International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 8 (2018) pp. 6297-6305

Forgetting Factor Adjustment in RLS Adaptive Volterra Filter using a New Peak Value-RMS Product Method: Loudspeaker Distortion Modeling Case

[1]Abdelmoudjib Benkada, [2]Abdelhafid Kaddour, [3]Saïd Karoui

[1]*Laboratory of Microsystems and Embedded Systems, University of Science and Technology of Oran "Mohamed Boudiaf", Algeria.*

[2]*Laboratory Data Systems Signals, University of Science and Technology of Oran "Mohamed Boudiaf", Algeria.*

[3]*University of Science and Technology of Oran "Mohamed Boudiaf", Algeria.*

Abstract

The aim of this work is to describe a new post-estimation method called here, the Peak Value-RMS Product (PVRP) method, for the adjustment of the forgetting factor value in RLS adaptive Volterra filter. This method is in the form of an algorithm based on the PVRP metric. This PVRP metric exploits the error signal obtained from the estimation of the Volterra kernels of a nonlinear system. The proposed method improves the choice of the forgetting factor value in the RLS adaptation algorithm. It is developed for the particular case of time-varying nonlinear systems, where the tracking of input variations causes disturbances in estimation error signals, and where metrics such as the mean-squared error (MSE) are limited. Finally, the efficiency of the PVRP method is evaluated on a loudspeaker system, by comparing estimated distortion curves with experimental curves ones.

Keywords: Peak Value-RMS Product, RLS adaptive Volterra filter, Time-varying nonlinear system

NOMENCLATURE

AVF	Adaptive Volterra Filter
DC	Direct Current
MSE	Mean-Squared Error
RLS	Recursive Least Squares
RMS	Root Mean Square
$d(n)$	desired signal
$e(n)$	error signal
i	index for general purpose
$\mathbf{I}$	identity matrix
$J(n)$	cost function
k	discrete time index
$\mathbf{K}(n)$	Kalman gain vector
L	filter length
n	time index
M	memory length
N	nonlinearity order
$P(n)$	Inverse correlation matrix of the input signal
$\mathbf{x}(n)$	input signal vector
$x(n)$	input signal of a system
$y(n)$	output signal
$\mathbf{w}(n)$	adaptive filter coefficient vector
w	adaptive filter coefficient
w_k	k -th order Volterra kernel
δ	regularization factor
λ	forgetting factor
$(\cdot)^{-1}$	inverse $(\cdot)$
$(\cdot)^T$	transpose $(\cdot)$

INTRODUCTION

All physical systems describable in classical equations of motion terms are nonlinear, therefore the study of nonlinear dynamics has become so important in order to develop new mathematical models [1].

Since Vito Volterra's development of the formalism that bears his name in 1887 [2], and the first use by Nobert Wiener of Volterra series for the analysis of non-linear circuits in 1942 [3], the nonlinear model of Volterra was at the origin of the majority of the other nonlinear models that have emerged. The study of nonlinear dynamic systems is more complicated if they are described in terms of at least one time-varying parameter.

Faced with the reality of time-varying nonlinear systems, new identification techniques have been developed, such as the adaptive methods that appeared in the middle of the last century. The efficiency of adaptive methods applied to the identification of nonlinear systems is now an irrefutable fact in different fields.

Among these different adaptive methods, recursive parameter estimation techniques became established in the case of time-varying or nonlinear stochastic systems, or when the offline estimation requires too much computing time or memory space [4]. The Recursive Least Squares (RLS) algorithm is the most well-known and used recursive estimator in this category of recursive identification methods, whose basic principle is the minimization of the error signal, which is none other than the difference between the filter response and the response of the real system.

M. Schetzen in [5] has emphasized the importance of error analysis in the characterization theory of a given system.

Since the beginning of the estimation theory, the most trivial choice of an error function allowing the assessment of estimators is the MSE.

However, only some works focusing exclusively on MSE and its role in RLS adaptive methods. B. Lindoff is among the few authors who have studied this question but only for linear time-invariant systems. He proposes in in [4] and [6], a choice criterion of the forgetting factor in the RLS method, depending on the number of observations, making it possible to minimize the MSE for the case of linear time-invariant systems.

In [7], the author advances a basic relationship between the RLS method and the number of previous measurements, with a recommendation on the typical range of forgetting factor values.

In real system modeling schemes, such as most acoustic systems, which are nonlinear and time-varying, the conventional metrics previously used to assess the quality of nonlinear adaptive filters must be reconsidered.

The questioning of purely energy statistical methods as the only means of assessing estimators is not recent. Indeed, in several areas of signal processing, and particularly in image processing, a wave of criticism of these metrics appeared in the late 1980s. We can consult as such, the remarkable treatise made by B. Girod under his famous title-formula "What's Wrong with Mean-squared Error?", which has been the subject of a book chapter on digital imaging [8], and has since been reused many times by several researchers. Following this episode, a wide range of new metrics emerged, taking into account the human perception and agreeing more closely with the subjective quality of images.

This fact led us to think of a solution that makes it possible to take better account of the particular nature of the error signals resulting from nonlinear adaptive filters used for the identification of nonlinear time-varying systems.

