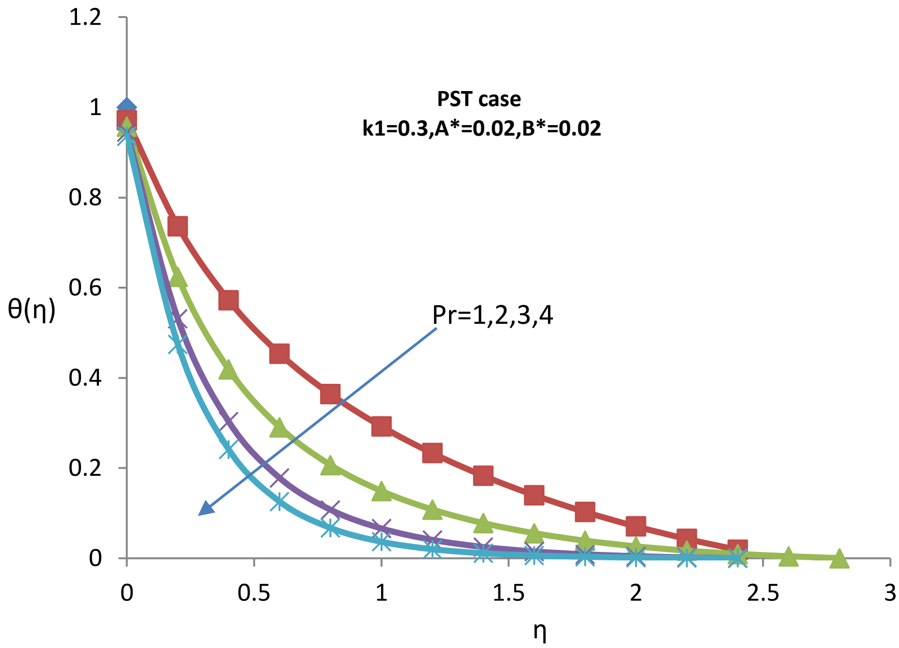

Figure 3(a). Effect of Prandtl number on temperature distribution in PST case

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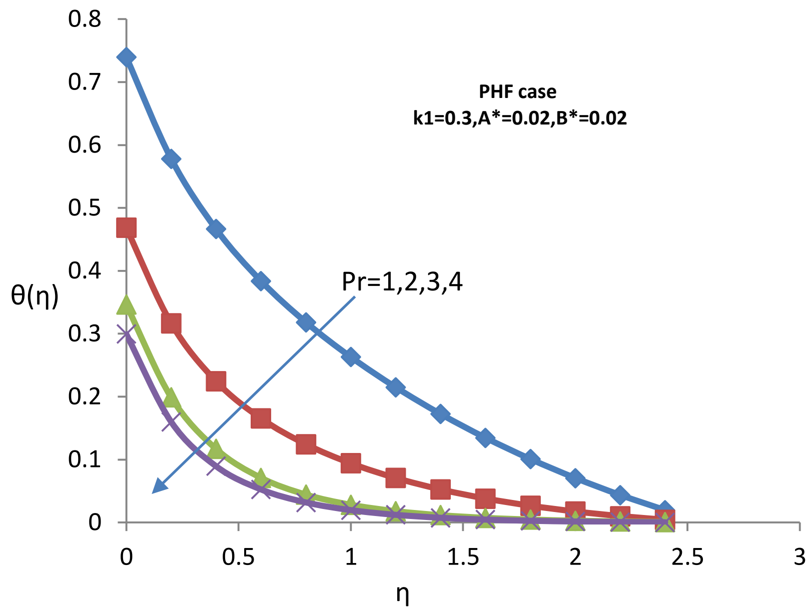