We thus developed a new method applicable to the error signal obtained with an RLS adaptive Volterra filter (AVF) based on an exponentially weighted least-squares cost function. This solution does not concern the estimation operation strictly speaking, but the use of its result, i.e. in post-estimation. The proposed method is developed from the examination of the obtained modeling results. It is able to provide a strong correlation between the accuracy of the models and the parameters of the RLS AVF used, including the forgetting factor of the RLS adaptation algorithm, which plays a crucial role in the filter convergence and the quality of the obtained Volterra kernels. Consequently, the proposed method was used for the identification of the nonlinear time-varying loudspeaker system. It is on this basis, the developed method has made it possible to accurately adjust the value of the forgetting factor in the RLS AVF, in order to improve the quality of the estimated Volterra kernels, and to obtain the modeled distortion responses, which are very close to the experimentally measured real distortion.

The paper is organized as follows. In Section II, we begin with the theoretical development of the RLS AVF, necessary for the construction of the identification procedure chosen for the considered system.

Section III concerns the development of the proposed PVRP method. We begin this section with an explanation of the context used to develop our method. The construction of the PVRP metric, which is at the center of the proposed method, is then presented, to finally consider the PVRP minimization algorithm.
Section IV is an application of the PVRP method on a loudspeaker system for distortion modeling. The experimental procedure used for identification is described. The formalism of the RLS AVF used as a model for the loudspeaker system is then given. In sub-section IV.3 the adjusted value of the forgetting factor for the considered application is determined.

In section V, we present the results of the simulations and their discussion, and the conclusions are parts of section VI.

The RLS Adaptive Volterra Filter

The electrodynamic loudspeaker considered as a weakly nonlinear, time-invariant and causal dynamical system can be represented by a discrete finite-order and finite memory length Volterra series, based on a Taylor series expansion [9]. This class of the finite memory truncated Volterra series has been established following the work of [10] and [11], and has been used effectively for audio systems in general [12], [13], [14], [15] in the following form

$$y(n) = w_0 + \sum_{k=1}^{N}\left\{\sum_{i_1=0}^{M-1}\cdots\sum_{i_k=i_{k-1}}^{M-1} w_k\left(i_1,\cdots,i_k\right)\prod_{j=1}^{k}x(n-i_j)\right\} \tag{1}$$

Where $x(n)$ is the input signal, $y(n)$ the output signal, $w_k(i_1,\cdots,i_k)$ is the k-th order symmetric discrete Volterra kernel, w_0 is the zero-order term (DC component), assumed equal to zero here, M is the memory length and N is the nonlinearity order. Most often only the lowest order kernels (first, second, or third) are dominant and the highest order kernels have a negligible effect on the response of the system, such systems are called weakly nonlinear [14]. However, even if the system is not weakly nonlinear, for the feasibility of the calculations the Volterra series is truncated to a maximum order of the finite nonlinearity N [12], [14], [15]. The complexity of the Volterra model depends on both the memory lengths M and the order of nonlinearity N.

Several experiments have been conducted for the identification of the loudspeaker system, using Volterra filters combined with different adaptive algorithms such as LMS, RLS or NLMS. The third order of non-linearity N was used, with memory lengths M (number of kernels coefficients) of 1 to 4. Considering the obtained results with the designed filters, only the case when M = 2 was selected for the work presented in this article. This filter proved to be the best method of identification among all the methods used, with the best convergence and able to generate the most accurate distortion models of the system under test (SUT). For its performance, this filter was chosen to study the convergence of the adaptation algorithms and the accuracy of the resulting models as functions of different estimators of the error signal.

A typical topology of system modeling using RLS AVF is presented in Fig. 1.

The adaptive RLS algorithm provides an exact solution by solving a deterministic least-squares problem. Based on an exponentially weighted least-squares cost function $J(n)$, the RLS algorithm calculates each time instant, the cost function $J(n)$, and tends to minimize it to adjust the filter coefficients $w(n)$ as the previous ones $w(n-1)$ are available [16], [17].

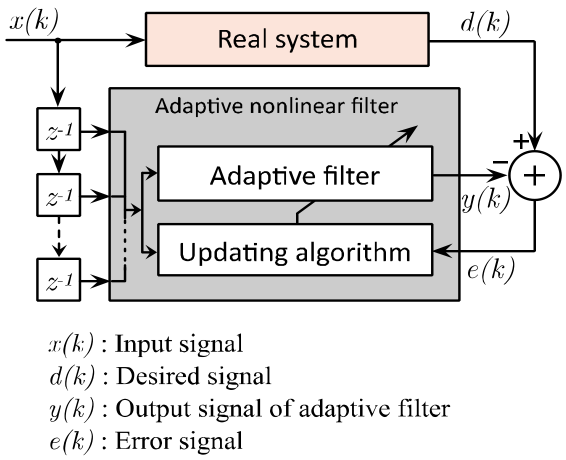

Figure 1: Diagram of system modeling using RLS AVF

$$J(n) = \sum_{i=0}^{n} \lambda^{n-i} [d(i) - \mathbf{w}^T(n)\mathbf{x}(i)]^2 \tag{2}$$

$$J(n) = \sum_{i=0}^{n} \lambda^{n-i} e^2(i) \tag{3}$$

Where λ is the forgetting factor ($0 \ll \lambda \le 1$), and $e(n)$ is the *a posteriori* error.

The value of the forgetting factor λ defines the system memory and is directly related to:

- The convergence of the adaptation algorithm and the aptitude of the filter to track time variations in the input signal;
- The stability of the estimated filter.

The forgetting factor λ, if it is roughly a measure of the adaptation algorithm memory; it is however a crucial design parameter of the RLS adaptation filter. Its value should be less than one, and must be chosen carefully [14], [18].

In the criterion given by equation 3, a forgetting factor less than one weights old samples of the error less than new ones, this is useful in time-varying situation where instantaneous variations in the input signal make the inclusion of old samples less appropriate [14]. Thereby, a low value of λ leads to a fast tracking of time-varying parameters but high noise sensitivity [19]. Usually λ = 1 is appropriate for time-invariant signals [14].

In the absence of rigorous criteria for the choice of forgetting factor, each author recommends a range of typical values valid in the conditions of application of the RLS method for a given system. Thus, for example, some recommend 0.95 < λ < 0.9995 [14], where others suggest 0.98 < λ < 0.995 [7], or 0.95 < λ < 0.995 [6], and finally 0.94 < λ < 0.999 [19].

The minimization problem is given by

$$w(n+1) = \arg\min_{w} J(n) \tag{4}$$

The RLS algorithm operations can be summarized as [18] [20] [21]:

Algorithm: RLS algorithm

Parameters:

$\mathbf{w}(n)$ adaptive filter coefficients vector ($L \times 1$)

$P(n)$ inverse correlation matrix of the input signal ($N \times N$)

δ regularization factor (can be a small positive constant)

$\mathbf{I}$ identity matrix ($N \times N$)

$\mathbf{K}(n)$ Kalman gain vector ($N \times 1$)

λ forgetting factor ($0 \ll \lambda \le 1$)

Initialize:

$$P(0) = \delta^{-1}\mathbf{I}$$

$$\mathbf{x}(0) = \mathbf{w}(0) = 0$$

For each instant of time $n = 1, 2, \cdots, L$, compute

$$\mathbf{K}(n) = \frac{P(n).\mathbf{x}(n)}{\lambda + \mathbf{x}^T(n).P(n).\mathbf{x}(n)}$$

$$\mathbf{w}(n) = \mathbf{w}(n-1) + e(n-1).\mathbf{K}(n-1)$$

$$P(n) = \lambda^{-1}P(n-1) - \lambda^{-1}\mathbf{K}(n-1).\mathbf{x}^T(n-1).P(n-1)$$

$$y(n) = \mathbf{w}^T(n)\mathbf{x}(n)$$

$$e(n) = d(n) - y(n)$$

The learning curve was used to evaluate the convergence speed of the adaptive filter, which consists of the MSE of the *a posteriori* error signal $e(n)$, calculated with the following equation [22]

$$MSE = \frac{1}{L}\sum_{k=1}^{L} e_k^2(n) \tag{5}$$

THE PVRP METHOD DEVELOPMENT

Development Context

The examination of the distortion curves modeled with the various RLS AVFs, generally showed a lack of correlation between the accuracy of the obtained models and the used forgetting factor values, which were optimized using the learning curve based on the MSE of the error signal $e(n)$, given in equation 5.

Thus the close observation of the various parameters of the error signals from the RLS AVFs with the different forgetting factor values, such as the amplitude or more precisely the envelope and the waveform, revealed a total difference with graphs of traditional error signals obtained with adaptive identification methods for linear or nonlinear time-invariant systems. Fig. 2. is a typical example of the shape of obtained error signals, with a patent impulse character, and an irregularity of the envelope inherent to the time variations of the real identified system.

In order to take into account the singular nature of the obtained error signals, we develop in this section a novel method whose role is to improve the choice of the forgetting factor λ by adjusting its value with great accuracy around the optimized forgetting factor.

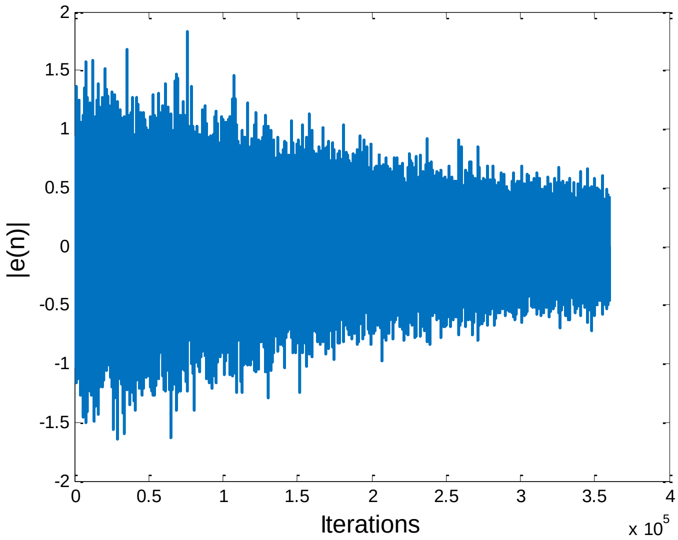

Figure 2: Error signal of the RLS AVF with λ = 0.99

An exact value of the forgetting factor in the RLS algorithm is very important in the estimation quality using the RLS AVF. This value adjustment implies the ability to ensure a correlation between the accuracy of the estimated distortion curves of the SUT and the chosen value of the forgetting factor.