Figure 3(b). Effect of Prandtl number on temperature distribution in PHF case

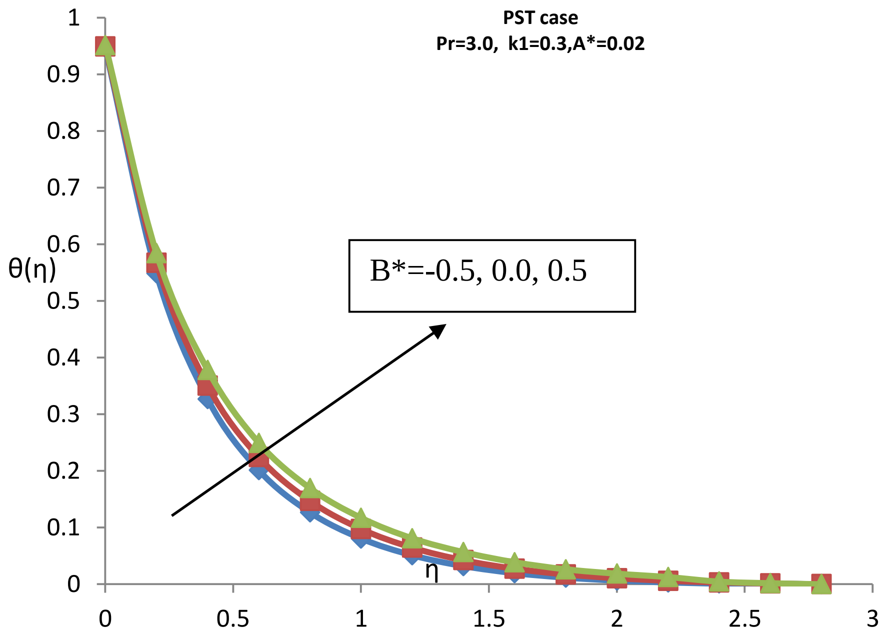

Figure 4(a). Effect of B* on temperature distribution in PST case

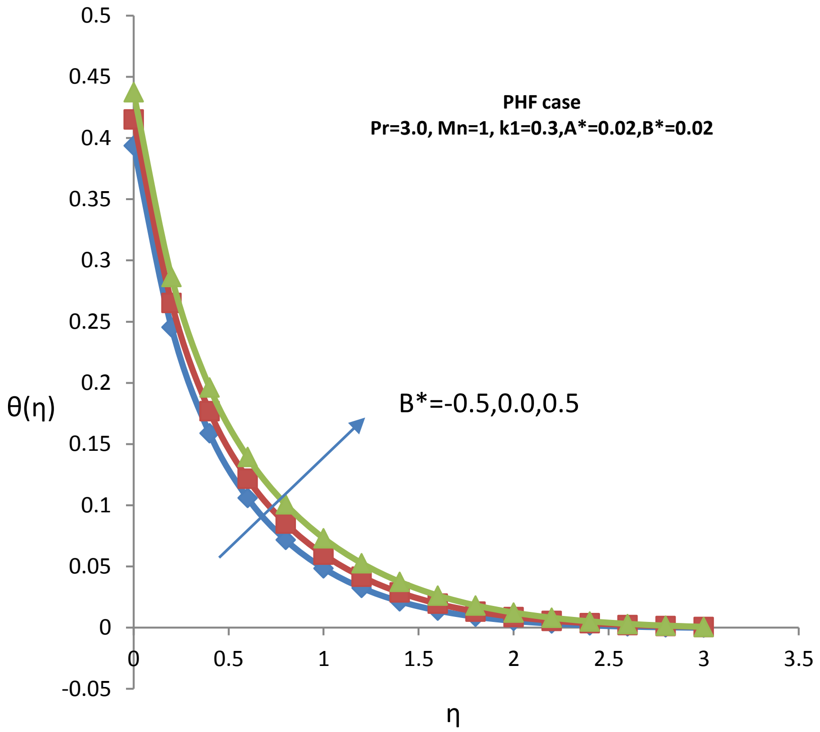

Figure 4(b). Effect of B* on temperature distribution in PHF case

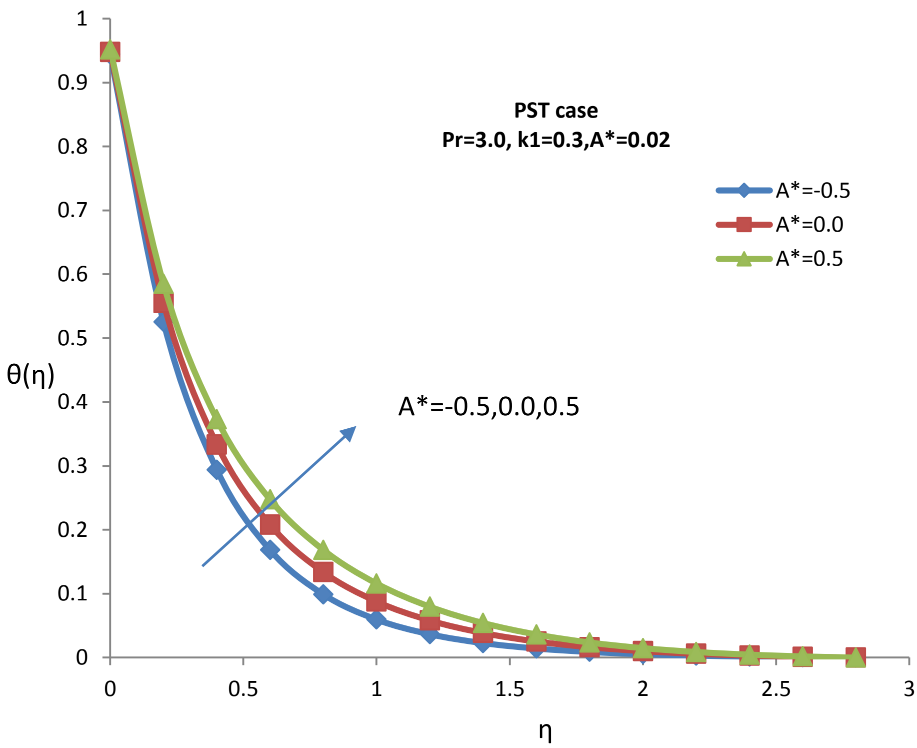

Figure 5(a). Effect of A* on temperature distribution in PST case

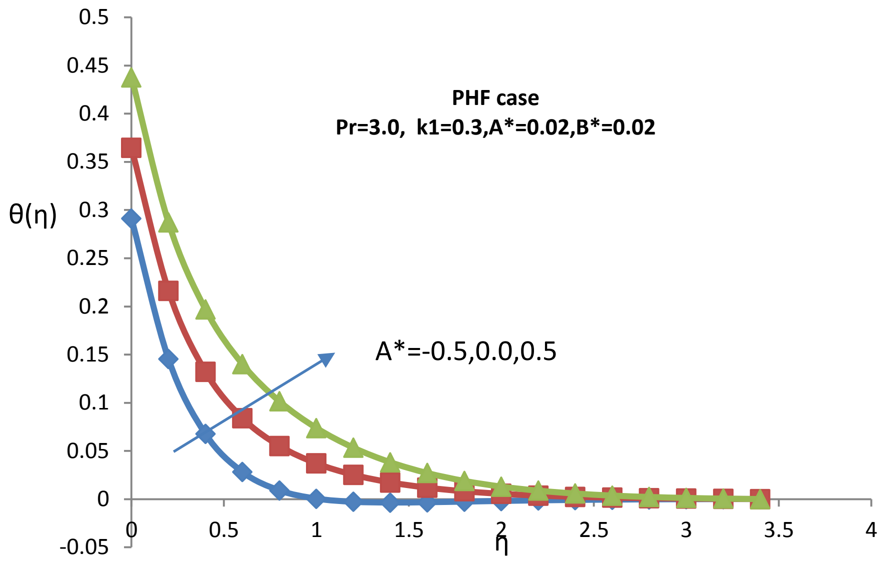

Figure 5(b). Effect of A* on temperature distribution in PHF case

Table 1: Values of wall temperature gradient $-\theta'(0)$ (PST case) and wall temperature g (0) (PHF case) for different values of K_1, Pr, A^* and B^*

K_1	Pr	A*	B*	PST Case $-\theta'(0)$	PHF Case g(0)
0.3	3	0.02	0.02	2.68289	0.34635
				2.56441	0.35432
				2.51125	0.35952
0.3	3	0.02	0.02	2.68289	0.34635
0.5				2.23409	0.4137
0.7				1.8898	0.47553
0.3	3	-0.5	0.5	2.71412	0.29118
		0.0		2.53177	0.36451
		0.5		2.34942	0.43784
0.3	1	0.02	0.02	1.3891	0.73992
	2			2.07872	0.46856
	3			2.68289	0.29956
0.3	3	0.5	-0.5	2.58476	0.39388
			0.0	2.4623	0.41514
			0.5	2.38197	0.43738

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