The PVRP Metric

Our forgetting factor adjustment method is primarily founded on a new metric. Based on simple time descriptors of the signal, this proposed metric, allows from the product of the peak value and the RMS value to enhance the choice of the forgetting factor value used in the RLS AVF algorithm.

The error signal, which is the difference between the output signal of the adaptive filter and the desired signal, response of the system to be identified, is ideally a finite energy signal with fading amplitude. In practice, the error signal rarely takes this theoretical form, albeit with a pulse aspect more or less important and an infinite energy.

These last two attributes of the signal, namely, the pulse character and the signal energy, represent the main degrading characteristics of the error signal due to default in tracking fast variations in the input signal. The proposed metric is based on the empirical observation of the obtained results. It combines the role of the two degrading characteristics in the deterioration of the error signal, which is formally reflected by the product of the peak value for the impulsive aspect, and the RMS value representing the energy of the error signal. This Peak Value-RMS Product, that we will note PVRP, is given by

$$\text{PVRP} = e_{pv} \times e_{rms} \tag{6}$$

With the peak value e_{pv} given by

$$e_{pv} = \frac{e_{\max} - e_{\min}}{2} \tag{7}$$

With $e_{\min}$, $e_{\max}$ minimum and maximum error respectively.

The RMSE is given by

$$RMSE = \sqrt{\frac{1}{L}\sum_{k=1}^{L} e_k^2(n)} \tag{8}$$

The PVRP Minimization Algorithm

The PVRP metric forms the basis of an algorithm allowing the use of the proposed method in improving the choice of the forgetting factor, which is a crucial step for the realization of the adaptive filters in general. A very well chosen value of λ would give coefficients of the identified filters, allowing a refined model of the SUT, and a better prediction of its responses.

This amounts to minimizing the error signal at the output of the adaptive filter, both from an energetic point of view by minimizing one of the energy parameters of the error (RMS value, variance, MSE, etc.), and also minimizing its impulsive character which remains present even with a low energy of the error signal if the system to be identified is not quite time-invariant. The following algorithm, written using Matlab™ commands, minimizes the value of the PVRP metric with a sweeping over the most likely range of the forgetting factor depending on the application.

Algorithm: The PVRP minimization algorithm

Parameters:

P number of sweeping points of λ

e_{min}, e_{max} minimum and maximum error

e_{pv}, e_{rms} peak value and RMSE

idx_PVRP_{min} index of the minimum value of PVRP

λ_{adj} adjusted value of the forgetting factor

Initialize:

λ =linspace(λ_{min} , λ_{max} , P) % P >>

For i=1: P

For j=1: L

Execute RLS algorithm with λ_i

$e_{max}(\lambda_i) = \max[e(\lambda_i)]$

$e_{min}(\lambda_i) = \min[e(\lambda_i)]$

$e_{pv}(\lambda_i) = (e_{max}(\lambda_i) - e_{min}(\lambda_i))/2$

$e_{rms}(\lambda_i) = \text{sqrt}(\text{mean}(e(\lambda_i).\hat{}\,2))$

end

$e_{pv}(\lambda) = [e_{pv}(\lambda_{min}) \cdots e_{pv}(\lambda_i) \cdots e_{pv}(\lambda_{max})]$

$e_{rms}(\lambda) = [e_{rms}(\lambda_{min}) \cdots e_{rms}(\lambda_i) \cdots e_{rms}(\lambda_{max})]$

end

$\text{PVRP}(\lambda) = e_{pv} \times e_{rms}$

$\text{PVRP}_{min} = \min[\text{PVRP}(\lambda)]$

idx_PVRP_{min} = find(PVRP(λ) == PVRP_{min})

$\lambda_{adj} = \lambda(idx\_PVRP_{min})$

This sweeping over the values of λ is a high-resolution sweeping, allowing a maximum coverage of the pulses of the error signal, whose occurrence is unpredictable and which are distributed over the entire range of λ values, and thus getting the adjusted value of the forgetting factor λ_{adj}.

APPLICATION: LOUDSPEAKER DISTORTION MODELING

In this section, we describe the experimental process used for the identification of the loudspeaker. The identification makes it possible to obtain the Volterra kernels required for the subsequent modeling of the different distortion curves. We also detail the writing of the different sequences used for the RLS AVF used in our work. In the last part, we apply the PVRP minimization algorithm to the error signal obtained after the Volterra kernel estimation operation, in order to deduce the adjusted value of the forgetting factor, capable of provide us with the desired estimates of the loudspeaker system distortions.

Experimental Set Up

The system to be identified is an 8-inch electrodynamic woofer-midrange Nuandi Electronic model ND-MW08M50 loudspeaker, with a frequency response between 90 and 8800 Hz.

The input signal used for identification is a zero-mean white Gaussian noise. This Gaussian signal is band-limited between 47 and 10500 Hz, corresponding to the usable frequency range of the loudspeaker, and is sampled at a 96 kHz rate. Despite the drawbacks related to technical and computational requirements, the use of such oversampling is justified by the need for very high-resolution Volterra kernels, which make it possible to use a high-resolution sweeping of the error signal as a function of the forgetting factor.

Third-order RLS AVF used for Loudspeaker System Identification

As explained in the first section, the identification of the loudspeaker system is carried out for the third order of non-linearity $N = 3$, and memory length $M = 2$.

The output vector of the adapted system is

$$\begin{aligned} y(n) = w_0 &+ \sum_{i_1=0}^{1} w_1(i_1)x(n-i_1) \\ &+ \sum_{i_1=0}^{1}\sum_{i_2=0}^{1} w_2(i_1,i_2)x(n-i_1)\,x(n-i_2) \\ &+ \sum_{i_1=0}^{1}\sum_{i_2=0}^{1}\sum_{i_3=0}^{1} w_3(i_1,i_2,i_3)x(n-i_1)\,. \\ &\quad x(n-i_2)x(n-i_3) \end{aligned} \tag{9}$$

The identification approaches and different distortion simulations were performed using Matlab™ programming environment.

The choice of oversampling and high-resolution sweeping required the use of supercomputers for simulations.

International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 8 (2018) pp. 6297-6305

Forgetting Factor Adjustment

According to the PVRP minimization algorithm developed above, a PVRP sweeping was performed as a function of the forgetting factor.

The PVRP sweeping was achieved in several stages with an increasing number of iterations. We first performed a sweeping for values of λ between 0 and 1, using 100 points (number of iterations of the sweeping loop), in order to get the general appearance of the graph PVRP (λ). After using a number of iterations up to 1000 points, we found that the values of the PVRP metric for $\lambda < 0.8$, were not represented on Matlab graphs, corresponding to either infinite values (inf), or NaN Matlab values ("Not a Number"), characteristics of the instability of this λ interval [0-0.8]. These preliminary sweeping have thus made it possible to carry out a final high-resolution sweeping with 3000 iterations, centered on the interval [0.8 - 1] of λ (Fig 3).

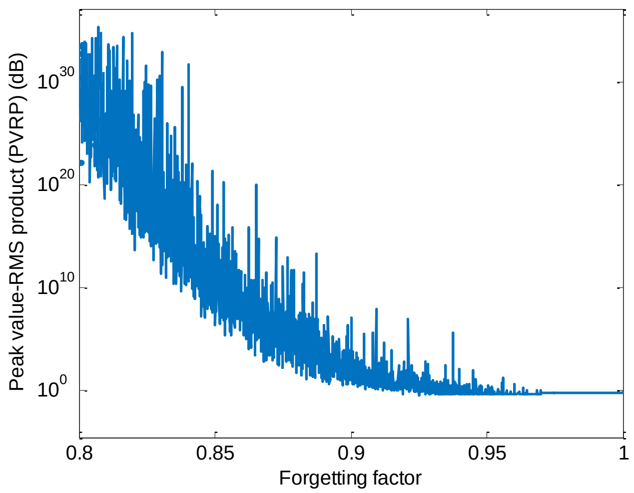

Figure 3: Peak Value-RMS Product (PVRP) vs. forgetting factor sweeping between 0.8 and 1, for third-order RLS adaptive Volterra filter with *M* = 2.

The examination of Fig. 3, has shown that using this high-resolution sweeping yielded PVRP values of high accuracy up to 6.66e-5. The PVRP starts with very large values starting from λ ≈ 0.8, then decreasing to a value of 0.5 for λ = 1, passing through a range of values of the curve's trough between λ = 0.9 and 0.94, with a minimum for the value λ = 0.924975, which represents the adjusted value of the forgetting factor λ_{adj}.

SIMULATION RESULTS & DISCUSSION

In this section, we examine the results of the different models obtained with different values of the forgetting factor λ. These results are compared with the experimental curves of the total harmonic distortion THD (fundamental component), and the second d2 and third d3 components of the harmonic distortion of the loudspeaker completed according to the standard IEC 60268-5 [23], which are given in Fig. 4.

Figs. 5 to 9, show the estimated distortion curves for the forgetting factor values around the adjusted value λ_{adj} obtained in the last section. The examination of the five figures shows an incontestable agreement between the estimated graphs of Fig. 7 (λ = 0.924975) and the experimental graphs of Fig. 4.

Moreover, excluding the orders of magnitude of the distortion percentages, if the graphs of Figs 6, 8 and 9 (λ = 0.90256, λ = 0.9414 and λ = 0.98 respectively) show some similarity with the experimental graphs, however those of Fig. 5 (λ = 0.85) are simply rejected.

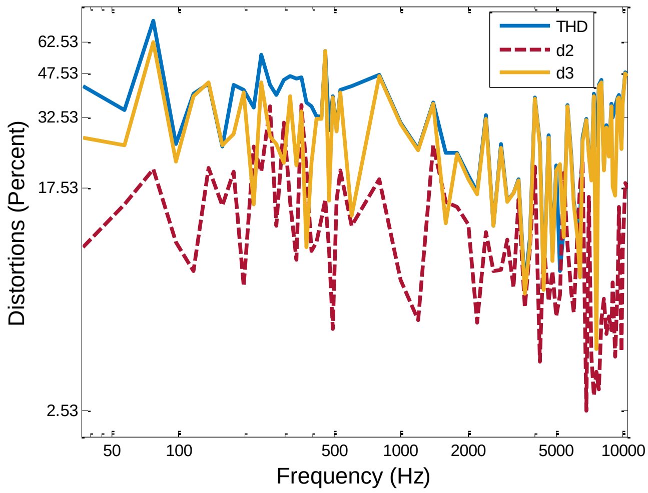

Figure 4: Experimental graphs of THD and harmonic distortion components d_2 and d_3

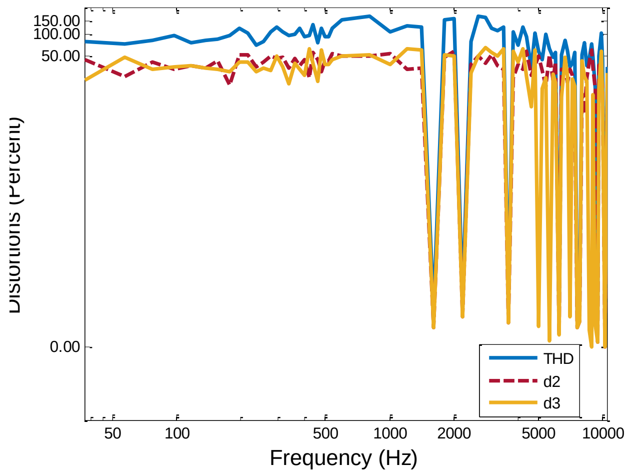

Figure 5: Estimated THD and harmonic distortion components d_2 and d_3 graphs (λ = 0.85, MSE = 3.5381e+12 and PVRP = 1.2478e+15)

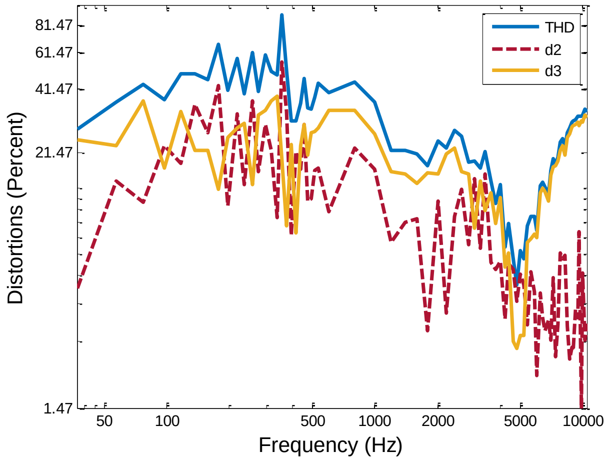

Figure 6: Estimated THD and harmonic distortion components d_2 and d_3 graphs (λ = 0.90256, MSE = 0.0373 and PVRP = 13.1876)

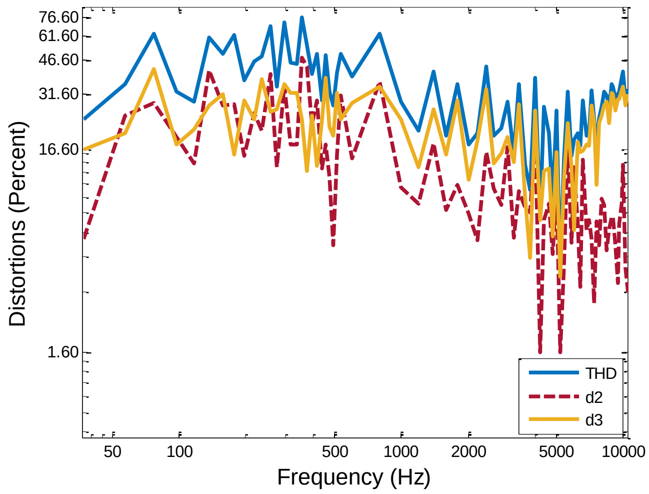

Figure 7: Estimated THD and harmonic distortion components d_2 and d_3 graphs (λ = 0.924975, MSE = 0.0463 and PVRP = 0.4146)

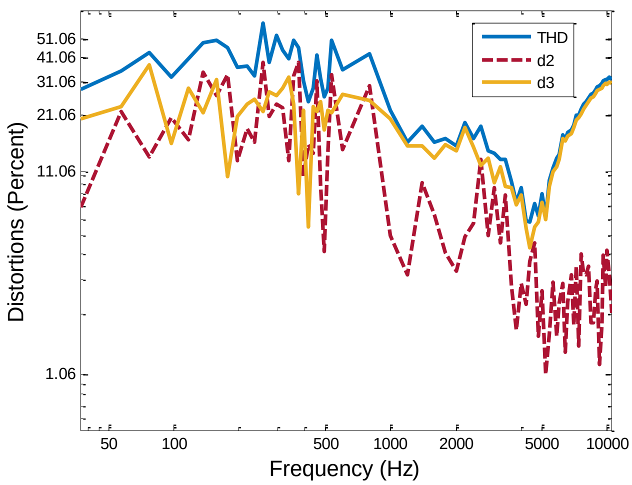

Figure 8: Estimated THD and harmonic distortion components d_2 and d_3 graphs (λ = 0.9414, MSE = 0.0574 and PVRP = 0.4626)

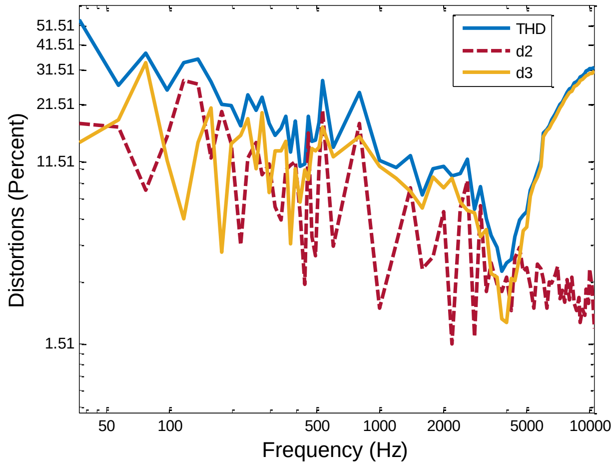

Figure 9: Estimated THD and harmonic distortion components d_2 and d_3 graphs (λ = 0.98, MSE = 0.0961 and PVRP = 0.6)

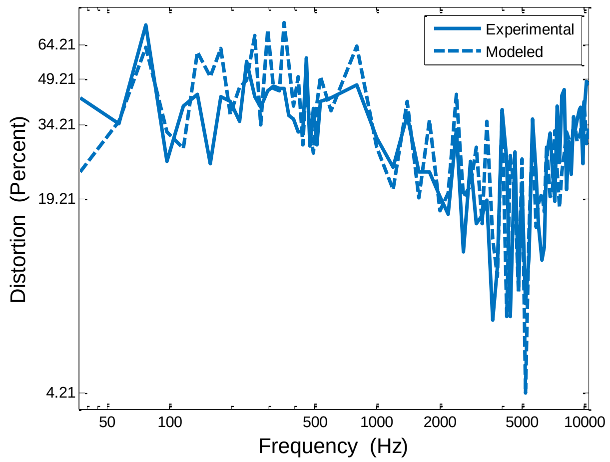

Figure 10: Experimental and estimated (λ = 0.924975) THD graphs

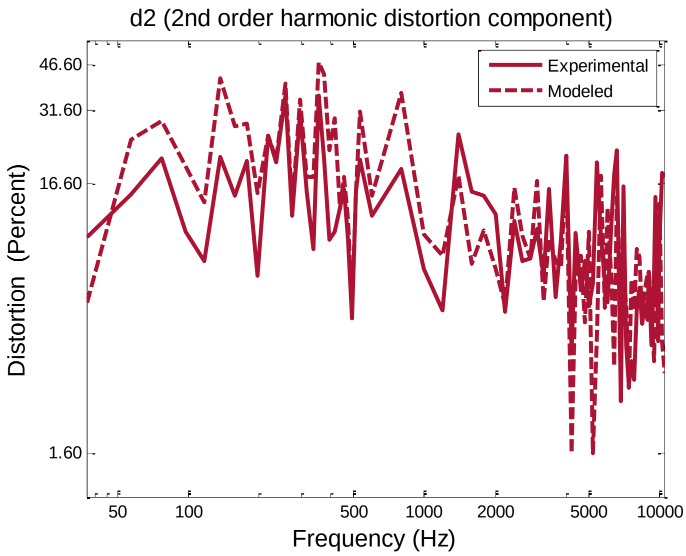

Figure 11: Experimental and estimated (λ = 0.924975) d_2 graphs

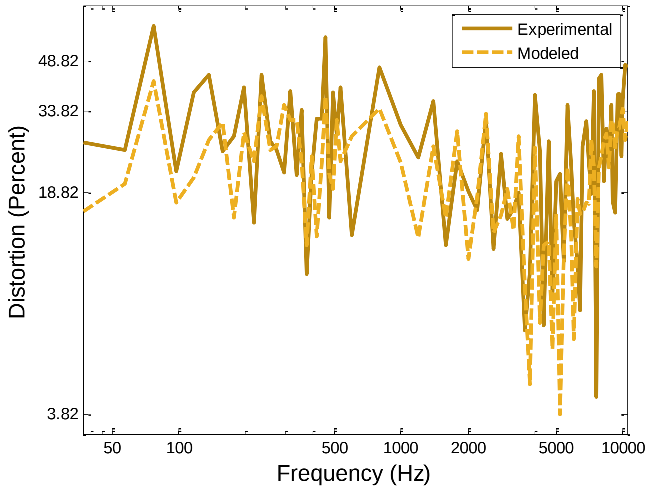

Figure 12: Experimental and estimated (λ = 0.924975) d_3 graphs

Table 1: Values of MSE and PVRP for relevant values of the forgetting factor

λ	MSE	PVRP
0.85	3.5381e+12	1.2478e+15
0.9026	0.0373	13.1876
0.924975	0.0463	0.4146
0.9414	0.0574	0.4626
0.98	0.0961	0.6

For a better comparison of the estimation results obtained with λ = 0.924975, with the experimental graphs, the corresponding curves of each distortion are superimposed and presented in Figs. 10 to 12.

The overall results are summarized in Table 1. It is clear from this table that the best estimated graphs, i.e., the closest to the experimental obtained graphs on the loudspeaker, correspond to the adjusted value of the forgetting factor λ_{adj} = 0.924975, and specifically to the minimum value of the PVRP (0.4146). The minimum value of the MSE (0.0373), gave a value of the forgetting factor equal to 0.9026, and thus allowed estimated graphs quite far from the experimental curves (Fig. 6).

These results show the effectiveness of our method, using a high accuracy adjustment of the forgetting factor, which allowed taking into account the peculiarity of the nature of the estimation error signal in the case of nonlinear time-varying dynamic systems.

CONCLUSION

In this paper, a new method for the adjustment of the forgetting factor value in RLS AVF has been presented.

The proposed method, namely the Peak Value-RMS Product method, is built from simple time metrics of the signal, i.e., the peak value and the RMS value, used in an adjustment algorithm allowing the minimization of the PVRP metric. This method enhances the choice of the forgetting factor value used in the RLS AVF algorithm.

An electrodynamic loudspeaker has been used as a nonlinear time-varying dynamic system, to study the efficiency of the proposed method for modeling distortion from Volterra kernels, estimated by identification using an RLS AVF.

Finally, thanks to an algorithm based on a very large number of iterations to allow a very high accuracy in the sweeping of the forgetting factor, combined with oversampled signals, the results obtained show a very good agreement between the estimated distortion and the experimental distortion curves. This is proof of the effectiveness of the proposed method for adjusting the value of the forgetting factor in adaptive RLS Volterra filters in the case of nonlinear time-varying systems.

ACKNOWLEDGEMENTS

Support by IBN BAJA Supercomputer time at USTOMB HPC Center is gratefully acknowledged.

REFERENCES

[1] J.E.S. Socolar, *Nonlinear Dynamical Systems* (In: Deisboeck T.S., Kresh J.Y. (eds) Complex Systems Science in Biomedicine. Topics in Biomedical Engineering International Book Series. Springer, Boston, MA, 2006).

[2] V. Volterra, *Theories of Functionals and of Integral and Integro-differential Equations* (Authorized translation by M. Long, Edited by Luigi Fantappiè, Blackie & Son Limited, London, Glasgow, 1930).

[3] N. Wiener, *Response of a nonlinear device to noise* (Report V-129, Radiation Laboratory, MIT, U.S. Department of Commerce PB-58087, April 1942).

[4] B. Lindoff and J. Holst, "Convergence analysis of the RLS identification algorithm with exponential forgetting in stationary ARX-structures," *Int. J. Adapt. Control Signal Process., 13: 1-22, 1999.*

[5] M. Schetzen, "A theory of non-linear system identification," International Journal of Control, *Vol. 20,* (Issue 4) *: 577-592, 1974.*

[6] B. Lindoff, "On the Optimal Choice of the Forgetting Factor in the Recursive Least Squares Estimator", *CiteSeerX, April 1997.*

[7] E. Mosca, *Optimal Predictive, and Adaptive Control* (Prentice Hall International Edition, 1995).

[8] B. Girod, What's Wrong with Mean-squared Error (*In: Digital Images and Human Vision, (Bradford Books), Ed. Andrew B. Watson, The MIT Press, London, 1993, 207-220).*

[9] A. J. M. Kaizer, *"Modeling of the Nonlinear Response of an Electrodynamic Loudspeaker by a Volterra Series Expansion," Journal of the Audio Engineering Society -*

Loudspeakers Vol 4 Transducers, measurement and evaluation vol. 32 vol 39 1984-1991 : 84-96, June 1987.

[10] W. J. Rugh, *Nonlinear system theory: The Volterra/Wiener approach* (Battimore: Johns Hopkins University Press, 1981.)

[11] M. Schetzen, *The Volterra and Wiener theories of nonlinear systems* (New York: 1989.)

[12] M. J. Reed and M. O. Hawksford, "Practical modelling of nonlinear audio systems using the Volterra series," *AES Convention:100, p. 4264, May 1, 1996.*

[13] M. Tsujikawa, T. Shiozaki, Y. Kajikawa and Y. Nomura, "Identification and elimination of second-order nonlinear distortion of loudspeaker systems using digital Volterra filter," *2000 10th European Signal Processing Conference, Tampere, Finland, pp. 1-4., April 2000*

[14] X. Lu, *Acoustic echo cancellation over nonlinear channels*, PhD thesis, Dept. Elect. & Comp. Eng. Univ. of Montreal, Canada, January 2004.

[15] J. Schattschneider and U. Zölzer , " Discrete-Time Models For Nonlinear Audio Systems," *Proceedings of the 2nd COST G-6 Workshop on Digital Audio Effects (DAFx99), NTNU, Trondheim, December 9-11, 1999.*

[16] T. Ogunfunmi, *Adaptive Nonlinear System Identification: The Volterra and Wiener Model* (Approaches, Springer, Santa Clara, USA, 2007).

[17] H. Nascimento, *Adaptive Filters*, (In: R. Chellappa and S. Theodoridis, eds., Signal Processing Theory and Machine Learning, Volume 1, Elsevier Ltd., Waltham, Academic Press Library in Signal Processing, 2014, pp. 619-761).

[18] P. S. R. Diniz, *Adaptive filtering: Algorithms and practical implementation* (3rd ed., Springer: 2008.).

[19] B. Carlsson, *Recursive identification* (Uppsala Universitet Institutionen För Informationsteknologi, September 2016).

[20] V. J. Mathews, "Adaptive polynomial filters, (1991). Adaptive Polynomial Filters," *IEEE Signal Processing Magazine*, Volume: 8, Issue: 3, July 1991, pp. 10 - 26.

[21] S. Haykin, *Adaptive filter theory* (3rd ed., Prentice-Hall, New Jersey, 1996.).

[22] A. D. Poularikas and Z. M. Ramadan, *Adaptive filtering primer with Matlab®* (CRC Press, Taylor & Francis, 2006.).

[23] International Standard IEC 60268-5, "Sound system equipment – Part 5: Loudspeakers," Edition 3.1, 2007-09.